False vacuum decay in kink scattering

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Abstract

In this work we consider kink-antikink and antikink-kink collisions in a modified ϕ^4 model with a false vacuum characterized by a dimensionless parameter ϵ . The usual ϕ^4 model is recovered for $\epsilon = 0$. We investigate the $\epsilon \ll 1$ regime where the kink in the presence of false vacuum can be understood as a small deformation of the standard kink for the ϕ^4 model. We show that the attractive interaction between the kink-antikink pair leads to a rich scattering pattern, in some cases delaying considerably the false vacuum decay.

PACS numbers: 04.60.Kz, 11.27.+d

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I. INTRODUCTION

False vacua play an important role in fundamental physics, in particular in the context of inflationary models [1–3], electroweak phase transition and baryogenesis [4–8], bubble collisions [9–15], signals for the possible metastability of Higgs vacuum [16–18] and dark energy models [19, 20]. Soliton solutions in nonlinear field theory describe localized energy concentrations which are able to propagate without modifying their shape. They have large applicability in condensed matter physics [21], optics [22], quantum field theory [23], nuclear physics [24–26] and cosmology [27–30]. The simplest soliton solutions are the kink and antikink in (1, 1)-dimensions.

Kink scattering processes in nonintegrable models have intrincate structures. This has been studied not only in the simple ϕ^4 model [31–40], but also in potentials of higher selfinteraction [41–48] and non-polynomial potentials [49–55]. Further interesting directions of investigation include the interaction of a kink with an impurity [56–58], kinks in models of two scalar fields [59–62], multi-kink collisions [63–65], boundary scattering [66, 67] and models with generalized dynamics [68].

Embedded in higher dimensional spaces, kinks give rise to domain walls and branes [69]. In some cyclic universe scenarios, domain walls were considered as a possibility to generate the Big Bang conditions [70–73], where planar symmetry is assumed and the dynamics is effectively (1, 1) dimensional. Solitons are also studied in the context of bubble collisions in the early universe. In the limit of a high nucleation rate per unit four-volume in comparison to H^4 (where H is the Hubble expansion rate), one can neglect the spacial curvature of the universe and consider the bubbles as in flat spacetime [74]. In this regime, collisions of two bubbles have SO(2, 1) symmetry and are described by a (1, 1) dimensional wave equation.

The mechanism of false vacuum decay in scalar field theories is well understood [75, 76]. In particular, for an asymmetric double well potential, domain wall collisions with planar and SO(2, 1) symmetry were considered in Ref. [77]. Domain wall collision with such high degree of symmetry can be described as collisions between a kink and an antikink in (1, 1) dimensions. In this background the effects of small initial planar [77] and nonplanar [78] small fluctuations were investigated. However, Ref. [77] only provided some examples for the output of background collisions, with no description of their dependence either with initial velocity or with the parameter controlling the difference in potential energy between

the two wells.

The present work intends to fill this gap, with the aim of investigating the kink scattering and the corresponding false vacuum evolution, considering the situation where the false vacuum initially occupies the restricted region between the kink-antikink pair and that where the false vacuum occupies the unbounded region outside the area defined by the antikink-kink system. We consider symmetric kink-antikink and antikink-kink collisions, with the pair initially at equal distances from the center of mass and initial velocities with same modulus. We show how the attractive interaction between the kink and antikink is the main factor affecting the structure of the scattering and, in some cases, delaying considerably the complete decay of the false vacuum even in a restricted region around the center of mass of the kink-antikink pair.

In the next section we review some of the main properties of the ϕ^4 model. The numerical results are presented in the Sect. III, and we conclude in the Sect. IV.

II. THE MODEL

We consider the action in (1, 1)-dimensions in Minkowski spacetime with a Lagrangian with standard dynamics

$$S = \int dt dx \left(\frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi - V(\phi)\right)$$
(1)

and a modified ϕ^4 potential

$$V(\phi) = \frac{\lambda}{4} \left(\phi^2 - \frac{M^2}{\lambda}\right)^2 - \frac{1}{2} \epsilon \frac{M^3}{\sqrt{\lambda}} \phi, \qquad (2)$$

where ϵ is a dimensionless self-coupling parameter. From here on we will consider $\epsilon \geq 0$ without loosing generality. For $\epsilon > 0$ the potential has a local minimum at $\phi = -1$ (the false vacuum), a local maximum at $\phi = 0$ and a global minimum at $\phi = 1$ (the true vacuum). Here we are interested in the $|\epsilon| \ll 1$ limit, where the presence of the linear term in Eq. (2) leads to a small force on the ϕ^4 kink that favors the expansion of the true vacuum region [81]. For sake of definiteness, we take $\lambda = M^2 = 2$. In this case equation of motion is given by

$$\ddot{\phi} - \phi'' - 2\phi + 2\phi^3 - \epsilon = 0 \tag{3}$$

Linear stability analysis around the static kink solution with $\phi(x,t) = \phi_K(x) + \cos(\omega t)\eta(x,t)$ leads to a Schrödinger-like equation:

$$-\eta'' + (-2 + 6\phi_K^2)\eta = \omega^2\eta.$$
 (4)

