Non-perturbative inputs for gluon distributions in the hadrons

B.I. Ermolaev

Ioffe Physico-Technical Institute, 194021

S.I. Troyan

St.Petersburg Institute of Nuclear Physics, 188300 Gatchina, Russia

Description of hadronic reactions at high energies is conventionally done in the framework of QCD factorization. All factorization convolutions comprise non-perturbative inputs mimicking non-perturbative contributions and perturbative evolution of those inputs. We construct inputs for the gluon-hadron scattering amplitudes in the forward kinematics and, using the Optical theorem, convert them into inputs for gluon distributions in the hadrons, embracing the cases of polarized and unpolarized hadrons. In the first place, we formulate mathematical criteria which any model for the inputs should obey and then suggest a model satisfying those criteria. This model is based on a simple reasoning: after emitting an active parton off the hadron, the remaining set of spectators becomes unstable and therefore it can be described through factors of the resonance type, so we call it Resonance Model. We use it to obtain non-perturbative inputs for gluon distributions in unpolarized and polarized hadrons for all available types of QCD factorization: Basic, K_T- and Collinear Factorizations.

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I. INTRODUCTION

Description of hadronic reactions at high energies requires the use of QCD in the perturbative and non-perturbative domains. It is well-known that such calculations cannot be carried out in the straightforward way because the non-perturbative QCD is poorly-known. The most efficient approximation approach to theory of hadronic reactions is QCD factorization, where perturbative QCD calculations are complemented by phenomenological fits or model expressions which mimic non-perturbative QCD contributions. Throughout the present paper we will address those expressions as non-perturbative inputs. Technically speaking, such inputs act as initial conditions for the evolution equations which account for perturbative contributions. The inputs are supposed to approximately describe short-time dissociations of each of the interacting hadrons into active partons and spectators. Depending on the number of the active partons emitted by each hadron, there can be Single-Parton Scattering (SPS) and Multi-Parton Scattering (MPS). An example of the factorized amplitude for the scattering of hadrons under the SPS approximation is depicted in Fig. 1. When such amplitudes are conjugated with the mirror amplitudes, the intermediate states consist of two partons. For instance, QCD factorization of the hadronic tensor $W_{\mu\nu}$ for DIS off a hadron with momentum p is represented in Fig. 2, where the s-cut is implied. The lowest blob stands for a non-perturbative input describing emition/absorption of the active partons with momentum k from the initial hadron while the upper blob corresponds to DIS off the active partons.
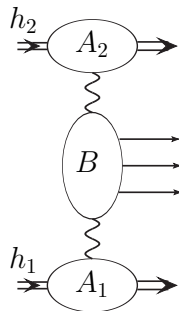

FIG. 1. Amplitude for the Single-Parton Scattering of hadrons h_1 and h_2 , with active partons being gluons. Blobs $A_{1,2}$ denote emission of the active gluons. Interaction of those gluons is depicted by blob B, where the outgoing arrows denote the produced partons. The outgoing double arrows on blobs $A_{1,2}$ stand for the final state spectators.

When the s-cut $(s = (p + q)^2)$ in Fig. 2 is not implied, Fig. 2 represents factorization of the amplitude $A_{\mu\nu}$ of the elastic Compton scattering off the hadron in the forward kinematics, with two-parton t-channel intermediate states between the blobs.
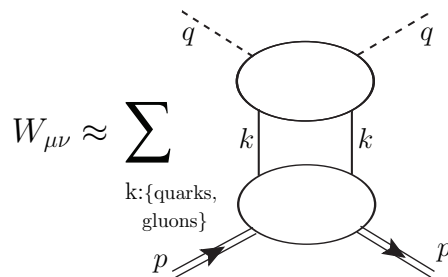

FIG. 2. QCD factorization for the DIS hadronic tensor. The s-cut of the graph is implied. The lowest blob includes the non-perturbative input while the upper blob corresponds to DIS off the active parton.

The Optical theorem relates the elastic scattering amplitudes in the forward kinematics to cross-sections. For example, it relates $A_{\mu\nu}$ to $W_{\mu\nu}$:

$$W_{\mu\nu} = \frac{1}{\pi} \Im_s A_{\mu\nu}. \tag{1}$$

So, in order to calculate cross-sections or parton distributions under the SPS approximation, it is sufficient to calculate the related elastic amplitudes with two-parton intermediate partons in the t-channel only. In order to

avoid misunderstanding, we note that t-channel intermediate states inside the upper (perturbative) blob can involve unlimited number of partons. Despite that there is a quite extensive literature on MPS scenario, the SPS scenario is still the most popular. By this reason we focus on it in the present paper.

There are different kinds of QCD factorization in the literature and each of them is tailored to a specific perturbative approach. For example, when DGLAP[1] or its generalization[2] to the small-x region are used to calculate $W_{\mu\nu}$, the first step is to introduce an arbitrary mass scale μ acting as a starting point of the perturbative $k_\perp^2$-evolution from μ^2 to Q^2, with $Q^2 \gg \mu^2$. This scale is called the factorization scale. At the same moment, μ can act as a cut-off for infrared-divergent perturbative contributions. In Collinear Factorization[3], transverse components of momentum k in Fig. 2 are neglected, so these partons are regarded as collinear to the incoming hadrons and therefore one-dimensional non-perturbative inputs can be used.

In contrast, BFKL[4] is free of infrared divergences and because of that integrations over transverse momenta in the BFKL ladders run down to zero. This approach operates with essentially non-collinear initial partons, which excludes neglecting their transverse components and as a result, it excludes a simple matching of BFKL with Collinear Factorization. By this reason, K_T-Factorization[5] named also High-Energy Factorization[6] was suggested. These factorizations use different ways of parametrization of momenta k of the connecting partons. In Collinear Factorization, the parametrization of k is one-dimensional:

$$k = \beta p, \tag{2}$$

with β being the longitudinal momentum fraction. K_T-Factorization involves the same longitudinal parameter β and, in addition, accounts for the transverse momentum $k_\perp$:

$$k = \beta p + k_\perp. \tag{3}$$

There are different ways to construct the non-perturbative inputs in Collinear and K_T- Factorizations. Quite often, see e.g. Ref. [7], the inputs applied in the context in Collinear Factorization are introduced entirely on basis of practical reasons without any theoretical grounds. In contrast, there also are the models with solid theoretical background. In particular, the models in Ref. [8] are actually based on various theoretical approaches to approximate the problem of confinement: the chiral quark-soliton model, diquark model, etc. An overview of the most popular models of hadrons can be found in Ref. [9]. The features of the saturation model[10] are used in Refs. [11, 12] for modeling the inputs in the context of K_T- Factorization while the model in Ref. [13] combines features of a variety of models for the Fock space wave function on the light cone. Some features of this model are similar to the results obtained in Ref. [14]. Another interesting approach is the lattice calculations. They are a complementary method to study the non-perturbative inputs for parton distributions. Some of the recent results of the lattice calculations can be found is Ref. [15].

