On the new wave behavior to the longitudinal wave equation in a magneto-electro-elastic circular rod

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Abstract

With the aid of the symbolic computations software; Wolfram Mathematica 9, the powerful sine-Gordon expansion method is used in examining the analytical solution of the longitudinal wave equation in a magneto-electro-elastic circular rod. Sine-Gordon expansion method is based on the well-known sine-Gordon equation and a wave transformation. The longitudinal wave equation is an equation that arises in mathematical physics with dispersion caused by the transverse Poisson's effect in a magneto-electro-elastic circular rod. We successfully get some solutions with the complex, trigonometric and hyperbolic function structure. We present the numerical simulations of all the obtained solutions by choosing appropriate values of the parameters. We give the physical meanings of some of the obtained analytical solutions which significantly explain some practical physical problems.

Keywords: The SGEM; longitudinal wave equation in a MEE circular rod; complex, hyperbolic, trigonometric function solutions

1 Introduction

Searching for the new analytical solutions to nonlinear evolution equations (NEEs) plays a vital role in the study of nonlinear physical aspects. Nonlinear evolution equation are often used to express complex models that arise in the various fields of nonlinear sciences, such as; plasma physics, quantum mechanics, biological sciences and so on. For the past two decades, various analytical techniques have been invested to explore the search for the new solutions to different type of NLEs such as the new generalized algebra method [\[1\]](#page-9-0), the tan($\frac{F(\xi)}{2}$)-expansion method [\[2,](#page-9-1) [3\]](#page-10-0), the extended tanh method [\[4\]](#page-10-1), the $[1]$, the cand $[2]$ expansion method $[2]$, $[3]$, the extended tail method $[4]$, the jacobi elliptic function method $[5]$, the homogeneous balance method $[6]$, the generalized Kudryashov method [\[7,](#page-10-4) [8\]](#page-10-5), the generalized (G'/G) method [\[9\]](#page-10-6), the extended homoclinic test function method [\[10\]](#page-10-7), the improved Bernoulli sub-equation function method [\[11\]](#page-10-8), the modified exp $(-\Omega(\xi))$ -expansion function method [\[12,](#page-10-9) [27\]](#page-12-0) and so on. Generally, many more analytical techniques have been designed and used in obtaining analytical solutions of various NLEs [\[13–](#page-10-10) [24\]](#page-11-0).

In this work, the powerful sine-Gordon expansion method (SGEM) [\[25,](#page-12-1) [26\]](#page-12-2) is invested to search for some new solutions to the longitudinal wave equation in a magneto-electro-elastic (MEE) circular rod [\[27\]](#page-12-0). The longitudinal wave equation is an equation with dispersion caused by the transverse Poisson's effect in a MEE circular rod which is derived by [\[28\]](#page-12-3).

The longitudinal wave equation in a MEE circular rod is given by [\[27\]](#page-12-0);

$$
u_{tt} - v_0^2 u_{xx} - \left(\frac{v_0}{2}u^2 + M u_{tt}\right)_{xx} = 0, \tag{1.1}
$$

where v_0 is the linear longitudinal wave velocity for a MEE circular rod and M is the dispersion parameter, all of them depend on the material property and the geometry of the rod [\[28\]](#page-12-3). Various analytical approaches have been invested to seek for the solutions of the longitudinal wave equation in a magneto-electroelastic MEE circular rod such as the modified (G'/G) -expansion method [\[29\]](#page-12-4), the functional variable method [\[30\]](#page-12-5), the ansatz method [\[31\]](#page-12-6) and so on.

2 The SGEM

In the present section, we give the general facts of SGEM.

Consider the following sine-Gordon equation [\[32,](#page-12-7) [33\]](#page-12-8):

$$
u_{xx} - u_{tt} = m^2 \sin(u). \tag{2.1}
$$

where $u = u(x, t)$ and $n \in \mathbb{R} \setminus \{0\}$.

