

Characterizing quantum phase transition by teleportation

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Abstract

In this paper we provide a novel way to explore the relation between quantum teleportation and quantum phase transition. We construct a quantum channel with a mixed state which is made from one dimensional quantum Ising chain with infinite length, and then consider the teleportation with the use of entangled Werner states as input qubits. The fidelity as a figure of merit to measure how well the quantum state is transferred is studied numerically. Remarkably we find the first-order derivative of the fidelity with respect to the parameter in quantum Ising chain exhibits a logarithmic divergence at the quantum critical point. The implications of this phenomenon and possible applications are also briefly discussed.

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I. INTRODUCTION

Quantum phase transition(QPT) is a prominent phenomenon caused by quantum fluctuations in a many-body system, reflecting the degeneracy of the ground states[1]. Unlike thermal phase transition which is caused by thermal fluctuations and can always be characterized by some order parameters due to the symmetry breaking, quantum phase transition is very hard to be diagnosed when the system is lack of classical order parameters or manifest symmetry breaking can not be found. In this circumstance it has been suggested that the entanglement may play a key role in characterizing quantum phase transition. Therefore, the critical behavior of some typical quantities which can be used to measure the degree of entanglement has been extensively investigated in literature, including the quantum concurrence[2], entanglement entropy[3] as well as the fidelity. In particular, the fidelity as a very crucial notion in quantum information science, which measures the quality of information transformation, has been widely used to investigate the occurrence of quantum phase transition [4–10]. Nevertheless, as far as we know, in all previous literature the fidelity used in this context is the Hilbert-Schmidt fidelity which is defined as the overlap between two pure quantum states $F(\lambda, \lambda + \delta\lambda) = |\langle\varphi(\lambda)|\varphi(\lambda + \delta\lambda)\rangle|$, where $|\varphi(\lambda)\rangle$ is a ground state of a many-body Hamiltonian $\hat{H}(\lambda)$, and λ is an external field parameter. Roughly speaking, in those papers the fidelity just measure the difference between two ground states when the system parameter is shifted from λ to $\lambda + \delta\lambda$, while the concept of information loss during the transmission as described in information science is absent in this context. Without surprise, in this setup one can find that the fidelity itself would exhibit extremal behavior at the critical point since two ground states at the critical point are orthogonal to each other in the thermodynamic limit, which is also known as the Anderson orthogonality catastrophe[4, 11].

In this paper we intend to provide a novel way to diagnose the occurrence of quantum phase transition by quantum teleportation. In contrast to the strategy as mentioned above, we will construct a specific quantum channel by picking up two qubits in a one dimensional quantum Ising chain with infinite length, and then consider the fidelity when a specific quantum state is teleported through this channel. Now, the fidelity becomes a figure of merit[12] to characterize the quality of transmission indeed. Usually the quantum channel is described by a mixed state with density matrix ρ_c and the fidelity is given by $F(\rho_{in}, \rho_{out}) = Tr(\sqrt{\sqrt{\rho_{in}}\rho_{out}\sqrt{\rho_{in}}})$, where ρ_{in} denotes the density matrix of input mixed state while ρ_{out}

corresponds to the density matrix of output.

Quantum teleportation was originally proposed by C.H. Bennett *et.al.* in 1993[13]. An unknown quantum state can successfully be transferred through a quantum channel which is made of a pure but entangled state, given that a classical information channel could also instruct local observers taking appropriate operations. Next applying arbitrary mixed state as the quantum channel associated with the standard teleportation protocol has been demonstrated in [14]. Later on a more specific scheme was proposed to teleport entangled Werner state[15] via thermally entangled states of two-qubit Heisenberg XX chain in [16]. Inspired by this scheme we will provide a novel way to construct the quantum channel with a quantum mixed state which is made from the ground state of the quantum Ising chain. It is this key point that makes it plausible to link quantum teleportation to quantum critical phenomenon in our paper. Thanks to the tensor network techniques recently developed in [17–21], we will numerically find the ground states of quantum Ising chain in terms of matrix product states(MPS), then construct the quantum channel by picking up two qubits which could be nearest neighboring or next-nearest neighboring to each other in the quantum Ising chain. By tracing out all the other qubits in the chain the quantum channel will be a mixed state described by a reduced-density matrix.

Our paper is organized as follows. In next section we will present the setup for the construction of quantum channel with MPS. Then in section III we will numerically calculate the entanglement entropy and the fidelity of the quantum channel when a Werner state is transferred. More importantly, we will demonstrate that the first order derivative of the fidelity to the system parameter will display a logarithmic divergence at the critical point. We conclude this paper with some discussion on the implications and possible applications of this phenomenon.