Static solutions can be found for $\epsilon = 0$, given by the usual ϕ^4 kink $\phi_K(x) = \tanh(x)$ and antikink $\phi_{\bar{K}}(x) = -\tanh(x)$. These solutions, of minimal energy, can also be obtained by first-order BPS equations [79, 80]. Stability analysis of the ϕ^4 kink have eigenvalues (and corresponding eigenfunctions) [31]: $\omega_0^2 = 0$ (the zero-mode or translational mode) and $\omega_1^2 = 3$ (the vibrational mode), followed by a continuum of states with $\omega_k^2 = 4 + k^2$. The existence of vibrational modes is important for the comprehension of the complex behavior of kinkantikink scattering. One such effect is known as two-bounces, and it is characterized by a collision process where the translational energy is stored in the kink-antikink pair during an amount of time. As a result, the pair, after being scattered, oscillates around the contact point and retrocede for a second collision. This mechanism was described in the Ref. [33] as an exchange of energy between the translational and vibrational modes. Despite applicable in the present case, we remark there are some known exceptions to this mechanism: in the ϕ^6 model, two-bounces occur even in the absence of a vibrational mode [41]; the presence of more than one vibrational mode can result in the suppression of two-bounce windows [51]. Also, quasinormal modes can store energy during a collision in the ϕ^4 model [40]. Now back to our problem for $\epsilon \neq 0$, as far as we know, there are no explicit solutions of the equation of motion. In particular, corrections ϕ_n to the kink field in *n*th order in ϵ were presented in Ref. [81], and are described in terms of the basis of eigenfunctions $\{\eta_n\}$.

III. NUMERICAL RESULTS

The kink scattering processes have a strong dependence on whether the initial region with false vacuum is finite or not. Then in the following we will consider separately kink-antikink and antikink-kink collisions.

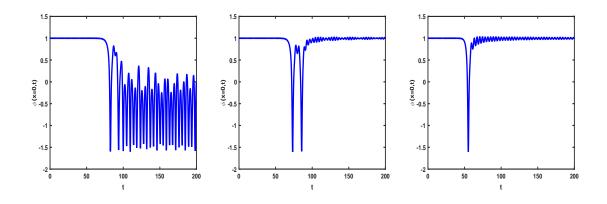


FIG. 1: False vacuum in an unbounded interval: kink-antikink collisions. Scalar field at the center of mass, $\phi(0, t)$, showing a) the formation of bion state (for v = 0.17), b) two-bounce collision (for v = 0.1926) and c) one-bounce collision (for v = 0.26). Results are for the ϕ^4 model, corresponding to $\epsilon = 0$.

A. False vacuum in an unbounded interval: kink-antikink collisions

First of all let us consider a symmetric kink-antinkink collision. We take as initial conditions a ϕ^4 kink with velocity v and a ϕ^4 antikink with velocity -v:

$$\phi(x,0) = \phi_K(x+x_0,v,0) - \phi_K(x-x_0,-v,0) - 1, \qquad (5)$$

$$\dot{\phi}(x,0) = \dot{\phi}_K(x+x_0,v,0) - \dot{\phi}_K(x-x_0,-v,0).$$
 (6)

We fixed $x_0 = 15$ as the initial kink position, i.e., the kink solution centered at $-x_0$ and the antikink at x_0 . Note that the initial condition is only a solution for $\epsilon = 0$. We apply this even for $\epsilon \ll 1$ considering that the system will relax by emitting scalar radiation during their free propagation toward the collision region. To solve Eq. (3) we use a pseudospectral method on a grid with 2048 nodes with periodic boundary conditions for ϕ and $\dot{\phi}$ and we set the grid boundary at $x_{max} = 200$. A sympletic method with the Dirichlet condition imposed at $x = \pm x_{max}$ was also applied to double check our numerical results. We used a 4th order finite-difference method with spatial step $\delta x = 0.09$ and a 6th order symplectic integrator with time step $\delta t = 0.04$.