Utilizing the wave transformation $u = u(x, t) = U(\zeta), \zeta = \alpha(x - kt)$ on Eq. [\(2.1\)](#page-1-0), produces the following nonlinear ordinary differential equation (NODE):

$$
U'' = \frac{n^2}{\alpha^2 (1 - k^2)} sin(U),
$$
\n(2.2)

where $U = U(\zeta)$, ζ is the amplitude of the travelling wave and k is the speed of the travelling wave. Integrating Eq. (2.2) , we obtain the following equation:

$$
\left[\left(\frac{U}{2}\right)'\right]^2 = \frac{n^2}{\alpha^2(1-k^2)}sin^2\left(\frac{U}{2}\right) + Q,\tag{2.3}
$$

where Q is the integration constant.

Substituting
$$
Q = 0
$$
, $\omega(\zeta) = \frac{U}{2}$ and $b^2 = \frac{n^2}{\alpha^2(1-k^2)}$ in Eq. (2.3), gives:

$$
\omega^{'} = b\sin(\omega), \qquad (2.4)
$$

inserting $b = 1$ into Eq. [\(2.4\)](#page-2-0), produces:

$$
\omega' = \sin(\omega),\tag{2.5}
$$

simplifying Eq. (2.5) , gives the following two significant equations;

$$
sin(\omega) = sin(\omega(\zeta)) = \frac{2de^{\zeta}}{d^2e^{2\zeta} + 1}\Big|_{d=1} = sech(\zeta), \tag{2.6}
$$

$$
cos(\omega) = cos(\omega(\zeta)) = \frac{d^2 e^{2\zeta} - 1}{d^2 e^{2\zeta} + 1} \bigg|_{d=1} = tanh(\zeta), \tag{2.7}
$$

where d is the integral constant.

For the given nonlinear partial differential equation Eq. [\(2.8\)](#page-2-2);

$$
P(u, uu_x, u^2u_t, \ldots), \qquad (2.8)
$$

we look its solution in the form;

$$
U(\zeta) = \sum_{i=1}^{m} \tanh^{i-1}(\zeta) \left[B_i \operatorname{sech}(\zeta) + A_i \tanh(\zeta) \right] + A_0. \tag{2.9}
$$

Equation (2.9) may be given according to Eq. (2.6) and (2.7) as;

$$
U(\omega) = \sum_{i=1}^{m} \cos^{i-1}(\omega) \left[B_i \sin(\omega) + A_i \cos(\omega) \right] + A_0. \tag{2.10}
$$

We determine m by balancing the highest power nonlinear term and the highest derivative in the transformed NODE. Taking each summation of the coefficients of $sin^i(w)cos^j(w)$, $0 \le i, j \le m$ to be zero, produces a set of equations. Solving this set of equation with the symbolic computational software like Wolfram Mathematica 9, yields the values of the coefficients A_i , B_i , μ and c . Finally, inserting the obtained values of these coefficients into Eq. [\(2.9\)](#page-2-3) along with the value of m , gives the new travelling wave solutions to Eq. (2.8) .

3 Applications

In this section, the SGEM is used in searching the new solutions to Eq. [\(1.1\)](#page-1-3). Considering Eq. [\(1.1\)](#page-1-3), we derive the following NODE by utilizing the wave transformation; $u = U(\zeta), \zeta = \mu(x - kt);$

$$
2pk^{2}\mu^{2}U'' - 2(k^{2} - c_{0}^{2})U + c_{0}^{2}U^{2} = 0,
$$
\n(3.1)

we get $m = 2$ by balancing U'' and U^2 in Eq. [\(3.1\)](#page-3-0).

Using Eq. (2.10) together with the value $m = 2$, we get the following equation;

$$
U(\omega) = B_1 \sin(\omega) + A_1 \cos(\omega) + B_2 \cos(\omega) \sin(\omega) + A_2 \cos^2(\omega) + A_0, \quad (3.2)
$$

differentiating Eq. [\(3.2\)](#page-3-1) twice, we get:

$$
U''(\omega) = B_1 \cos^2(\omega) \sin(\omega) - B_1 \sin^3(\omega) - 2A_1 \sin^2(\omega) \cos(\omega) + B_2 \cos^3(\omega) \sin(\omega) - 5B_2 \sin^3(\omega) \cos(\omega) - 4A_2 \cos^2(\omega) \sin^{\omega}(\omega) + 2A_2 \sin^4(\omega),
$$
\n(3.3)

Putting Eq. [\(3.2\)](#page-3-1) and [\(3.4\)](#page-4-0) into Eq. [\(3.1\)](#page-3-0), yields an equation in trigonometric functions. After making some trigonometric identities substitutions into the trigonometric functions equation, we collect a set of algebraic equations by setting each summation of the coefficients of the trigonometric functions of the same power to zero. We solve the set of equations with the aid of the symbolic software; Mathematica or Maple to get the various cases for the values of the coefficients. We insert the values of the coefficients for each case into Eq. [\(2.9\)](#page-2-3) along with the value $m = 2$, this gives us new solution to Eq. Eq. [\(1.1\)](#page-1-3).