II. BASIC SETUP

A. The ground states of quantum Ising chain in terms of MPS

In this subsection we will present the setup for the quantum channel of teleportation. We start with the one-dimensional Ising model composed of an infinite spin chain, which is one of the simplest models in many-body physics and exactly solvable [22]. The Hamiltonian of

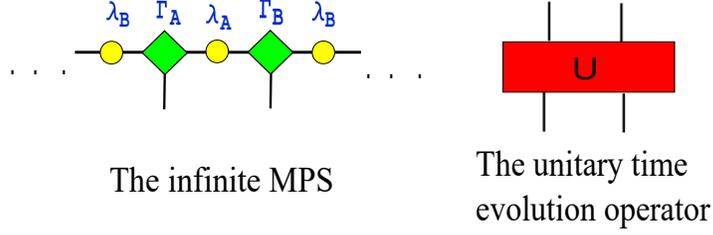


FIG. 1: The left is the infinite MPS, and the right is unitary time evolution operator $U = e^{-\hat{H}\delta\tau}$.

the quantum Ising chain considered in our paper is given by

$$\hat{H} = \sum_{j=1}^{\infty} \sigma_1^j \sigma_1^{j+1} + \lambda \sum_{j=1}^{\infty} \sigma_3^j. \quad (1)$$

which only involves the neighboring interactions of spins and $\sigma_1 = \sigma_x$, $\sigma_3 = \sigma_z$ are ordinary Pauli matrices.

The ground states of above quantum Ising chain with infinite length can be described by matrix product states(MPS) very efficiently [23]. For an MPS with infinite qubits, we will employ infinite time evolving block decimation(iTEBD) algorithm to simulate the ground states of quantum Ising chain [17][24]. This algorithm tells us that starting from any random MPS and performing an imaginary time evolution by acting the Hamiltonian operators on MPS, one could finally reach the ground state of the system provided that the time lasts long enough.

Next we demonstrate the algorithm of iTEBD in our paper briefly, closely following the logic presented in [17]. First, we construct the MPS with infinite length. Because the quantum Ising chain in Eq.(1) has \mathbb{Z}_2 symmetry, the infinite chain of MPS is only composed of two distinct pairs of tensors $\{\Gamma_A, \lambda_A, \Gamma_B, \lambda_B\}$ which could be viewed as the unit cells of the system, where λ_A, λ_B are diagonal matrices with non-negative diagonal elements, as shown in Fig.1. Second, we build the unitary time evolution operator $U = e^{-\hat{H}\delta\tau}$ with the use of the Hamiltonian in Eq.(1), where $\delta\tau$ is a tiny time step. Third, we perform the unitary operation by acting U on the infinite MPS and then contract them into a new tensor Θ , as illustrated in Fig.2. Fourth, singular value decomposition(SVD) is used to decompose Θ into individual tensors X and Y , as shown in Fig.2. Lastly, we contract X and Y using matrix λ_B^{-1} and obtain updated Γ_A and Γ_B , as shown in Fig.2. So far, we have finished the process of updating the unit cells of MPS except for λ_B . By exchanging λ_A and λ_B ,

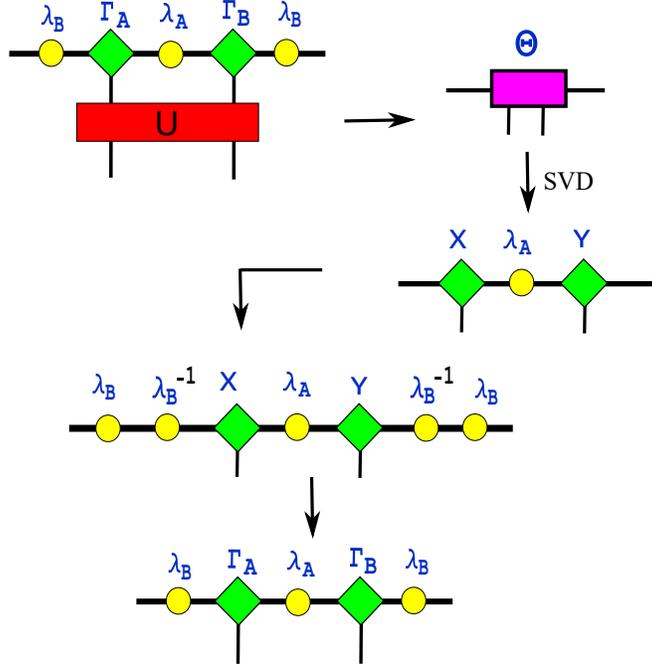


FIG. 2: The process of iTEBD algorithm.

we repeatedly perform above process until the ground state is reached within a precision setting.