Although the parametrization of momentum k in K_T- Factorization is more general than in Collinear Factorization, it misses one more parameter for the longitudinal component of k, which can be seen from comparison of Eq. (3) to the standard Sudakov parametrization[16]:

$$k = -\alpha p' + \beta q' + k_\perp, \tag{4}$$

where the light-cone (i.e. $p'^2 = q'^2 = 0$) momenta p', q' are made of the external momenta p and q satisfying the inequality $|pq| \gg |p^2|, |q^2|$ (for instance, they can be the momenta p, q introduced in Fig. 2):

$$p' = p - x_p q, \ q' = q - x_q p, \ x_p \approx p^2/w, \ x_q \approx q^2/w, \ w = 2p'q' \approx 2pq. \tag{5}$$

In Ref. [17] we presented a new kind of QCD factorization: Basic Factorization. It accounts for dependence of the factorized blobs on all components of momentum k and because of that it is the most general form of factorization. We proved that Basic Factorization can step-by-step be reduced to K_T-Factorization and to Collinear Factorization. Doing so, we considered the non-perturbative inputs in the most general form, without specifying them. In Ref. [17] we inferred general theoretical constraints on the non-perturbative inputs. They follow from the obvious requirement: despite the integrands in the factorization convolutions for forward scattering amplitudes have both infra-red (IR) and ultra-violet (UV) singularities, integrations over k in the factorization convolutions must yield finite results. This is possible only if the non-perturbative inputs kill all divergences, which leads to restrictions on the non-perturbative inputs. In Ref. [18] we suggested a model for the non-perturbative inputs to quark distributions in hadrons. This model is based on a simple observation: after emitting an active parton off the hadron, the set of remaining quarks and gluons (usually named spectators) becomes unstable, so it can be described by expressions of the resonance type. Because of that we named this model in Ref, [18] the Resonance Model. We used the Resonance Model model so

as to obtain the quark distributions in the hadrons whereas the very important case of gluon distribution was left uninspected in Ref. [18].

There is a certain similarity between handling quarks and gluons in Perturbative QCD. It allows us to believe that main features of non-perturbative inputs for gluons and quarks can be alike. However, the quark and gluon inputs cannot coincide. Firstly, they have different polarization structure: the quark input should be a spinor whereas the gluon one should be a tensor. Secondly, the well-known difference between the high-energy behavior of the gluon and quark perturbative amplitudes can lead to an essential difference between the quark and gluon inputs.

We think that this issue needs a thorough investigation and do it in the present paper. In the first place we obtain restrictions on the gluon inputs which guarantee both IR and UV stability of the factorization convolutions in Basic Factorization and then use those restrictions so as to extend the Resonance Model to description of the non-perturbative inputs for the gluon distributions in the hadrons. We consider here both polarized and unpolarized hadrons. Our strategy is to calculate the gluon-hadron elastic scattering amplitudes in the forward kinematics and, using the Optical theorem, to arrive at the gluon distributions in the hadrons in Basic Factorization. Then we reduce them down to the expressions which can be used in K_T- and Collinear Factorizations. As is well-known, K_T-Factorization (High-Energy Factorization) by definition can be used in the Regge (small-x) kinematics, so throughout the present paper we will consider the parton distributions in this region only.

Our paper is organized as follows: In Section II we study the elastic gluon-hadron amplitudes for the forward kinematic region in the Born approximation, and then analyze the impact of radiative corrections. We investigate the convergence of the factorization convolutions, using a general form for the non-perturbative inputs, to determine constraints on the non-perturbative inputs. In Section III we formulate general criteria for the non-perturbative inputs in Basic Factorization and use them to construct a Resonance Model. Transition from the Basic Factorization to K_T- Factorization is considered in Sect. IV. In this Sect. we also compare our results with the ones available in the literature. In Sect. V we reduce the inputs in K_T- Factorization down to the ones in Collinear Factorization. We focus here on comparing our results with the standard DGLAP-fits. At last, Sect. VI is for our concluding remarks.

II.   ELASTIC GLUON-HADRON SCATTERING AMPLITUDES IN THE FORWARD KINEMATICS

In this Sect. we consider the elastic gluon-hadron amplitude A in the forward kinematics and inspect integrability of convolutions for such amplitude in Basic Factorization. We study the convolutions involving two-gluon intermediate t-channel states only because we presume the approximation of Single-Parton Scattering for the gluon distributions. The convolutions describing the gluon-hadron amplitude are depicted by the sum of two graphs[1]: the graph in Fig. 3 and a similar graph, where q is replaced by $-q$.
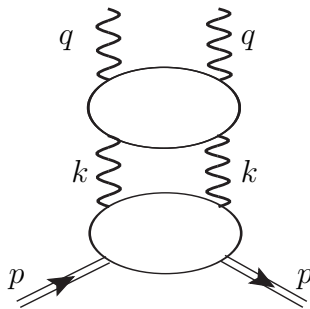

FIG. 3. Factorization of amplitudes of hadron-gluon scattering in the forward kinematics, with intermediate partons being gluons. The upper blob corresponds to amplitudes of the elastic gluon scattering.

Our aim here is to obtain mathematical restrictions on the non-perturbative inputs. In order to do it, we use the obvious reasoning: Integration over momentum k in the factorization convolutions (see Fig. 3) runs over the whole phase space and it should yield a finite result. Features of perturbative components of the convolutions are known, at least qualitatively, so integrability requirement can be used to establish necessary restrictions on the non-perturbative inputs in a general form. Then any model for the inputs should meet these restrictions. Therefore, the restrictions can be used as criteria for accepting or rejecting the models. To begin with, we consider amplitude A in the Born

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[1] Throughout the paper we consider the t-channel color singlets only.

approximation and then examine an impact of the radiative corrections. We show that, in contrast to the concepts of K_T- and Collinear Factorizations, Basic Factorization allows the simple scenario, where the upper blob in in Fig. 3 can be regarded as altogether perturbative one while the lowest blob includes non-perturbative contributions only.

A. Gluon-hadron scattering amplitudes in the Born approximation

Let us consider the simplest case of the factorization convolutions for the gluon-hadron amplitude, where the non-perturbative input is convoluted with the gluon-gluon perturbative amplitude in the Born approximation. In this case the upper blob in Fig. 3 is represented by two simple graphs, each with the single-gluon exchange. The first graph, with a non-zero imaginary part in s, is depicted in Fig. 4 and the second graph, with a non-zero imaginary part in u, can be obtained from the first one by replacement $q \to -q$.
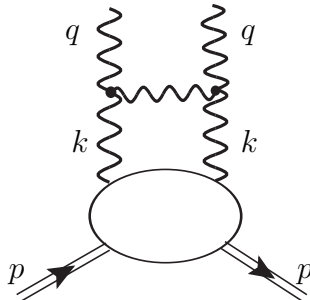

FIG. 4. Factorization for amplitude of hadron-gluon scattering in the Born approximation.