We start our analysis by reviewing some aspects of the $\epsilon = 0$ case, corresponding to the ϕ^4 theory, and well described in the literature [31–40]. Figs. 1a-1c depict three profiles of scalar field at the center of mass as a function of time, $\phi(0, t)$, for different values of the initial velocity. Initially the point x = 0 is at the vacuum $\phi = 1$. For small velocities, the

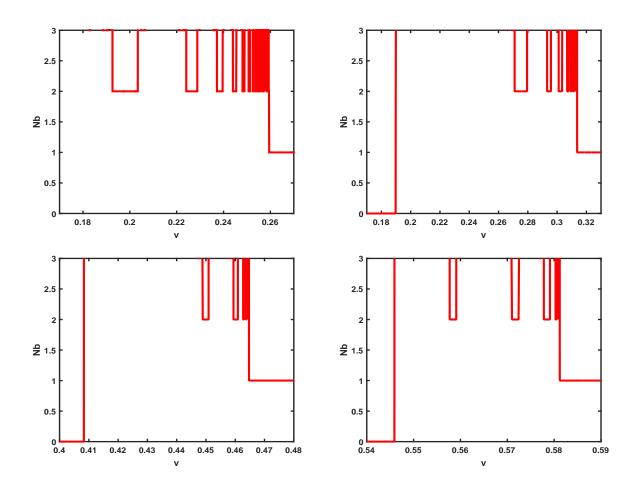


FIG. 2: False vacuum in an unbounded interval: kink-antikink collisions. Number of bounces versus initial velocity v for (a) $\epsilon = 0$, (b) $\epsilon = 0.001$, (c) $\epsilon = 0.005$ and (d) $\epsilon = 0.01$.

output of the scattering process is a bion state - see an example in Fig. 1a - where the pair radiates with the scalar field oscillating without a recognizable pattern until, in the long run, finishing at the vacuum $\phi = -1$. Velocities above a critical velocity $v_{crit} \sim 0.26$ show an inelastic scattering, where the output field retains the initial value $\phi = 1$ after one bounce, as in the example of Fig. 1c. We could characterize this inelastic scattering as having $N_b = 1$ bounces. For smaller velocities with $v \leq v_{crit}$ there are some velocity windows where the scalar field presents two-bounces (then $N_b = 2$) during the collision process, as shown in Fig. 1b.

The structure of such two-bounce windows is better visualized in Fig. 2a, which shows the number of bounces as a function of the initial velocity. As shown in Fig. 1a, bion states have a large number of oscillations of $\phi(x, 0)$. Large values of N_b are not directly represented in Fig. 2, but there one represent bion states as making frontier with the two-bounce windows.

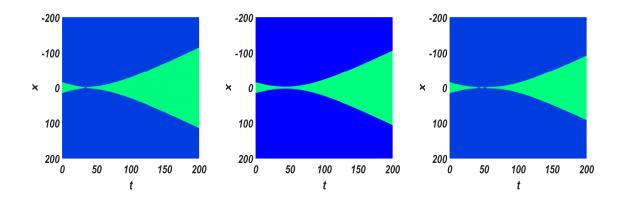


FIG. 3: False vacuum in an unbounded interval: kink-antikink collision for $\epsilon = 0.01$ with (a) v = 0.59, (b) v = 0.54 and (c) v = 0.558. Compare with diagram of Fig.2d. Blue represents the false vacuum $\phi = -1$ and the true vacuum $\phi = +1$ is represented in green.

Each two-bounce window is labeled by an integer m, the number of oscillations of $\phi(0,t)$ between the bounces. For instance, in Fig. 1b m = 1, meaning that this collision belong to the first two-bounce window from Fig. 2a.

Now we turn to the effect of ϵ on kink scattering. Figs. 2b-2d show the number of bounces as a function of the initial velocity for several values of $\epsilon \neq 0$. They show the inelastic scattering behavior with 1-bounce for $v > v_{crit}$, but with v_{crit} growing with ϵ . This is a direct effect of the tendency of the false vacuum to decay. One example of 1-bounce collision is presented in Fig. 3a. We observed an effect that appears only for $\epsilon \neq 0$, that is the lack of bounces or even bion states for $v < v^*$: the kink-antikink pair has not enought energy to encounter at the center of mass before the false vacuum decay.

An example of this effect is shown in Fig. 3b. Also, 2-bounce windows are still present, but are thinner in velocity for larger values of ϵ . See an example of this in Fig. 3c.

Another interesting aspect of kink-antikink collisions with the growing of ϵ is the appearing of a two-bounce window of zero order for $\epsilon \gtrsim 0.008$. An example of such effect is presented in Fig. 4. There one can see that there is no oscillations of $\phi(x = 0, t)$ between the bounces, meaning that m = 0. The appearing of zero order two-bounce windows was reported before in models with degenerate vacua in the context of boundary scattering [66] and in the study of transition.