Case-1:
\n
$$
A_0 = \frac{4}{c_0^2} (c_0^2 - k^2), A_1 = 0, B_1 = 0, A_2 = -\frac{6}{c_0^2} (c_0^2 - k^2), B_2 = 6 \left(\frac{k^2}{c_0^2} - 1 \right) i,
$$
\n
$$
p = \frac{1}{k^2 \mu^2} (k^2 - c_0^2).
$$

Case-2:
\n
$$
A_0 = 4\left(1 - \frac{1}{1 + p\mu^2}\right), A_1 = 0, B_1 = 0, A_2 = 6\left(\frac{1}{1 + p\mu^2} - 1\right), B_2 = \frac{6p\mu^2(p\mu^2 - 1)}{p^2\mu^4 - 1}i,
$$
\n
$$
c_0 = -k\sqrt{1 + p\mu^2}.
$$

Case-3:
\n
$$
A_0 = 6\left(\frac{k^2}{c_0^2} - 1\right), A_1 = 0, B_1 = 0, A_2 = 6\left(1 - \frac{k^2}{c_0^2}\right), B_2 = 6\left(1 - \frac{k^2}{c_0^2}\right)i,
$$

\n $\mu = -\frac{1}{k\sqrt{p}}(k^2 - c_0^2).$

Case-4:

$$
A_0 = \frac{1}{c_0^2} (c_0^2 - k^2), A_1 = 0, B_1 = 0, A_2 = -\frac{3}{c_0^2} (c_0^2 - k^2), B_2 = 0, p = \frac{k^2 - c_0^2}{4k^2\mu^2}.
$$

Case-5:

Case-6:
\n
$$
A_0 = 1 - \frac{1}{4p\mu^2 + 1}, A_1 = 0, B_1 = 0, A_2 = 3\left(\frac{1}{4p\mu^2 + 1} - 1\right), B_2 = 0,
$$
\n
$$
c_0 = k\sqrt{4p\mu^2 + 1}.
$$
\nCase-6:

$$
A_0 = 1 - \frac{k^2}{c_0^2}, A_1 = 0, B_1 = 0, A_2 = 3\left(\frac{k^2}{c_0^2} - 1\right), B_2 = 0, \mu = \frac{1}{2k\sqrt{p}}(k^2 - c_0^2)i.
$$

With case-1, we get the following solution;

$$
u_1(x,t) = \frac{(k^2 - c_0^2)}{c_0^2} \Big(6 + 6i \ sech[\mu(x - kt)]tanh[\mu(x - kt)] -6tanh^2[\mu(x - kt)]\Big).
$$
\n(3.4)

Figure 1: The 3D and 2D shape for the imaginary part of Eq. [\(3.4\)](#page-4-0) with the values $k = 2$, $c_0 = 1$, $\mu = 3$, $-3 < x < 3$, $-5 < t < 5$ and $t = 0$ for the 2D graphic.

Figure 2: The 3D and 2D shape for the real part of Eq. [\(3.4\)](#page-4-0) with the values $k = 2, c_0 = 1, \mu = 3, -13 < x < 13, -5 < t < 5$ and $t = 0$ for the 2D graphic.

With case-2, we get the following solution;

$$
u_2(x,t) = \left(1 - \frac{1}{1 + p\mu^2}\right) \left(4 - 6tanh^2[\mu(x - kt)]\right)
$$

+
$$
\frac{1}{p^2\mu^2 - 1} \left(6p\mu^2(p\mu^2 - 1)i \ sech[\mu(x - kt)]tanh[\mu(x - kt)]\right).
$$
 (3.5)

Figure 3: The 3D and 2D shape for the real part of Eq. [\(3.5\)](#page-5-0) with the values $k = 2, p = 1, \mu = 3, -5 < x < 8, 0 < t < 2$ and $t = 0$ for the 2D graphics.