A remarkable feature of Basic Factorization is that the analytic expression corresponding to the convolution graphs can be obtained with the use of the standard Fetnman rules. Applying these rules to the graph in Fig. 4 and complementing the result by the exchange $q \to -q$, we arrive at the following expression for the Born gluon-hadron amplitude A^B in Basic Factorization:

$$A^B = \frac{4\pi\alpha_s N}{(2\pi)^4} \int d^4k \; l_\mu l'_\nu \left[\frac{H_{\mu\nu\lambda\rho}(q,k)}{s_2 + \imath\epsilon} + \frac{H_{\nu\mu\lambda\rho}(-q,k)}{u_2 + \imath\epsilon} \right] \frac{1}{k^2 k^2} T_{\lambda\rho}(k,p,S), \tag{6}$$

where

$$s_2 = (q+k)^2, \; u_2 = (q-k)^2, \tag{7}$$

$N = 3$ is the color factor and S denotes the hadron spin. The term in squared brackets corresponds to the Born amplitude for gluon-gluon scattering, $l_\mu(q)$ and $l'_\nu(q)$ denote the polarization vectors of the external gluons, the terms $1/k^2$ correspond to propagators of the connecting gluons and the input $T_{\lambda\rho}$ contains non-perturbative contributions only. Throughout the paper we use the Feynman gauge. Let us notice that both $T_{\lambda\rho}$ and the term in the squared brackets are dimensionless. The perturbative term $H_{\mu\nu\lambda\rho}$ in Eq. (6) is

$$H_{\mu\nu\lambda\rho} = [(-q-2k)_\mu g_{\lambda\sigma} + (2q+k)_\lambda g_{\mu\sigma} + (k-q)_\sigma g_{\mu\lambda}] [(q+2k)_\nu g_{\rho\sigma} + (-2q-k)_\rho g_{\nu\sigma} + (q-k)_\sigma g_{\nu\rho}]. \tag{8}$$

We remind that in the present paper we consider the gluon-hadron scattering amplitudes and parton distributions in the small-x region. The point is that we are going to reduce Basic Factorization to K_T- Factorization which is defined in the small-x region only. It is easy to check that A^B with $H_{\mu\nu\lambda\rho}$ given by Eq. (8) is gauge-invariant in the small-x, though approximately[2]

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[2] The gauge invariance for the quark-hadron scattering amplitudes was considered in Ref. [18].

In order to make further progress we should specify $T_{\lambda\rho}$. First of all, we should fix its polarization (tensor) structure. For instance, when the hadron and the gluons are unpolarized, $T_{\lambda\rho}$ should be symmetric in λ, ρ. The general form of such a tensor, $T_{\lambda\rho}^{(gen)}$ is

$$T_{\lambda\rho}^{(gen)} = g_{\lambda\rho}A + p_\lambda p_\rho B + (p_\lambda k_\rho + k_\lambda p_\rho)C + k_\lambda k_\rho D, \tag{9}$$

but it involves four arbitrary invariant amplitudes $A, .., D$. Trying to diminish their number, we recall an instance from perturbative QCD, namely that, when the hadron is replaced by a bare quark, $T_{\lambda\rho}$ is replaced by the quark-gluon Born amplitude $Q_{\lambda\rho}$. It consists of the unpolarized part $Q_{\lambda\rho}^U$ and the spin-dependent part $Q_{\lambda\rho}^S$:

$$Q_{\lambda\rho} = Q_{\lambda\rho}^U + Q_{\lambda\rho}^S, \tag{10}$$

with

$$Q_{\lambda\rho}^U = (2p_\lambda p_\rho - k_\lambda p_\rho + p_\lambda k_\rho - pk\ g_{\lambda\rho}) \left(\frac{-8\pi\alpha_s C_F}{(p-k)^2 - m_q^2 + \imath\epsilon} \right), \tag{11}$$

$$Q_{\lambda\rho}^S = \imath m_q \epsilon_{\lambda\rho\sigma\tau} k_\sigma (S_q)_\tau \left(\frac{-8\pi\alpha_s C_F}{(p-k)^2 - m_q^2 + \imath\epsilon} \right),$$

where m_q is the quark mass, S_q is the quark spin and $C_F = 4/3$. We suggest the simplest generalization of Eqs. (10,11), for the hadrons: we presume that $T_{\lambda\rho}$ keeps the polarization structure of $Q_{\lambda\rho}$. It means that

$$T_{\lambda\rho} = T_{\lambda\rho}^U + T_{\lambda\rho}^S, \tag{12}$$

with

$$T_{\lambda\rho}^U = (2p_\lambda p_\rho - k_\lambda p_\rho - p_\lambda k_\rho + pk\ g_{\lambda\rho})\ M_U(s_1, k^2), \tag{13}$$
$$T_{\lambda\rho}^S = \imath M_h \epsilon_{\lambda\rho\sigma\tau} k_\sigma S_\tau M_S(s_1, k^2),$$

where M_h, S are the hadron mass and spin respectively, and

$$s_1 = (p-k)^2. \tag{14}$$

The invariant amplitudes M_U and M_S may either coincide or differ from each other. Substituting $T_{\lambda\rho}^{U,S}$ in Eq. (6), we obtain:

$$A_U^B = \frac{4\pi\alpha_s N}{(2\pi)^4} \int \frac{d^4k}{k^2 k^2} \left[\frac{N_s^{(1)}}{s_2 + \imath\epsilon} + \frac{N_u^{(1)}}{u_2 + \imath\epsilon} \right] M_U(s_1, k^2), \tag{15}$$

$$A_S^B = \frac{4\pi\alpha_s N}{(2\pi)^4} \int \frac{d^4k}{k^2 k^2} \left[\frac{N_s^{(2)}}{s_2 + \imath\epsilon} + \frac{N_u^{(2)}}{u_2 + \imath\epsilon} \right] M_S(s_1, k^2), \tag{16}$$

with

$$N_s^{(1)} = 4k^2(2m_h^2 + 2pk) - 16(2(pq)^2 + 2(pq)(pk) - q^2(pk)) + u_2(2M_h^2 - 2pk), \tag{17}$$
$$N_s^{(2)} = \imath M_h \epsilon_{\lambda\rho\sigma\tau} S_\tau \left[8q_\rho \left(l_\lambda' k_\sigma(kl) - l_\lambda k_\sigma(kl') \right) - u_2 k_\sigma l_\lambda l_\rho' \right].$$