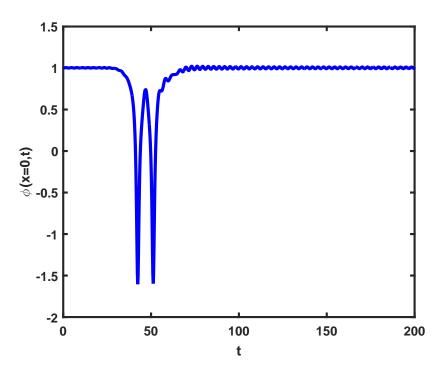


FIG. 4: False vacuum in an unbounded interval: kink-antikink collision showing a zero order (m = 0) two-bounce for $\epsilon = 0.01$ with v = 0.558.

B. False vacuum in a bounded interval: antikink-kink collisions

We take as initial condition a ϕ^4 antikink with velocity v and a ϕ^4 kink with velocity -v:

$$\phi(x,0) = -\phi_K(x+x_0,v,0) + \phi_K(x-x_0,-v,0) + 1, \tag{7}$$

$$\dot{\phi}(x,0) = -\dot{\phi}_K(x+x_0,v,0) + \dot{\phi}_K(x-x_0,-v,0).$$
(8)

In the context of false vacuum, the output for antikink-kink collisions is much simpler than the one described above for kink-antikink. This is due to the fact that both the kink-antikink interaction and the tendency of the false vacuum to decay favor a kink-antikink pair to form.

The output of mechanism is a two-step process: firstly the field bounces around the false vacuum with low frequency. Then the antikink-kink pair radiates and the field oscillates in a high-frequency, approaching in the long run to the true vacuum state. This is showed in Figs. 5a-5d for $\epsilon = 0.001$. Surprisingly larger initial velocities are not more effective in inducing the process of vacuum decay. On the contrary, with the growing of v the lowfrequency oscillation lasts even more before the decaying process to the false vacuum. It seems that, with false vacuum, larger initial velocities excite the vibrational state of the kink

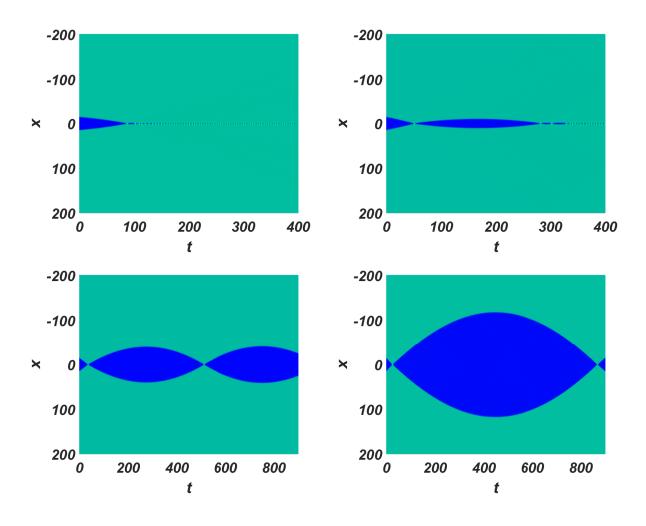


FIG. 5: False vacuum in a bounded interval: antikink-kink collision for $\epsilon = 0.001$ and with (a) v = 0.1 (upper left), (b) v = 0.25 (uper right), (c) v = 0.4 (lower left), (d) v = 0.6 (lower right). The figures show slices in the plane (x, t) of the scalar field. Blue represents the false vacuum $\phi = -1$ and the true vacuum $\phi = +1$ is represented in green.

and antikink in a way that does not occurs for the degenerate vacuum, and the antikinkkink pair can bounce several times and be even more departed during such low-frequency oscillations (compare Figs. 5a-5b).

The relation between high- and low-frequency oscillations is more evident in a plot of the scalar field at the center of mass, $\phi(0, t)$, as shown in Figs. 6a-b. There one can see that for a too low initial velocity the field is not even able to bounce and the high-frequency process already takes place, as can be see in Fig. 6a for v = 0.1. As an example for larger velocities we take v = 0.25 as in Fig. 6b. There one can see three low-frequency oscillations around the false vacuum $\phi = -1$ before the field jumps and acquires the high-frequency