B. Teleportation via mixed entangled states

In this subsection we will outline our strategy to construct a quantum channel for teleportation with ground states of quantum Ising chain. Our main purpose is to teleport a specific mixed state with a quantum channel which is also made from a mixed but entangled state. The standard teleportation protocol has previously been appeared in [14, 16, 25] and we will briefly review their setup as follows. In [14], it is originally shown that the standard teleportation with an arbitrary entangled mixed state χ_{AB} as quantum channel is equivalent to a generalized depolarizing channel $\Lambda(\chi_{AB})$ with probabilities given by the maximally entangled components of the quantum channel χ_{AB} , i.e.

$$\Lambda(\chi_{AB})\rho = \sum_i Tr[P_i\chi_{AB}]\sigma_i\rho\sigma_i, \quad (2)$$

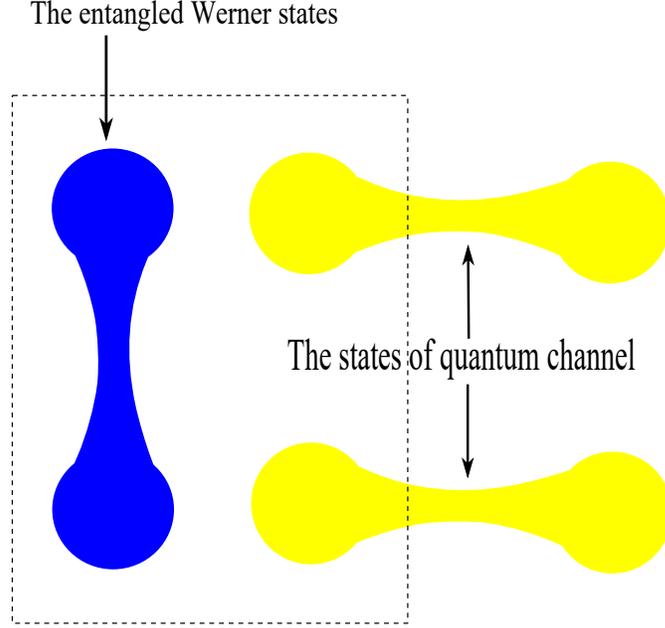


FIG. 3: The sketch of the quantum teleportation by taking entangled Werner state as input and using two copies of the mixed states as quantum channel.

where $P_i = \sigma_i P_0 \sigma_i$ ($i = 0, 1, 2, 3$) with $P_0 = |\Phi^+\rangle\langle\Phi^+|$ and $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. σ_0 is the identity matrix and $\sigma_1 = \sigma_x$, $\sigma_2 = \sigma_y$, $\sigma_3 = \sigma_z$ are Pauli matrices. ρ is the single qubit that we wish to teleport.

Now in this paper, we intend to teleport entangled Werner states with two qubits $\rho_W = \frac{1}{4}(\sigma_0 \otimes \sigma_0 - \frac{2\gamma+1}{3} \sum_{i=1}^3 \sigma_i \otimes \sigma_i)$, where $0 < \gamma \leq 1$. In paper[16], thermally entangled states of two-qubit Heisenberg XX chain are employed to construct the quantum channel. Given the Hamiltonian of two-qubit Heisenberg XX chain \hat{H} , one can write down the density matrix of the thermal entangled state as $\rho_c = \frac{1}{Z} e^{-\hat{H}/kT}$, where $Z = \text{Tr}(e^{-\hat{H}/kT})$ is the partition function, while T is the equilibrium temperature and k is Boltzmann constant. Now, taking entangled Werner state as the input and using two copies of the above thermal states as quantum channel, see Fig.3, the standard teleportation protocol tells us that the density matrix of the output by teleportation can be written as:

$$\rho_{out} = \sum_{i,j} \text{Tr}[(E^i \otimes E^j)(\rho_c \otimes \rho_c)](\sigma_i \otimes \sigma_j) \rho_{in}(\sigma_i \otimes \sigma_j), \quad (3)$$

where $E^0 = |\Psi^-\rangle\langle\Psi^-|$, $E^1 = |\Phi^-\rangle\langle\Phi^-|$, $E^2 = |\Phi^+\rangle\langle\Phi^+|$, $E^3 = |\Psi^+\rangle\langle\Psi^+|$, and $|\Phi^\pm\rangle =$