We have done summation over the gluon polarizations in the expression for $N_s^{(1)}$ in Eq. (17). In what follows we will use the Sudakov variables defined in Eq. (4). In their terms

$$2pk = -\alpha w + \beta x_p w, \quad 2qk = \beta w - \alpha x w, \quad k^2 = -\alpha\beta w + k_\perp^2. \tag{18}$$

B. Analysis of IR and UV singularities of the Born amplitudes

Let us consider integrability of the Born amplitudes in Eqs. (15,16). Integration over momentum k in both convolutions runs over the whole phase space and result of the integration must be finite despite the integrands may have singularities which must be regulated. First of all, let us note that the expressions in parentheses in Eqs. (15,16) stand for the gluon perturbative amplitudes in the Born approximation, which are obviously free of divergences. In contrast, the factors $1/k^2$ in Eqs. (15,16) become divergent at $k^2 = 0$. We cannot implement any IR cut-off to regulate those singularities because there is no physical reason to restrict the integrations regions. It leaves us with the only option: amplitudes $M_{U,S}$ in Eqs. (15,16) at small k^2 must decrease rapidly enough in order to kill these IR singularities. Therefore we infer that $M_{U,S}$ should satisfy the following restrictions at $k^2 \to 0$:

$$M_{U,S}(s_1, k^2) \sim \left(k^2\right)^{1+\eta}, \tag{19}$$

with $\eta > 0$. Eq. (19) sets IR stability for the integrals in Eqs. (15,16). Then, there may be UV singularities in Eqs. (15,16) at large $|k|$, i.e. at large $|\alpha|$ in terms of the Sudakov variables, when the integration over α goes first. It is easy to see that the integrands in Eqs. (15,16) at large $|\alpha|$ behave as

$$\sim \frac{\alpha^2}{\alpha^3} M_{U,S} \tag{20}$$

which can be UV-divergent. In order to kill this divergence, $M_{U,S}$ should decrease at large α:

$$M_{U,S} \sim \alpha^{-\chi}, \tag{21}$$

with $\chi > 0$. In other words, the non-perturbative inputs should decrease, when their invariant energy s_1 grows. Also, we note that the restrictions in Eqs. (19,21) are valid at any choice of the polarization structure of $T_{\lambda\rho}$, including the most general parametrization in Eq. (9).

C. Gluon-hadron scattering amplitudes beyond the Born approximation

Diagrammatically, accounting for the radiative corrections to the Born amplitudes $A_{U,S}^B$ of Eqs. (15,16) means adding more quark and gluon propagators to the single-gluon exchange in Fig. 4 and summing up contributions of such graphs. This procedure converts the Born perturbative amplitudes of Eqs. (15,16) into amplitudes $A_{U,S}^{(pert)}$ (the upper blob in Fig. 3) but keeps unchanged the non-perturbative inputs $M_{U,S}$ (the lowest blob in Fig. 3). Such natural separation of the perturbative and non-perturbative contributions is a remarkable feature of our approach and perfectly agrees with the factorization concept.

Replacing the Born amplitudes in Eqs. (15,16) by the amplitudes $A_{U,S}^{(pert)}$, we arrive at the gluon-hadron amplitudes A_U, A_S beyond the Born approximation:

$$A_{U,S} = \int \frac{d^4k}{k^2 k^2} A_{U,S}^{(pert)}\left(s_2/q^2, s_2/k^2\right) M_{U,S}(s_1, k^2). \tag{22}$$

Applying the Optical theorem to Eq. (22), we obtain the spin-independent, D_U, and polarized, D_S, distributions of gluons in the hadrons:

$$D_{U,S} = \int \frac{d^4k}{k^2 k^2} D_{U,S}^{(pert)}\left(s_2/q^2, s_2/k^2\right) G_{U,S}(s_1, k^2), \tag{23}$$

where

$$D_{U,S}^{(pert)}(s_2, q^2, k^2) = \Im_{s_2} A_{U,S}^{(pert)}, \quad G_{U,S}(s_1, k^2) = \Im_{s_1} M_{U,S}(s_1, k^2). \tag{24}$$

We remind that the invariant sub-energies $s_{1,2}$ are defined as follows: $s_1 = (q+p)^2, s_2 = (k-p)^2$.

Now we prove that the difference between perturbative amplitudes $A_{U,S}^{(pert)}$ and their Born values does not changes the conditions (19,21) of IR and UV stability. Before doing it, let us notice that $A_{U,S}^{(pert)}$ depends on s_2, k^2, q^2, so, being dimensionless, it can be parameterized, for instance, as follows:

$$A_{U,S}^{(pert)}(s_2, q^2, k^2) = A_{U,S}^{(pert)}\left(s_2/q^2, s_2/k^2\right). \tag{25}$$

Perturbative higher-loop contributions can induce both IR and UV divergences in $A_{U,S}^{(pert)}$. All those divergences should be regulated before using $A_{U,S}^{(pert)}$ in the factorization convolutions (22). We start with considering regularization of IR divergences in $A_{U,S}^{(pert)}$. They appear from integrations of gluon loops over the regions, where some of the virtual gluons are soft. In addition, the problem of IR divergences is essential for soft quarks when their masses are neglected. The ways of regulating IR divergences are different for the amplitudes with on-shell and off-shell external partons. We will address such amplitudes as the on-shell and off-shell amplitudes respectively. For the on-shell amplitudes, the IR divergences are regulated by suitable IR cut-offs, while virtualities of the external partons play the role of natural IR cut-offs for the off-shell amplitudes. The latter was shown first in Ref. [16] and then was used in numerous calculations. Therefore, q^2 and k^2 can be used as IR cut-offs for $A_{U,S}^{(pert)}$.

Now let us consider regulation of the UV divergences in $A_{U,S}^{(pert)}$. This problem is solved easily. The point is that QCD is a renormalizable theory, so all UV divergences are automatically absorbed by appropriate redefinitions of α_s and masses of involved quarks and gluons.