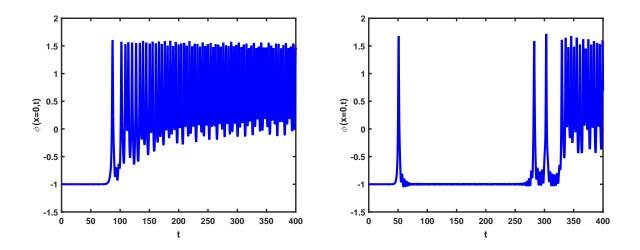


FIG. 6: False vacuum in a bounded interval: antikink-kink collision. Scalar field at the center of mass, $\phi(0, t)$ as a function of time for $\epsilon = 0.001$ with (a) v = 0.1 and (b) v = 0.25, corresponding to Figs. 5a-b

oscillations that approach the true vacuum $\phi = +1$. As seen in Figs. 5c-5d, a too larger initial velocity would require a much longer running time of simulations to see the final highfrequency decay of the false vacuum. For the values $x_0 = 15$ considered in the simulations, the pattern of collisions described here is valid up to $\epsilon \sim 5 \times 10^{-3}$. Larger values of ϵ leads to a non-recognizable pattern, meaning that the nonlinear coupling for antikink-kink collisions is already too large.

IV. CONCLUSIONS

In this work we have studied the effect of kink scattering in the false vacuum decay. Kinkantikink collisions are characterized by two critical velocities, v^* (not present for degenerate vacua) and v_{crit} . For $0 < v < v^*$ we have inelastic scattering of the pair without contact. For $v^* < v < v_{crit}$ the scalar field at the center of mass $\phi(x, 0)$ can show bion or two-bounce around the true vacuum. In a diagram in velocities we have the usual structure of bion states and a sequence of two-bounce windows accumulating around v_{crit} . For $\epsilon \gtrsim \bar{\epsilon}$ there is the appearance of zero-order two-bounce windows. For $v > v_{crit}$ we have inelastic scattering, and $\phi(x, 0)$ presents one-bounce around the true vacuum. Larger choices of initial position of the kink-antikink pair up to $x_0 = 25$ mean more time for the false vacuum to decay during the free propagation of the kink-antikink pair, leading to an increasing of v^* and v_{crit} We found that it is relatively easier for a finite region in a true vacuum to expand through the false vacuum region, as described here in kink-antikink collisions. This is evident for low initial velocities, where the kink-antikink pair scatters at non-null distance. In comparison, it demands considerably more time for a finite region in false vacuum to decay completely to the true vacuum. We showed this for antikink-kink collisions. That is, the attractive kinkantikink interaction favors the formation of a long-lived bion state that inhibits the vacuum decay. The antikink-kink collisions were described in a two-step process with a low-frequency oscillation around the false vacuum followed by a high-frequency oscillation tending to the true vacuum in the long run. This two-step process resembles the behavior of what is known in the literature as false n-bounce-windows. In the end such bounce windows must be false since the scalar field will not remain in the initial state. We also investigated the effect of increasing the gap parameter ϵ . For kink-antikink collisions we have the increasing of both velocities v^* and v_{crit} .

For antikink-kink collisions the center of mass belongs to the false vacuum region and we have a two-step process for decay of the false vacuum: a low-frequency fast decaying bouncing state, followed by another long-lived high frequency bion state that radiates continuously. For antikink-kink collisions, the more notable effect of the increasing of ϵ is the reduction of the maximum separation of the antikink-kink pair during the low-frequency oscillations. This shows the stronger tendency of the false vacuum to decay. The final state is the same, with the field oscillating fastly around the true vacuum and radiating.

V. ACKNOWLEDGEMENTS

The authors thank FAPEMA and CNPq (brazilian agencies) for financial support. Gomes thanks CAUP from University of Porto for the hospitality. P.P.A. acknowledges the support provided by the FCT grant UID/FIS/04434/2013.

J. Garcia-Bellido, M. Garcia Perez, A. Gonzalez-Arroyo, Symmetry breaking and false vacuum decay after hybrid inflation, Phys. Rev. D 67, 103501 (2003);

^[2] H. Firouzjahi, S. Jazayeri, A. Karami, T. Rostami, Bubble nucleation and inflationary perturbations, JCAP 1712 (2017) 029;