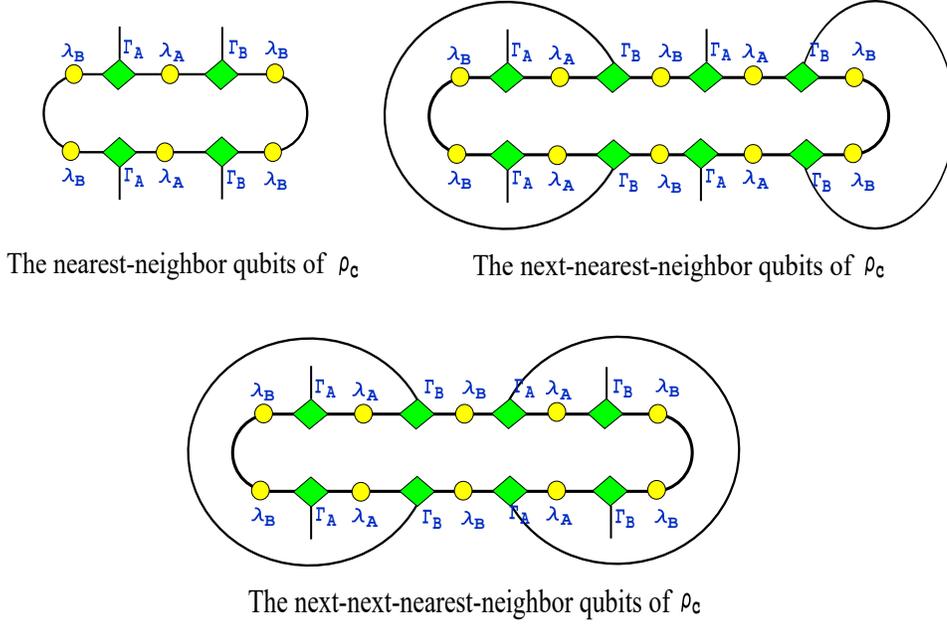


FIG. 4: The nearest-neighbor qubits, the next-nearest-neighbor qubits and the next-next-nearest-neighbor qubits of ρ_c .

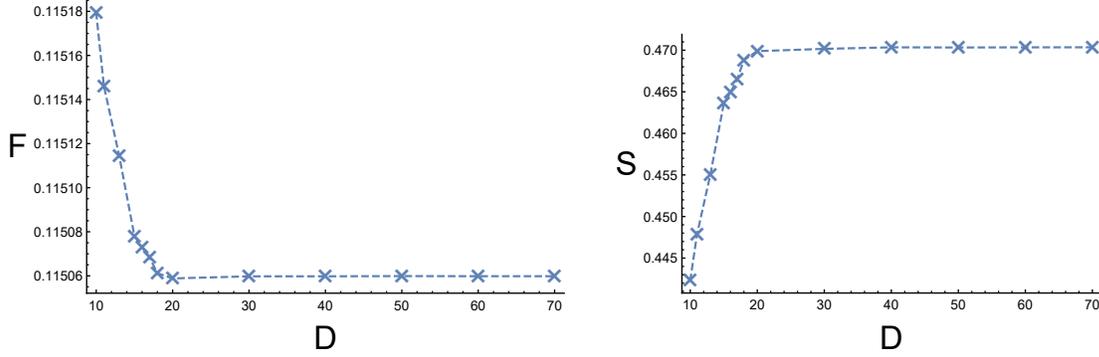


FIG. 5: The variations of the fidelity F and the entropy S with the truncation dimension D at the critical point for the nearest-neighbor qubits with $\gamma = 1$.

$$\frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), |\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle).$$

In our paper one significant change will be considered in order to investigate the relation between quantum phase transition and teleportation. Rather than employing a thermal state to construct the quantum channel, we will apply quantum mixed states of quantum Ising chain to construct the channel. Explicitly, given a ground state of quantum Ising chain in terms of MPS, we pick up two qubits which could be the nearest neighboring or next-

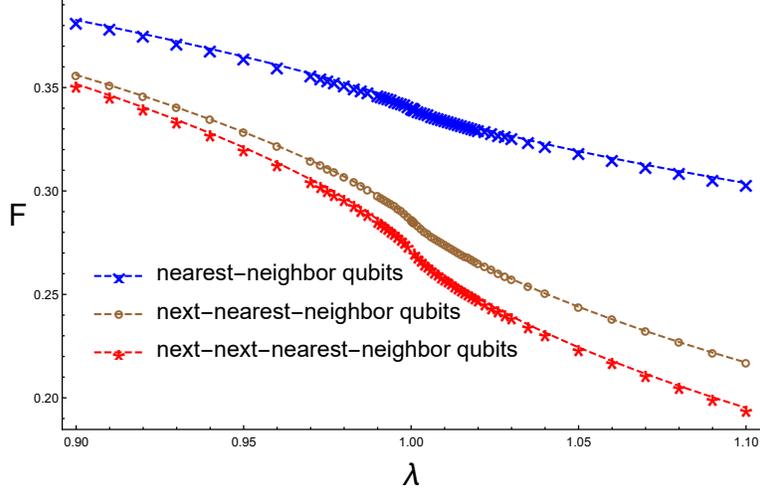


FIG. 6: The fidelity as functions of λ for the nearest-neighbor qubits, the next-nearest-neighbor qubits and the next-next-nearest-neighbor qubits with $\gamma = 1$.