Unfortunately, regulating IR singularities of $A_{U,S}^{(pert)}$ does not eliminate the IR divergences of the factorization convolutions (22). Indeed, the IR-sensitive contributions in $A_{U,S}^{(pert)}$ are $\sim \ln^n(w\beta/k^2)$ (or $\sim \ln^n(q^2/k^2)$), so these contributions are IR stable at $k^2 \neq 0$ but become divergent at the point $k^2 = 0$. Integration in Eqs. (22) covers the whole phase space, so the integration region includes the point $k^2 = 0$. Such logarithmic divergences were addressed as mass singularities in the pioneer studies[3] of QCD hard processes in the framework of Collinear Factorization, where the mass singularities were moved from the hard part to universal structure functions. However, such a treatment of the mass singularities becomes unproductive in the Regge (small-x) kinematics which we pursuit throughout this paper. Furthermore, the logarithmic singularities are the only kind of IR divergences in Collinear Factorization but in Basic Factorization we should handle both them and the power singularity $\sim 1/(k^2 k^2)$ (see Eqs. (22)). In Sect. IIB we demonstrated that the power singularity $1/(k^2 k^2)$ can be regulated by the condition (19). Obviously, this condition persists when the singularity $1/(k^2 k^2)$ is accompanied by the logarithmically divergent terms. It is worth noticing here that the well-known power factor s/k^2 in the spin-independent amplitude appears in the lowest order in α_s, borrowing k^2 from the denominator in Eqs. (22), so its impact on IR stability was taken into account in Eq. (19).

Finally, it is easy to prove that accounting for the perturbative radiative corrections does not change the condition (21) of UV stability of the factorization convolutions at large $|\alpha|$. Indeed, the fact that the perturbative QCD is renormalizable means that the fastest-growing with energy terms in amplitudes $A_{U,S}^{(pert)}$ are of logarithmic kind, i.e. $\sim \ln^n\left(w\alpha/q^2\right), \ln^n\left(w\alpha/k^2\right)$, so such logarithmic divergences can be regulated by the power restriction in Eq. (21).

III. MODELING THE NON-PERTURBATIVE INVARIANT AMPLITUDES $M_{U,S}$

In this Sect. we consider first the Resonance Model for non-perturbative amplitudes $M_{U,S}$ and then proceed to the non-perturbative inputs for the gluon distributions. Our models is based on simple and clear-cut physical arguments which we list below and call them criteria which any model for $M_{U,S}$ should satisfy. Although constructing such a model cannot be done unambiguously, any proposed model should satisfy the following criteria:

A. General criteria for modeling non-perturbative inputs

Criterion (i) Expressions for $M_{U,S}$ should satisfy the requirements of IR and UV stability of the factorization convolutions given in Eqs. (19,21).

Criterion (ii) Expressions for $M_{U,S}$ should have non-zero imaginary parts in s_1 to make possible the use of the Optical theorem. It is necessary in order to proceed from elastic gluon-hadron amplitudes to gluon distributions in the hadrons.

Criterion (iii) Expressions for $M_{U,S}$ should enable the step-by-step reduction of Basic Factorization down to K_T- and Collinear Factorizations. Such a reduction was suggested in general in Ref. [17].

B. Specifying non-perturbative gluon inputs in Basic Factorization

We suggest the following model expressions for the amplitudes $M_{U,S}(s_1, k^2)$:

$$M_U(s_1, k^2) = R_U\left(k^2\right) Z_U\left(s_1\right), \quad M_S(s_1, k^2) = R_S\left(k^2\right) Z_S\left(s_1\right). \tag{26}$$

Eq. (26) reads that dependences on s_1 and k^2 are separated. This is done for the sake of simplicity. We could assume that $R = R(k^2, s_1)$ but we find it unnecessary. As handling M_U and M_S is much the same, we skip the subscripts U, S and will work with the generic notations

$$M(s_1, k^2) = R(k^2)Z(s_1) \tag{27}$$

instead of Eq. (26). In order to satisfy Eq. (19), R in Eq. (27) should exhibit such a behavior:

$$R \sim \left(k^2\right)^{1+\eta} \tag{28}$$

at small k^2. According to Eq. (21), $RZ \sim \alpha^{-\chi}$ at large $|\alpha|$. A behavior of R at large $|\alpha|$ is unknown. However, it can be that the small-k^2 behavior at large $|\alpha|$ is again $R \sim (k^2)^{1+\eta}$, i.e. according to Eq. (18)

$$R \sim \alpha^{1+\eta}. \tag{29}$$

This is the most UV divergent case which should not be ignored, so in order to satisfy Eqs. (21,29), Z should behave as follows: at large $|\alpha|$

$$Z \sim \alpha^{-1-\eta-\chi}. \tag{30}$$

If R does not grow at large $|\alpha|$, we can all the same use Z obeying Eq. (30). In order to specify Z, we suggest the guiding idea: After emitting an active parton by the initial hadron, the remaining set of quark and gluons pick up a color and therefore it cannot be stable. Unstable states, in general, are known to be often modeled by expressions of the resonance type. In particular, we can approximate Z in Eq. (27) by the following expression:

$$Z(s_1) \approx Z_n(s_1) = \prod_{r=1}^{n} \frac{1}{(s_1 - M_r^2 + \imath\Gamma_r)}, \tag{31}$$

with arbitrary positive integer n. In order to get $Z_n(s_1)$ satisfying Eq. (30), we should choose $n > 1$. For the sake of simplicity, we consider the minimal value $n = 2$. Obviously, Z_2 can also be written as an interference of two resonances:

$$Z(s_1) = \frac{1}{(\Delta M_{12}^2 + \imath\Delta\Gamma_{12})} \left[\frac{1}{(s_1 - M_1^2 + \imath\Gamma_1)} - \frac{1}{(s_1 - M_2^2 + \imath\Gamma_2)} \right], \tag{32}$$

with $\Delta M_{12}^2 = M_1^2 - M_2^2$, $\Delta\Gamma_{12} = \Gamma_1 - \Gamma_2$. In terms of the Sudakov variables,

$$M(w\alpha, k^2) = \frac{R(k^2)}{C_Z} \left[\frac{1}{(w\alpha - \mu_1^2 + k^2 + \imath\Gamma_1)} - \frac{1}{(w\alpha - \mu_2^2 + k^2 + \imath\Gamma_2)} \right], \tag{33}$$

where $C_Z = \left(\Delta M_{12}^2 + \imath\Delta\Gamma_{12}\right)$ and $\mu_{1,2}^2 = M_{1,2}^2 - p^2$. Applying the Optical theorem to Eq. (33) allows us to obtain the non-perturbative contribution G to the gluon distributions in the hadrons:

$$G = -\Im_{s_1} T = \frac{R(k^2)}{C_Z} \left[\frac{\Gamma_1}{\left(w\alpha + k^2 - \mu_1^2\right)^2 + \Gamma_1^2} - \frac{\Gamma_2}{\left(w\alpha + k^2 - \mu_2^2\right)^2 + \Gamma_2^2} \right]. \tag{34}$$

Obviously, this expression is of the Breit-Wigner type.