- [3] R. Bousso, D. Harlow, L. Senatore, Inflation after false vacuum decay: Observational prospects after Planck, Phys. Rev. D 91, 083527 (2015);
- [4] M. Dine, R.G. Leigh, P. Huet, A. Linde, D. Linde, Towards the theory of the electroweak phase-transition, Phys. Rev. D 46, 550 (1992);
- [5] G.W. Anderson, L.J. Hall, *Electroweak phase-transition and baryogenesis*, Phys. Rev. D 45, 2685 (1992);
- [6] R. Apreda, M. Maggiore, A. Nicolis, A. Riotto, Gravitational waves from electroweak phase transitions, Nucl. Phys. B 631, 342 (2002);
- [7] A. Kobakhidze, A. Spencer-Smith, *Electroweak vacuum (in)stability in an inflationary uni*verse, Phys. Lett. B 722, 130 (2013);
- [8] L. Leitao, A. Megevand, A. D. Sanchez, Gravitational waves from the electroweak phase transition, JCAP10(2012)024.
- [9] A. Kosowsky, M.S. Turner, R. Watkins, Gravitational-radiation from colliding vacuum bubbles, Phys. Rev. D 45, 4514 (1992);
- [10] E.J. Copeland, M. Gleiser, H.R. Muller, oscillons resonant configurations during bubble collapse, Phys. Rev. D 52, 1920 (1995);
- [11] M. Kleban, Cosmic Bubble Collisions, Class. Quant. Grav. 28 (2011) 204008
- [12] A. Aguirre, M. C. Johnson, A status report on the observability of cosmic bubble collisions, Rept.Prog.Phys.74:074901,2011.
- [13] J.R. Bond, J. Braden, L. Mersini-Houghton, Cosmic bubble and domain wall instabilities III: the role of oscillons in three-dimensional bubble collisions, JCAP 1509 (2015) 004.
- [14] Matthew C. Johnson, Carroll L. Wainwright, Anthony Aguirre and Hiranya V. Peiris, Simulating the universe(s) III: observables for the full bubble collision spacetime, JCAP07(2016)020.
- [15] R. Jinno, S. Lee, H. Seong, M. Takimoto, Gravitational waves from first-order phase transitions: Towards model separation by bubble nucleation rate, JCAP 1711 (2017) 050.
- [16] A. Kusenko, P. Langacker, G. Segre, Phase transitions and vacuum tunneling into charge- and color-breaking minima in the MSSM, Phys. Rev. D 54, 5824 (1996).
- [17] G. Isidori, G. Ridolfi, A. Strumia, On the metastability of the Standard Model vacuum, Nucl. Phys. B 609, 387 (2001)
- [18] A. Rajantie, S. Stopyra, Standard Model vacuum decay with gravity, Phys. Rev. D 95, 025008 (2017).

- [19] R.G. Landim and E. Abdalla, *Metastable dark energy*, Phys. Lett. B 764, 271 (2017).
- [20] P.P. Avelino, C.J.A.P. Martins and L. Sousa, Dynamics of Biased Domain Walls and the Devaluation Mechanism, Phys. Rev. D 78 (2008) 043521.
- [21] A. R. Bishop and T. Schneider (eds.), Solitons and Condensed Matter Physics, Springer-Verlag, Berlin, 1978.
- [22] Yu. S. Kivschar and G. P. Agrawal, Optical Solitons From Fiber to Photonic Crystals, Academic Press, Burlington (2003).
- [23] N. Manton and P. Sutcliffe, *Topological Solitons*, Cambridge University Press, Cambridge, England, 2004.
- [24] M. C. Birse, Soliton models for nuclear physics, Progr. in Part. and Nucl. Phys. 25, 1 (1990).
- [25] A. Drago, V. M. Sarti, A chiral quark-soliton model with broken scale invariance for nuclear matter, Phys.Rev. C 86 (2012) 015211.
- [26] S. Baldino, S. Bolognesi, S. B. Gudnason, D. Koksal, Solitonic approach to holographic nuclear physics, Phys.Rev. D 96 (2017) 034008.
- [27] A. Vilenkin and E. P. S. Shellard, Cosmic Strings and Other Topological Defects, Cambridge University Press, Cambridge, England, 1994.
- [28] A. Aguirre and M. C. Johnson, A status report on the observability of cosmic bubble collisions, Rep. Prog. Phys. 74 (2011) 074901.
- [29] P.P. Avelino and R.P.L. Azevedo, Perfect fluid Lagrangian and its cosmological implications in theories of gravity with non-minimally coupled matter fields, arXiv:1802.04760.
- [30] P.P. Avelino and L. Sousa, On the matter Lagrangian of particles and fluids, arXiv:1802.03961.
- [31] Sugiyama, Kink-Antikink Collisions in the Two-Dimensional ϕ^4 Model, Prog. Theor. Phys. 61 (1979) 1550.
- [32] M. Moshir, Soliton-antisoliton scattering and capture in ϕ^4 theory, Nucl. Phys. B 185 (1981) 318
- [33] D. K. Campbell, J. S. Schonfeld, C. A. Wingate, Resonance Structure in Kink-antikink interactions in φ⁴ theory, Physica. D 9, 1 (1983).
- [34] C. A. Wingate, Numerical Search for a ϕ^4 Breather Mode, SIAM J. Appl. Math. 43(1), 120-140 (1983).
- [35] D.K. Campbell, Solitary wave collisions revisited, Physica D 18 (1986) 47.
- [36] T.I. Belova and A.E. Kudryavtsev, Quasi-periodic orbits in the scalar classical ϕ^4 field theory,

Physica D 32 (1988) 18.