nearest neighboring or next-next-nearest neighboring to form the quantum channel. Then by tracing out all other qubits of the density matrix, a reduced density matrix of this quantum mixed state ρ_c could be obtained for the quantum channel. We illustrate this process in Fig.4. Similarly, when we use entangled Werner state as input state ρ_{in} to teleport through this channel, an output state ρ_{out} with the same expression as in Eq.(3) can be obtained. The fidelity of this system now can be evaluated by the difference between ρ_{in} and ρ_{out} , which is given by:

$$F(\rho_{in}, \rho_{out}) = Tr \left(\sqrt{\sqrt{\rho_{in}} \rho_{out} \sqrt{\rho_{in}}} \right). \quad (4)$$

In next section we will report our numerical results about the fidelity with the variation of the system parameter λ , and disclose its critical behavior at the critical point of phase transition.

III. THE FIDELITY AT QUANTUM CRITICAL POINT

In this section we present the numerical results of fidelity with a focus on its behavior at the critical point of quantum Ising chain, which is located at $\lambda = 1$. In general, the fidelity $F(\rho_{in}, \rho_{out})$ depends on parameters λ and γ . As an example, in Fig.6 we demonstrate the fidelity as a function of λ for the nearest-neighbor qubits, the next-nearest-neighbor qubits

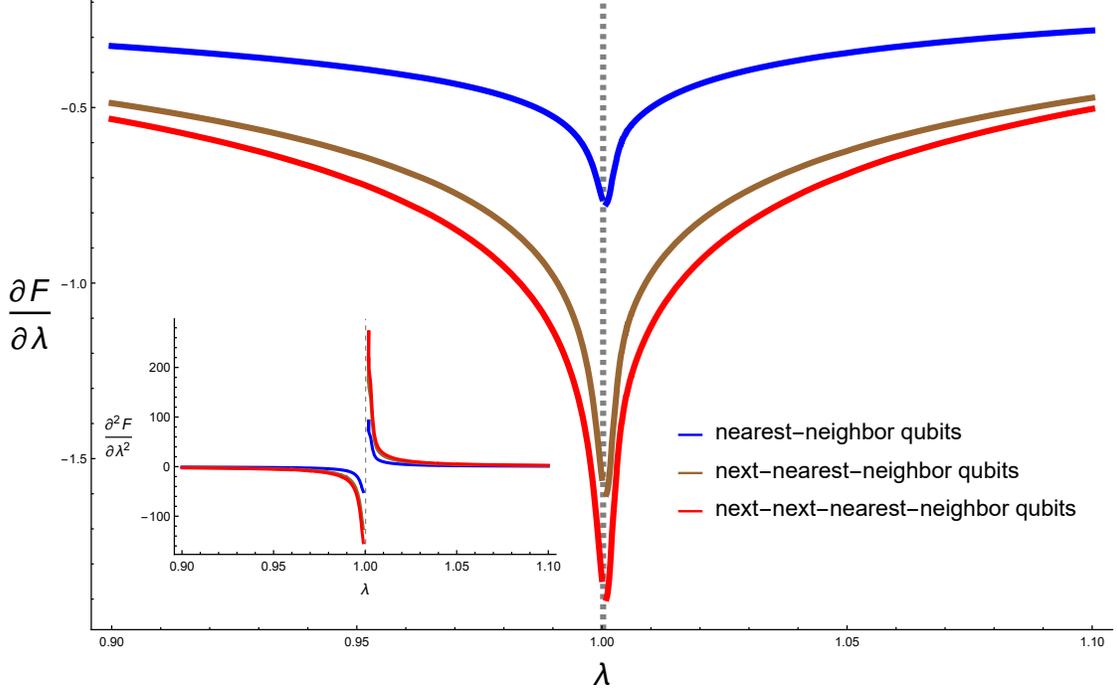


FIG. 7: The first order derivative of the fidelity $\frac{\partial F}{\partial \lambda}$ for the nearest-neighbor qubits, the next-nearest-neighbor qubits and the next-next-nearest-neighbor qubits with $\gamma = 1$.

and the next-next-nearest-neighbor qubits with $\gamma = 1$, respectively.