IV.   NON-PERTURBATIVE GLUON INPUTS IN K_T- FACTORIZATION

The expression for the gluon-hadron scattering amplitude $M(\alpha, \beta, k^2)$ in Eq. (33) corresponds to Basic Factorization. In order to reduce $M(\alpha, \beta, k^2)$ to the amplitude $M_{KT}(\beta, k_\perp^2)$ of the same process in K_T-Factorization, we should integrate out the α-dependence of $M(\alpha, \beta, k^2)$. In principle, as soon as However, in factorization convolutions (see e.g. Figs. 2,3,4) both the upper and the lowest blobs depend on α, so one cannot integrate $M(\alpha, \beta, k^2)$ only, without integration of the perturbative blob. Therefore, $M(\beta, k_\perp^2)$ cannot be derived from $M(\alpha, \beta, k^2)$ in the straightforward way, which makes impossible a straightforward reduction of Basic Factorization to K_T-Factorization. Nevertheless, it can be done approximately. The perturbative blobs (the upper blob in Figs. 2,3,4) approximately do not depend on α in the region

$$w|\alpha\beta| \ll k_\perp^2 \tag{35}$$

and the non-perturbative blobs in this region are independent of β. Indeed, the upper blob depends on α through k^2 only and in the region (35) $k^2 \approx -k_\perp^2$. At the same time, the region (35) is known to be the region for gaining the perturbative contributions most essential at small x. For instance, all BFKL logarithms come from this region. So, Eq. (35) makes possible to integrate $M\left(\alpha, \beta, k^2\right)$ independently of perturbative contributions. It is convenient to define the non-perturbative input M_{KT} in K_T Factorization as follows:

$$k_\perp^2 M_{KT}(\beta, k_\perp^2) = \int_{-\alpha_0}^{\alpha_0} d\alpha M(\alpha, \beta, k^2), \tag{36}$$

with

$$\alpha_0 \ll k_\perp^2/(w\beta). \tag{37}$$

So, Eq. (37) is satisfied when

$$\alpha_0 = \xi k_\perp^2/\beta, \tag{38}$$

with a positive ξ obeying the inequality $\xi \ll 1$. Combining Eqs. (33,36,38) we arrive at the following expression for the non-perturbative input M_{KT} in K_T -Factorization:

$$
\begin{aligned}
M_{KT} &\approx \frac{R\left(k_\perp^2\right)}{k_\perp^2} \left[\frac{1}{\xi k_\perp^2/\beta - \mu_1^2 + \imath\Gamma_1} + \frac{1}{\xi k_\perp^2/\beta - \mu_2^2 + \imath\Gamma_2} + \frac{1}{\xi k_\perp^2/\beta + \mu_1^2 - \imath\Gamma_1} + \frac{1}{\xi k_\perp^2/\beta + \mu_2^2 - \imath\Gamma_2} \right] \\
&= R_{KT}\left(k_\perp^2\right) \left[\frac{1}{k_\perp^2/\beta - \mu_1'^2 + \imath\Gamma_1'} + \frac{1}{k_\perp^2/\beta - \mu_2'^2 + \imath\Gamma_2'} + \frac{1}{k_\perp^2/\beta + \mu_1'^2 - \imath\Gamma_1'} + \frac{1}{k_\perp^2/\beta + \mu_2'^2 - \imath\Gamma_2'} \right],
\end{aligned} \tag{39}
$$

where $R_{KT} = R/(\xi k_\perp^2 C_Z)$ and

$$\mu'^2_{1,2} = \mu_{1,2}^2/\xi, \ \Gamma'_{1,2} = \Gamma_{1,2}/\xi. \tag{40}$$

We also used the well-known observation that the most essential region in the factorization convolutions is the small-β region, where ξ/β is not far from x. Eq. (39) is valid when

$$w\alpha_0 \gg \mu_{1,2}^2, \qquad \Gamma_{1,2} \gg \Delta\mu^2, \Delta\Gamma, \tag{41}$$

i.e. when $k_\perp^2/\beta$ is pretty far from the resonance region $k_\perp^2/\beta \sim \mu_{1,2}'^2$. The closer $k_\perp^2/\beta$ is to $\mu_{1,2}'^2$ the greater are corrections to Eq. (39). It means that transition from Basic Factorization to K_T-Factorization considered in Ref. [17] is straightforwardly done outside the resonance region. After that, Eq. (39) can be analytically continued in the resonance region. As a result, we arrive at the gluon-hadron amplitude A_{KT} and the gluon distribution D_{KT} in K_T-Factorization:

$$A_{KT} = \int \frac{d\beta}{\beta} \frac{dk_\perp^2}{k_\perp^2} A_{KT}^{(pert)}(w\beta, q^2, k_\perp^2) M_{KT}(\beta, k_\perp^2) \tag{42}$$

$$D_{KT} = \int \frac{d\beta}{\beta} \frac{dk_\perp^2}{k_\perp^2} D_{KT}^{(pert)}(w\beta, q^2, k_\perp^2) G_{KT}(\beta, k_\perp^2), \tag{43}$$

where $A_{KT}^{(pert)}$ and $D_{KT}^{(pert)} = \Im A_{KT}^{(pert)}$ are the perturbative contributions whereas M_{KT} and $G_{KT} = \Im M_{KT}$ (cf. Eq. (24)) are the non-perturbative inputs. It is convenient to represent G_{KT} in the following form:

$$G_{KT} = G_R + G_B, \tag{44}$$

with

$$G_R = R_{KT}\left(k_\perp^2\right) \left(\frac{\Gamma_1'}{(k_\perp^2/\beta - \mu_1'^2)^2 + \Gamma_1'^2} + \frac{\Gamma_2'}{(k_\perp^2/\beta - \mu_2'^2)^2 + \Gamma_2'^2} \right) \tag{45}$$

and

$$G_B = R_{KT}\left(k_\perp^2\right) \left(\frac{\Gamma_1'}{(k_\perp^2/\beta + \mu_1'^2)^2 + \Gamma_1'^2} + \frac{\Gamma_2'}{(k_\perp^2/\beta + \mu_2'^2)^2 + \Gamma_2'^2} \right). \tag{46}$$

G_R and G_B obviously consist of expressions of the Breit-Wigner type. Signs of $\mu_1'^2$ and $\mu_2'^2$ cannot be fixed a priory, but $G_R \leftrightarrows G_B$ when $\mu_{1,2}'^2 \to -\mu_{1,2}'^2$, so we can consider the case of positive $\mu_{1,2}'^2$ without loss of generality. We remind that $k_\perp^2/\beta > 0$, so G_R is within the resonant region $k_\perp^2/\beta \sim \mu_{1,2}'^2$ while G_B is out of that region. Eqs. (45,46) allows us to represent the gluon distribution D_{KT} in K_T-Factorization as the sum of its resonance part D_{KT}^R and the background contribution D_{KT}^B:

$$D_{KT} = D_{KT}^R + D_{KT}^B, \tag{47}$$

with

$$D_{KT}^R = \int \frac{d\beta}{\beta} \frac{dk_\perp^2}{k_\perp^2} D_{KT}^{(pert)}(w\beta, q^2, k_\perp^2) G_R(\beta, k_\perp^2) \tag{48}$$

and

$$D_{KT}^B = \int \frac{d\beta}{\beta} \frac{dk_\perp^2}{k_\perp^2} D_{KT}^{(pert)}(w\beta, q^2, k_\perp^2) G_B(\beta, k_\perp^2). \tag{49}$$

Representation of D_{KT} through the resonance D_{KT}^R and background D_{KT}^B contributions is similar to the structure of expressions in the Duality concept.