- [37] P. Anninos, S. Oliveira, R. A. Matzner, Fractal structure in the scalar $\lambda(\phi^2 1)^2$ theory, Phys. Rev. D 44, 1147 (1991).
- [38] R. H. Goodman, R. Haberman, Kink-Antikink Collisions in the φ⁴ Equation: The n-Bounce Resonance and the Separatrix Map, SIAM J. Appl. Dyn. Syst. 4, 1195 (2005).
- [39] D. Saadatmand, S. V. Dmitriev, D. I. Borisov, P. G. Kevrekidis, M. A. Fatykhov, K. Javidan, Effect of the φ⁴ kinks internal mode at scattering on a PT-symmetric defect, Pisma Zh. Eksp. Teor. Fiz. 101 (2015) 550 [JETP Lett. 101 (2015) 497].
- [40] Patrick Dorey and Tomasz Romanczukiewicz, Resonant kink-antikink scattering through quasinormal modes, J. Phys Let B (2018) 02 003.
- [41] P. Dorey, K. Mersh, T. Romanczukiewicz, Ya. Shnir, Kink-antikink collisions in the φ⁶ model, Phys. Rev. Lett. 107 (2011) 091602.
- [42] A. Demirkaya, R. Decker, P. G. Kevrekidis, I. C. Christov, A. Saxena, Kink Dynamics in a Parametric φ⁶ System: A Model With Controllably Many Internal Modes, JHEP 12(2017)071.
- [43] V.A. Gani, A.E. Kudryavtsev and M.A. Lizunova, Kink interactions in the (1+1)-dimensional φ⁶ model, Phys. Rev. D 89 (2014) 125009.
- [44] H. Weigel, Kink-Antikink Scattering in ϕ^4 and ϕ^6 Models, J. Phys.: Conf. Ser. 482 (2014) 012045.
- [45] T. Romanczukiewicz, Could the primordial radiation be responsible for vanishing of topological defects?, Phys. Lett. B 773 (2017) 295.
- [46] E. Belendryasova and Vakhid A. Gani, Resonance phenomena in the ϕ^8 kinks scattering, J. Phys. Conf. Ser. 934 (2017), 012059.
- [47] V.A. Gani, V. Lensky and M.A. Lizunova, Kink excitation spectra in the (1+1)-dimensional ϕ^8 model, JHEP 08 (2015) 147.
- [48] Ekaterina Belendryasova and Vakhid A. Gani, Scattering of the ϕ^8 kinks with power-law asymptotics, arXiv:1708.00403.
- [49] M. Peyrard, D. K. Campbell, Kink-antikink interactions in a modified sine-Gordon model, Physica D Nonlinear Phenomena 9 (1983) 3351.
- [50] V.A. Gani and A.E. Kudryavtsev, Kink-antikink interactions in the double sine-Gordon equation and the problem of resonance frequencies, Phys. Rev. E 60 (1999) 3305 [cond-mat/9809015].