From Fig.5, we obviously observe that the fidelity F and the entropy S are convergent when the the truncation dimension of MPS D is large enough. So we remark that throughout this paper we fix the truncation dimension of MPS $D = 70$ and the numerics will not change with the increase of the truncation dimension. Firstly, it is interesting to notice that the fidelity is monotonously going down with the increase of the parameter λ , implying that more information of input qubits is missing with the increase of λ . This is not surprising because we know the fidelity of teleportation depends on quality of the quantum channel. When $\lambda \rightarrow 0$ the ground states of quantum Ising chain is dominantly determined by the interaction and the entanglement between the qubits of the channel is stronger, while when $\lambda \rightarrow \infty$ the ground state is dominantly determined by the second term of the Hamiltonian in Eq.(1) such that two qubits of the channel disentangled, leading to a vanishing fidelity of teleportation.

Next we notice that the fidelity goes down more quickly around the critical point regard-

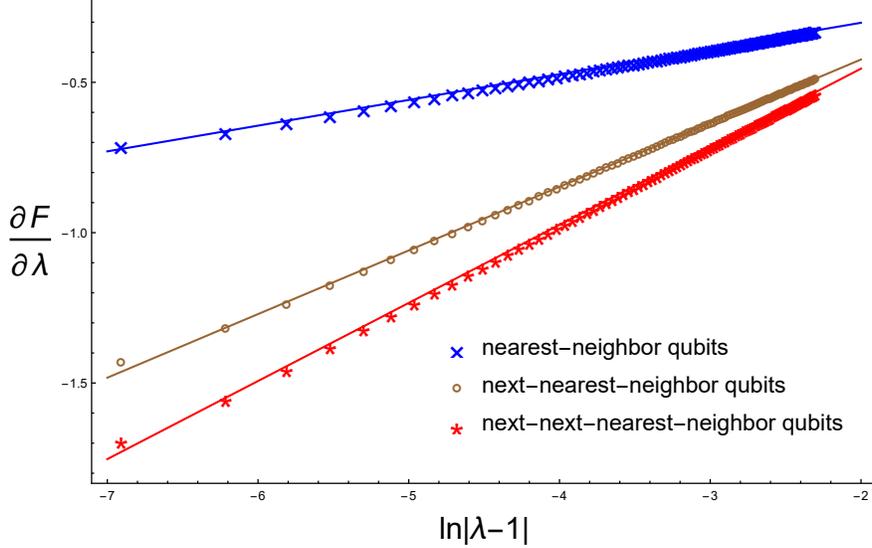


FIG. 8: The first order derivative of the fidelity $\frac{\partial F}{\partial \lambda}$ is in direct proportion to $\ln|\lambda-1|$ for the nearest-neighbor qubits, the next-nearest-neighbor qubits and the next-next-nearest-neighbor qubits with $\gamma = 1$.

less of whether two qubits of the channel are nearest neighboring or next-nearest neighboring or next-next-nearest neighboring in the chain. To describe this tendency more quantitatively we plot the first order derivative of the fidelity $\frac{\partial F}{\partial \lambda}$ near the critical point of the system in Fig.7. It is remarkable to observe that this quantity exhibits a divergent behavior at the critical point¹, which is the main results obtained in this paper. More precisely, we find that in critical region the first order derivative of the fidelity $\frac{\partial F}{\partial \lambda}$ is directly proportional to $\ln|\lambda-1|$, as illustrated in Fig.8. It implies that the quantity $\frac{\partial F}{\partial \lambda}$ has a logarithmic singularity at the critical point, which is just like the behavior of the concurrence as firstly disclosed in [26]. One could notice that the first order derivative of the fidelity for the nearest-neighbor qubits should more diverged at the critical point from Fig.7. We clarify that this phenomenon are caused by the limitation of computer precision. And we compute the second derivative of the fidelity near the critical point as shown in inset of Fig.7, and we find that it becomes discontinuous at the critical point. In addition, from Fig.8, we could see that when the value of λ tends to 1, i.e. $\ln|\lambda-1| \rightarrow -\infty$, the deviation between numerical values and the fitted

¹ One more evidence of this divergence is the the second-order derivative of the fidelity exhibits discontinuities at the critical point as shown in the insert of the Figure 7.