A. Discussion of the distribution $\mathbf{G_{KT}}$

First of all, let us compare the mass parameters $\mu_{1,2}$ of the gluon distribution D of Eq. (34) in Basic Factorization and the parameters $\mu_{1,2}'$ of the gluon distribution D_{KT} of Eq. (43) in K_T-Factorization. They are related by Eq. (40). As $\xi \ll 1$, Eq. (40) means that $\mu_1' \gg \mu_1$, $\mu_2' \gg \mu_2$. It means that despite the non-perturbative input G is defined in Basic Factorization within the non-perturbative domain, the resonance maximums of G_{KT} are located at perturbative values of $k_\perp^2/\beta > 0$. Then, we would like to comment on the factor R_{KT} in Eqs. (39,45,46). This factor describes a dependence of G_{KT} on $k_\perp^2$. Refs. [10–13] suggest the exponential form of R_{KT}:

$$R' \sim N \exp[-\lambda k_\perp^2], \tag{50}$$

with N and λ in Eq. (50) being parameters. However, Eq. (50) contradicts to the general requirement of IR stability (cf. Eq. (28)) stating that

$$R_{KT} \sim \left(k_\perp^2\right)^\eta \tag{51}$$

at $k_\perp^2 \to 0$, so R' in Eq. (50) must be modified. The simplest modification compatible with Eq. (51) is

$$R_{KT} = N \left(k_\perp^2\right)^\eta \exp[-\lambda k_\perp^2], \tag{52}$$

which agrees with the expressions used in Refs. [11, 12]. So, Eqs, (45,46,52) present the complete set of formulas to describe the non-perturbative input G_{KT} for the gluon distributions in K_T Factorization. It is interesting to note a further similarity of our expressions in Eqs. (45,46) and the model suggested in Ref. [13], where non-perturbative distributions for the five-quark state are studied. The quark distributions in Ref. [13] involve both a Gaussian (though without the power factor $\left(k_\perp^2\right)^\eta$) and propagators $\sim (k_\perp^2/\beta - m^2 + \iota\varepsilon)^{-1}$ of the five quarks. The latter resembles our factor Z in Eq. (32) save the difference between the factors $\iota\varepsilon$ and $\iota\Gamma_{1,2}$. The quarks in Ref. [13] are assumed to be free and stable, the gluon content is dropped. In contrast to Ref. [13], we do not specify the content of the spectators but describe the whole set of spectators through the resonances and background contributions.

V.   GLUON DISTRIBUTIONS IN COLLINEAR FACTORIZATION

Our next aim is to reduce D_{KT} of Eq. (43) to the gluon distribution D_{col} in Collinear Factorization and bring the gluon distribution to the following form:

$$D_{col} = \int \frac{d\beta}{\beta} D_{col}^{(pert)}(x/\beta, \mu_F^2)\varphi_{col}(\beta, \mu_F^2), \tag{53}$$

with $D_{col}^{(pert)}$ being the perturbative contribution and φ being the non-perturbative input; μ_F is the factorization scale. To this end, the $k_\perp$-dependence in Eq. (43) should be integrated out. At the first sight, such a reduction is impossible: $D_{KT}^{(pert)}$ and G_{KT} in Eq. (43) explicitly depend on $k_\perp$, so they both must be integrated and because of that the integration can yield some entangled mixture of the perturbative and non-perturbative terms instead of their product as represented in Eq. (53). However, the specific form of the inputs in Eq. (45,46) allows us to reduce D_{KT} to D_{col} though approximately. According to Eq. (47), D_{KT} consists of D_{KT}^R and D_{KT}^B. Let us consider first the resonance component, D_{KT}^R given by Eq. (48). Integration over ζ therein runs in the resonance region, so only G_R must be integrated:

$$
\begin{aligned}
D_{KT}^R &= \int_x^1 \frac{d\beta}{\beta} \int_0^{\zeta_0} \frac{d\zeta}{\zeta} D_{KT}^{(perp)}(x/\beta, q^2/\zeta)G_R(\beta, \zeta) \\
&\approx \int_x^1 \frac{d\beta}{\beta} D_{col}^{(perp)}(x/\beta, \mu_1'^2)\varphi_R(\beta, \mu_1'^2) + O\left(\Gamma_1'/\mu_1'^2\right) \\
&+ \int_x^1 \frac{d\beta}{\beta} D_{col}^{(pert)}(x/\beta, \mu_2'^2)\varphi_R(\beta, \mu_2'^2) + O\left(\Gamma_2'/\mu_2'^2\right),
\end{aligned}
\tag{54}
$$

where $\zeta_0 = w(1 - x/\beta) \approx w$, with $w = 2pq$ as defined in Eq. (5). We presume that the resonances are narrow:

$$\Gamma_1'/\mu_1'^2 \ll 1, \quad \Gamma_2'/\mu_2'^2 \ll 1. \tag{55}$$

In Eq. (54) the notations $D_{col}^{(pert)}(x/\beta, \mu_r'^2)$, with $r = 1, 2$, stand for the perturbative components while non-perturbative inputs $\varphi_R(\beta, \mu_r'^2)$ are

$$\varphi_R(\beta, \mu_r'^2) = R(\mu_r^2\beta)/\mu_r^2 = N \left(\mu_r^2\right)^{\eta-1} \beta^\eta e^{-\lambda\beta\mu_r^2} \approx N_r\beta^\eta \left(1 - C_r\beta\right). \tag{56}$$