With case-3, we get the following solution;

$$
u_3(x,t) = \frac{1}{c_0^2} (c_0^2 - k^2) \Big(-1 - i \sech\Big[\frac{1}{k\sqrt{p}} (k^2 - c_0^2)(x - kt)\Big] \times \tanh\Big[\frac{1}{k\sqrt{p}} (k^2 - c_0^2)(x - kt)\Big] + \tanh^2\Big[\frac{1}{k\sqrt{p}} (k^2 - c_0^2)(x - kt)\Big]\Big). \tag{3.6}
$$

Figure 4: The 3D and 2D shape for the real part of Eq. [\(3.6\)](#page-6-0) with the values $k = 2, p = 1, c_0 = 1, -5 < x < 5, 0 < t < 2$ and $t = 0$ for the 2D graphics.

With case-4, we get the following solution;

$$
u_4(x,t) = \frac{1}{c_0^2} \Big((c_0^2 - k^2) - 3(c_0^2 - k^2)tanh^2[\mu(x - kt)] \Big). \tag{3.7}
$$

Figure 5: The 3D and 2D shape for the real part of Eq. [\(3.7\)](#page-7-0) with the values $k=0.005,\,\mu=3,\,c_0=1,\,-1 < x < 1,\,0 < t < 2$ and $t=0$ for the 2D graphic.

With case-5, we get the following solution;

$$
u_5(x,t) = \frac{4p\mu^2}{1 + 4p\mu^2} \left(1 - 3tanh^2[\mu(x - kt)]\right).
$$
 (3.8)

Figure 6: The 3D and 2D shape for the real part of Eq. [\(3.8\)](#page-7-1) with the values $k=0.5,\, \mu=3,\, p=1,\, -0.5 < x < 1,\, 0 < t < 2$ and $t=0.7$ for the 2D graphic.

With case-6, we get the following solution;

$$
u_6(x,t) = -\frac{k^2 - c_0^2}{c_0^2} \left(1 + 3\tan^2 \left[\frac{\sqrt{k^2 - c_0^2}}{2k\sqrt{p}} (x - kt) \right] \right).
$$
 (3.9)

Figure 7: The 3D and 2D shape for the real part of Eq. [\(3.9\)](#page-8-0) with the values $k=2,\,c_0=1,\,p=1,\,-0.5 < x < 1,\,0 < t < 2$ and $t=0.7$ for the 2D graphic.

4 Discussion and Remarks

In [\[27\]](#page-12-0) the modified exp $(-\Omega(\xi))$ -expansion function method was developed and been utilized in solving the longitudinal wave equation in a magnetoelectro-elastic circular rod and various solutions in hyperbolic functions form were obtained. Secondly, the well-known modified (G'/G) -expansion method [\[29\]](#page-12-4) has been employed to this equation and some exact hyperbolic and trigonometric function were obtained. We observe that our results are new, but with the same solution structures when compared with the existing results obtained by using these two methods. On the other hand, we observe that in the numerical simulations of the solutions we presented; fig. 1, 2, 7 are singular soliton surfaces, fig. 3 is solitoff surface, fig. 4, 5, 6 are soliton surfaces. We observed that some solutions in this study have important physical meanings, like the hyperbolic tangent arises in the calculation of magnetic moment and rapidity of special relativity and the hyperbolic secant arises in the profile of a laminar jet [\[34\]](#page-12-9).

5 Conclusions

In this study, by utilizing the sine-Gordon expansion method with the help of Wolfram Mathematica 9, we investigated the solutions of the longitudinal wave equation in a magneto-electro-elastic circular rod. We obtained some new complex hyperbolic and trigonometric function solutions. All the obtained solutions in this study verified the longitudinal wave equation in a magnetoelectro-elastic circular rod, we checked this using the same program in Wolfram Mathematica 9. We performed the numerical simulations of all the obtained solutions in this article. We observed that our results might be helpful in detecting the transverse Poisson's effect in a magneto-electro-elastic circular rod. Sine-Gordon expansion method is powerful and efficient mathematical tool that can be used with the aid of symbolic software such as Maple or Mathematica in exploring search for the solutions of the various nonlinear equations arising in the various field of nonlinear sciences.

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