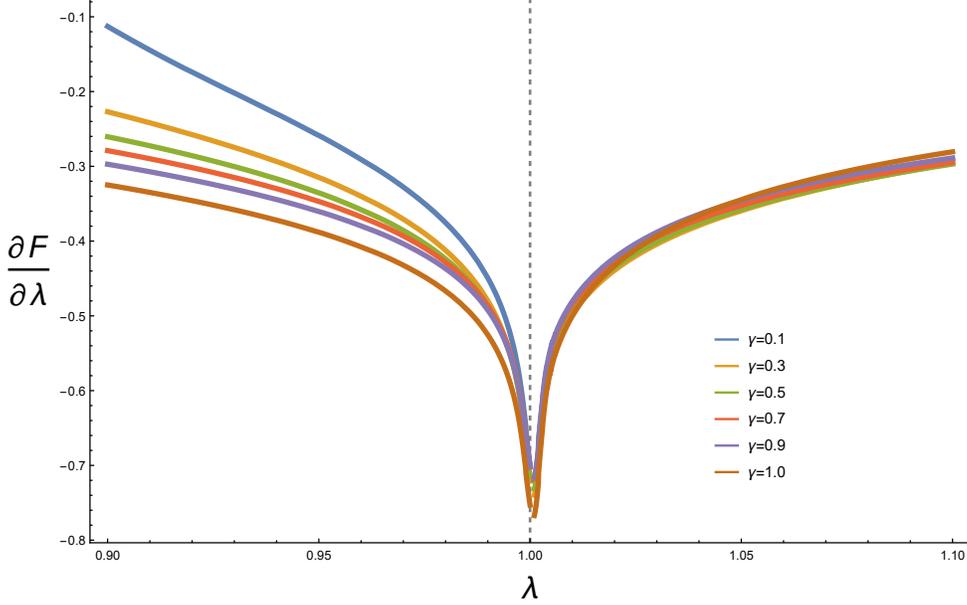


FIG. 9: The first order derivative of the fidelity $\frac{\partial F}{\partial \lambda}$ for nearest-neighbor qubits with different values of γ .

lines becomes more evident.

Finally, we argue that this phenomenon is general in the sense that it does not depend on the parameter γ in entangled Werner state. To show this we plot the first order derivative of the fidelity for nearest-neighbor qubits with different values of γ in Fig.9. The divergence of $\frac{\partial F}{\partial \lambda}$ can be understood from the perspective of entanglement. Basically, we know the efficiency of transmission through this quantum channel depends on two facets. One is the entanglement between these two qubits made of the channel, which may be called intrinsic entanglement; the other is the entanglement between the channel and the environment, which is composed of all other qubits in spin chain which have been traced out. We intend to call this external entanglement. Obviously, the stronger is the external entanglement, the lower is the transmission efficiency. Furthermore, the degree of external entanglement can be reflected by the entanglement entropy which is defined as $S = -Tr(\rho_c \ln \rho_c)$. It is well known that this quantity as well as its derivative displays a peak at the critical point, as illustrated in Fig.10. Therefore, when λ runs from zero to infinity, as a global tendency the fidelity is largely determined by the intrinsic entanglement and becomes smaller, while near the critical point the system undergoes the most prominent change with the parameter λ and the quality of the quantum channel is affected by this external entanglement with

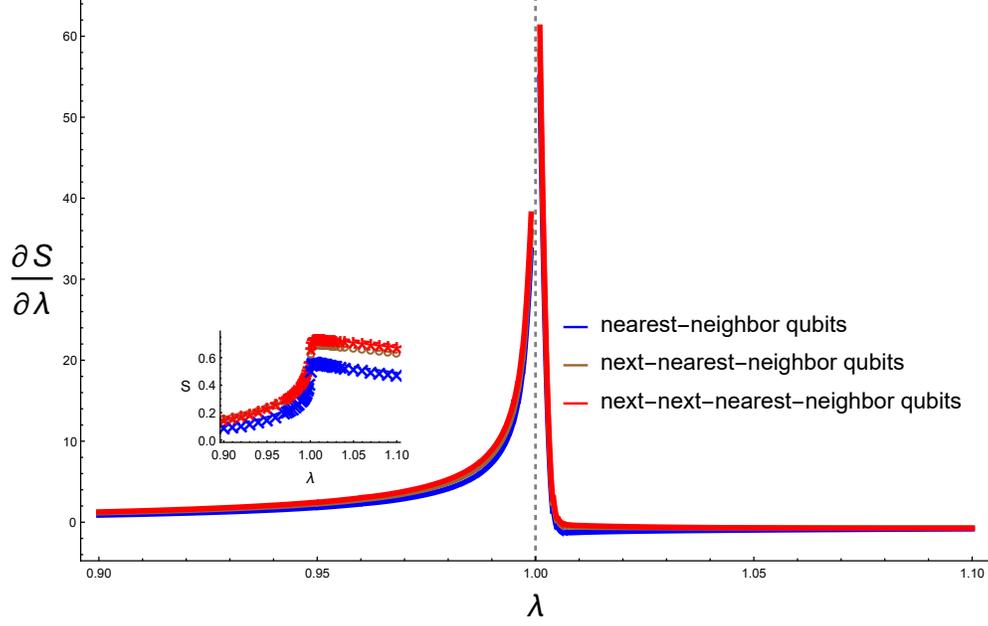


FIG. 10: The first order derivative of the external entanglement entropy $\frac{\partial S}{\partial \lambda}$ for the nearest-neighbor qubits, the next-nearest-neighbor qubits and the next-next-nearest-neighbor .

environment most severely. As a reflection, the fidelity of transmission goes down more quickly at the critical point, leading to the divergence of $\frac{\partial F}{\partial \lambda}$.