In Eq. (56) we have denoted $N_r = N \left(\mu_r^2\right)^{\eta-1}$ and $C_r = \lambda\mu_r^2$. Besides, we presumed that $\lambda\mu_r^2 < 1$ to truncate the series of the power expansion of the exponential. In contrast to D_{KT}^R, integration over $k_\perp$ (over ζ in fact) in Eq. (49)

runs outside the resonance region and therefore it necessitates integration of both G_B and $D_{KT}^{(perp)}$. Because of that the outcome scarcely can be represented in the factorized form of Eq. (53), which violates the reduction to Collinear Factorization. Fortunately, this violation is small. Indeed,

$$D_{KT}^B = \int \frac{d\beta}{\beta} \frac{d\zeta}{\zeta} D_{KT}^{(perp)}(x/\beta, q^2/\zeta) G_B(\beta, \zeta) \sim O\left(\Gamma_1'/\mu_1'^2, \Gamma_2'/\mu_2'^2\right), \tag{57}$$

so when the resonance input G_{KT}^R is narrow, i.e. when $\Gamma_{1,2}' \ll \mu_{1,2}'^2$, we can neglect all contributions $\sim \Gamma_1'/\mu_1'^2, \Gamma_2'/\mu_2'^2$ in Eqs. (54,57), thereby arriving at the following expression for the gluon distribution D_{col} in Collinear Factorization:

$$D_{col} \approx \int \frac{d\beta}{\beta} \left[D_{col}^{(pert)}(x/\beta, \mu_1'^2)\varphi_R(\beta, \mu_1'^2) + D_{col}^{(pert)}(x/\beta, \mu_2'^2)\varphi_R(\beta, \mu_2'^2) \right]. \tag{58}$$

The integrand in Eq. (58) looks similar to the conventional one in Eq. (53) but does not coincide with it. It is easy to bring Eq. (58) to the conventional form (53) with the perturbative evolution in the $k_\perp$-space. It can be done in two steps: First, let us introduce a scale μ_F so that $|q^2| > \mu_F > \mu_{1,2}'$. Then, using DGLAP (or another evolution approach) we evolve $\varphi_R(\beta, \mu_1'^2)$ and $\varphi_R(\beta, \mu_2'^2)$ in Eq. (58) from their scales up to the scale μ_F, keeping β fixed. This procedure automatically sets $D_{col}^{(pert)}(x/\beta, \mu_{1,2}'^2)$ in Eq. (58) on the scale μ_F. As a result, we convert Eq. (58) into Eq. (53), where φ_{col} is related to the non-perturbative input φ_R by the operator E of the DGLAP-evolution:

$$\varphi_{col}(\beta, \mu_F^2) = E(\mu_F^2, \mu_1^2) \otimes \varphi_R(\beta, \mu_1^2) + E(\mu_F^2, \mu_2^2) \otimes \varphi_R(\beta, \mu_2^2). \tag{59}$$

The evolution operator E in the Mellin (momentum) space is expressed in terms of the DGLAP anomalous dimensions. In contrast to the non-perturbative inputs $\varphi_R(\beta, \mu_r'^2)$, the input φ_{col} comprises both non-perturbative (through operators E) and perturbative (through φ_R) contributions.

A. Comparison of Eq. (56) to the standard DGLAP fits

Let us compare the non-perturbative input φ_R with the standard DGLAP fit $\widetilde{\varphi}$. Quite often (see e.g. Ref. [7]) $\widetilde{\varphi}$ is chosen in the following form:

$$\widetilde{\varphi}(\beta, \mu_F^2) = N\beta^{-a}(1 - \beta)^b(1 + c\beta^d), \tag{60}$$

where N, a, b, c, d are phenomenological parameters defined from analysis of experimental data. All these parameters implicitly depend on the factorization scale μ_F. Its value can be chosen arbitrary but factorization convolutions, where $\widetilde{\varphi}$ participates, do not depend on μ_F.

First of all, let us note that the singular factor β^{-a} in Eq. (60) provides the fast growth of the parton distributions at small x and leads to their Regge asymptotics. We proved (see e.g. the overview [2]) that the singular factors in the DGLAP fits mimic accounting for the total resummation of $\ln^n(1/x)$. When such resummation is taken into account, the factors β^{-a} become irrelevant and should be dropped. On the other hand, our analysis does not exclude the factor β^η in Eq. (56) because η is positive.

The factor $(1 - \beta)^b$ of Eq. (60) looks as the large-x asymptotics of of the parton distributions at $x \to 1$. By this reason, we suggest that the origin of this factor also is perturbative, so the factor $(1 - \beta)^b$ can be excluded from the fit when the total resummation of $\ln^n(1 - x)$ is taken into account.

After the factors β^{-a} and $(1 - \beta)^b$ have been excluded from Eq. (60), it becomes quite similar to the input φ_R.

VI. SUMMARY

In the present paper we have performed the detailed study of the general structure of the gluon non-perturbative inputs for the gluon distributions in hadrons and applied the Resonance Model to construct the gluon inputs. These inputs can also apply to description of various hadronic high-energy processes, including the DIS structure functions. We constructed the gluon inputs in Basic Factorization and then calculated the gluon inputs in the reduced forms so as to use them in K_T- and Collinear Factorizations.

We began with constructing the non-perturbative inputs for the elastic gluon-hadron scattering amplitudes in the forward kinematics, where we considered the cases of polarized and non-polarized hadrons. Then, the Optical theorem allowed us to proceed to the gluon distributions in the hadrons.

Before specifying our model, we derived general theoretical restrictions (19,21) on the non-perturbative inputs, which are obligatory for any model. These restrictions follow from the obvious requirement: integrating the factorization convolutions must yield finite results, i.e. the integrands of the convolutions must be free of IR and UV divergences.

We used those restrictions as criteria for modeling the inputs in Basic Factorization and reduced them step-by-step to K_T- and Collinear Factorizations. Our model presumes that the gluon non-perturbative inputs in Basic Factorization consist of the polarization structures and invariant amplitudes $M_{U,S}$. For the sake of simplicity, we suggested in Eq. (27) separation of the k^2- and s_1- dependence in the expressions for $M_{U,S}$, representing these amplitudes as products of the factors $R(k^2)$ and $Z(s_1)$. Then for specifying the factors $Z(s_1)$ we proposed the Resonance Model and represented in Eqs. (33,34) the inputs through superpositions of the resonances.

Transition from Basic Factorization to K_T- Factorization caused reducing the inputs of Eqs. (33,34) down to the expressions in Eqs. (39,44). Eqs. (39,44) demonstrate that the inputs in K_T-Factorization are again given by the terms of the resonance type. However, some of the resonances have maximums far beyond the region of integration over $k_\perp$, so they can be regarded as a background. Using the models suggested in Refs. [10–13], we assumed the exponential (Gaussian) form for the factor R introduced in Eq. (27). Confronting it to Eq. (51) following from the requirement of IR stability allowed us to exclude a part of these models.

Moving from K_T- Factorization to Collinear Factorization, we reduced the expression for the gluon non-perturbative input in Eq. (44) down to Eq. (56) and compared it with the conventional expression (60) for integrated parton distributions.

Finally, we conclude that our calculations prove that the only difference between the gluon inputs and quark inputs obtained in Ref. [18] is the difference between their polarization structures whereas the invariant amplitudes $M_{U,S}$ can be the same. Our study points out that there is a certain universality between the non-perturbative quark and gluon inputs in each of the available forms of QCD factorization.

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