- [51] F.C. Simas, Adalto R. Gomes, K.Z. Nobrega, J.C.R.E. Oliveira, Suppression of two-bounce windows in kink-antikink collisions, JHEP 1609 (2016) 104.
- [52] F.C. Simas, A.R. Gomes, K.Z. Nobrega, Degenerate vacua to vacuumless model and kinkantikink collisions, Phys. Lett. B 775 (2017) 290.
- [53] V. A. Gani, A. M. Marjaneh, A. Askari, E. Belendryasova and D. Saadatmand, Scattering of the double sine-Gordon kinks, arXiv:1711.01918.
- [54] D. Bazeia, E. Belendryasova, Vakhid A. Gani, Scattering of kinks in a non-polynomial model,
 J. Phys. Conf. Ser. 934 (2017) 012032.
- [55] D. Bazeia, E. Belendryasova, Vakhid A. Gani, Scattering of kinks of the sinh-deformed φ⁴ model, arXiv:1710.04993.
- [56] R. H. Goodman, R. Haberman, Interaction of sine-gordon kinks with defects: the two-bounce resonance, Physica D 195 (3) (2004) 303.
- [57] Z. Fei, Y. S. Kivshar, L. Vazquez, Resonant kink-impurity interactions in the sine-gordon model, Phys. Rev. A 45 (1992) 60196030.
- [58] M. I. W. Roy H Goodman, Philip J Holmes, Interaction of sine-Gordon kinks with defects: phase space transport in a two-mode model, Physica D 161 (2002) 21.
- [59] A. Halavanau, T. Romanczukiewicz, Ya. Shnir, Resonance structures in coupled twocomponent ϕ^4 model, Phys. Rev. D86 (2012) 085027.
- [60] A. Alonso-Izquierdo, Reflection, transmutation, annihilation and resonance in two-component kink collisions, Phys.Rev. D97 (2018) no.4, 045016.
- [61] A. Alonso-Izquierdo, Kink dynamics in a system of two coupled scalar fields in two space-time dimensions, Physica 365 (2018) 12-26.
- [62] Vakhid A. Gani, Alexander A. Kirillov, Sergey G. Rubin, Transitions between topologically non-trivial configurations, J. Phys. Conf. Ser. 934 (2017) no.1, 012046.
- [63] A. M. Marjaneh, V. A. Gani, D. Saadatmand, S. V. Dmitriev and K. Javidana, Multi-kink collisions in the φ⁶ model, JHEP 07 (2017) 028.
- [64] A. M. Marjaneh, A. Askari, D. Saadatmand, S. V. Dmitriev, Extreme values of elastic strain and energy in sine-Gordon multi-kink collisions, Eur. Phys. J. B (2018) 91: 22.
- [65] D. Saadatmand, S.V. Dmitriev and P.G. Kevrekidis, High energy density in multisoliton collisions, Phys. Rev. D 92 (2015) 056005.
- [66] P. Dorey, A. Halavanau, J. Mercer, T. Romanczukiewicz, Y. Shnir, Boundary scattering in

the ϕ^4 model, JHEP 1705 (2017) 107.

- [67] R. Arthur, P. Dorey, R. Parini, Breaking integrability at the boundary: the sine-gordon model with robin boundary conditions, Journ. of phys. A: mathematical and theoretical. 49 (16) (2016) 165205.
- [68] A.R. Gomes, R. Menezes, K.Z. Nobrega, F.C. Simas, Kink-antikink collisions for twin models, Phys.Rev. D90 (2014)065022.
- [69] L. Sousa and P.P. Avelino, p-brane dynamics in (N+1)-dimensional FRW universes: a unified framework, Phys. Rev. D 83 (2011) 103507.
- [70] J. Khoury, B.A. Ovrut, P.J. Steinhardt, N. Turok, The ekpyrotic universe: colliding branes and the origin of the hot big bang, Phys. Rev. D 64 (2001) 123522.
- [71] Yu-ichi Takamizu, Kei-ichi Maeda, Collision of domain walls and reheating of the brane universe, Phys. Rev. D 70 (2004) 123514.
- [72] Paul M. Saffin, Anders Tranberg, Particle transfer in braneworld collisions, J.High Energy Phys. 08 (2007) 072.
- [73] J. Omotani, P. M. Saffin, and J. Louko, *Colliding branes and big crunches*, Phys.Rev. D84 (2011) 063526.
- [74] S.W. Hawking, I.G. Moss and J.M. Stewart, Bubble Collisions in the Very Early Universe, Phys. Rev. D 26 (1982) 2681.
- [75] I. Kobzarev, L. Okuna and M. Voloshin, *Bubbles in metastable vacuum*, Sov. J. Nucl. Phys. 20 644 (1975) [Yad. Fiz. 20, 1229 (1974)].
- [76] S. Coleman, Fate of a false vacuum: Semiclassical theory, Phys. Rev. D 15, 2929 (1977).
- [77] J. Braden, J.R. Bond, L. Mersini-Houghton, Cosmic bubble and domain wall instabilities I: parametric amplification of linear fluctuations, J. Cosmol. Astropart. Phys. 03(2015)007.
- [78] J. Braden, J.R. Bond, L. Mersini-Houghton, Cosmic bubble and domain wall instabilities II: Fracturing of colliding walls, J. Cosmol. Astropart. Phys. 08(2015)048.
- [79] E. B. Bogomolny'i, The stability of classical solutions, Sov. J. Nucl. Phys. 24, 449 (1976).
- [80] M. K. Prasad, C. M. Sommerfield, Exact Classical Solution for the 't Hooft Monopole and the Julia-Zee Dyon, Phys. Rev. Lett. 35, 760 (1975).
- [81] V. G. Kiselev and Ya. M. Shnir, Forced topological nontrivial field configurations, Phys. Rev. D 57, 5174 (1998).