IV. CONCLUSION AND DISCUSSION

In this paper we have proposed a novel way to diagnose the quantum phase transition by constructing a quantum channel with mixed state for teleportation. In this circumstance we have found that the first-order derivative of fidelity exhibits a divergent behavior at the critical point. Firstly, we intend to stress that what we have observed may not be limited to the quantum Ising chain or the Werner state considered in this paper. Instead, the relations between the fidelity and quantum phase transition should be general and the experiment on teleportation may also play a key role in diagnosing quantum critical phenomenon. For instance, we may consider another state called X-state [27] as input state, whose form is given as $\rho_X = \frac{1}{4} (\sigma_0 \otimes \sigma_0 - \frac{2\gamma+1}{3} \sum_{i=1}^3 \sigma_i \otimes \sigma_i + \frac{2\epsilon+1}{3} (\sigma_1 \otimes \sigma_1 - \sigma_2 \otimes \sigma_2))$. For simplify, we also find the extremal behavior of $\frac{\partial F}{\partial \lambda}$ at the quantum critical point, as demonstrated in Fig.11. Secondly, in contrast to all the previous references on the relation between the fidelity

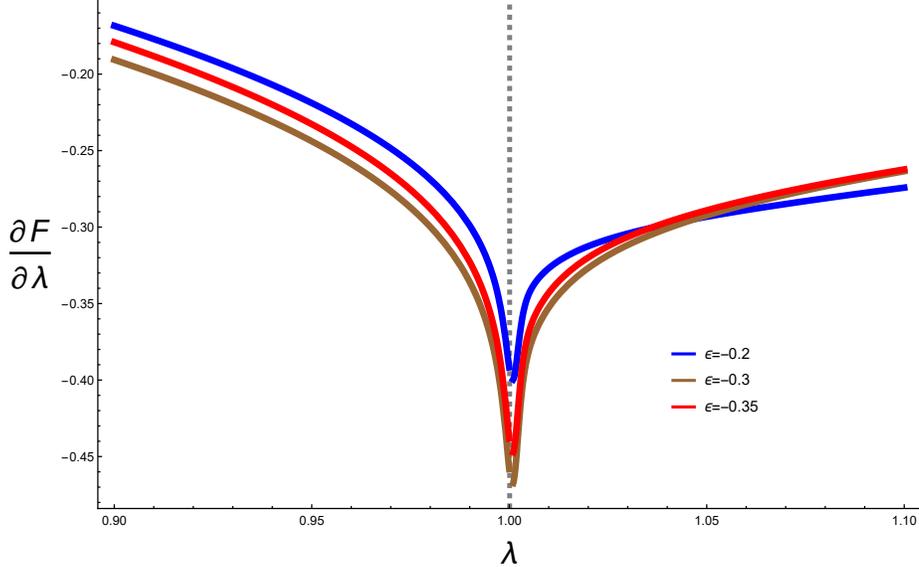


FIG. 11: The first order derivative of the fidelity $\frac{\partial F}{\partial \lambda}$ for the nearest-neighbor qubits with $\gamma = 0.6$ for different values of ϵ by using X-state as input state (truncation dimension $D = 70$).

and quantum phase transition, we have employed the fidelity to measure the information loss during the transmission rather than the Hilbert-Schmidt fidelity which only measures the difference between two ground states of a many body system. In this sense, our paper has paved a new bridge linking condensed matter physics to quantum information and may stimulate experimentalists to explore more exciting phenomena in laboratory such as the field of cold atoms.

Finally, remarkable progress has been made in recent years on the relation between condensed matter system and the geometry of space-time, in which the entanglement plays an essential role in describing the microscopic structure of space time and understanding the emergent signature of geometry[28–33]. However, it was pointed out by Susskind [34, 35] that entanglement maybe not enough and further quantum information quantities are needed in our understanding of holography. One recent effort is a conjectured duality[36] between fidelity susceptibility and the max volume of a codimension-one time slice in the Anti-de Sitter. The system under question is a priori conformal invariant at the critical point. This makes it very suitable for investigating its gravity dual of the involved fidelity within the framework of AdS/CFT correspondence. Moreover, the feature of the fidelity that we have found in this simple model may be helpful for us to investigate its role in the route of reconstructing geometry by holography.

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