

# Cosmology in scalar-vector-tensor theories

Lavinia Heisenberg<sup>1</sup>, Ryotaro Kase<sup>2</sup>, and Shinji Tsujikawa<sup>2</sup>

<sup>1</sup>*Institute for Theoretical Studies, ETH Zurich, Clausiusstrasse 47, 8092 Zurich, Switzerland*

<sup>2</sup>*Department of Physics, Faculty of Science, Tokyo University of Science,  
1-3, Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan*

(Dated: July 27, 2018)

We study the cosmology on the Friedmann-Lemaître-Robertson-Walker (FLRW) background in scalar-vector-tensor theories with a broken  $U(1)$  gauge symmetry. For parity-invariant interactions arising in scalar-vector-tensor theories with second-order equations of motion, we derive conditions for the absence of ghosts and Laplacian instabilities associated with tensor, vector, and scalar perturbations at linear order. This general result is applied to the computation of the primordial tensor power spectrum generated during inflation as well as to the speed of gravity relevant to the late-time cosmic acceleration. We also construct a concrete inflationary model in which a temporal vector component  $A_0$  contributes to the dynamics of cosmic acceleration besides a scalar field  $\phi$  through their kinetic mixings. In this model, we show that all the stability conditions of perturbations can be consistently satisfied during inflation and subsequent reheating.

## I. INTRODUCTION

Despite the tremendous progress of observational cosmology over the past two decades, there are several unsolved issues in theoretical cosmology. The observations of Cosmic Microwave Background (CMB) [1] and supernovae type Ia [2] have shown that our Universe exhibited two stages of cosmic acceleration: inflation and dark energy. Moreover, we know that dark matter played a crucial role for the large-scale structure formation [3]. The existing problems of inflation, dark energy, and dark matter imply that there may be some extra degrees of freedom (DOFs) beyond the paradigms of standard model of particle physics and General Relativity (GR) [4].

A scalar field  $\phi$  can be a natural candidate for addressing such problems. In theories aiming to unify quantum field theory and GR, the scalar field can generally have direct couplings to gravity. A dilaton field arising in string theory is one of such examples, in which case there is a nonminimal coupling of the form  $F(\phi)R$  with the Ricci scalar  $R$  [5]. One can also consider a derivative interaction in which the field kinetic energy  $-\partial_\mu\phi\partial^\mu\phi/2$  is directly coupled to  $R$  [6]. In such cases, however, the theories generally contain derivatives higher than second order, so they are plagued by the problem of so-called Ostrogradski instabilities [7]. It is possible to keep the equations of motion up to second order by adding counter terms in the Lagrangian to eliminate higher-order derivatives [8]. The most general scalar-tensor theories with second-order equations of motion are dubbed Horndeski theories [9–11], which have been widely applied to the construction of viable models of inflation and dark energy [12–21].

A vector field  $A_\mu$  can also be the source for cosmic acceleration. If the vector field coupled to gravity respects the  $U(1)$  gauge symmetry as well as the Lorentz invariance, it is not possible to construct nontrivial derivative interactions such as those appearing in scalar Horndeski theories [22]. The vector field with a broken  $U(1)$  symmetry (including a massive Proca field) allows Galileon-type derivative and nonminimal couplings to gravity. Unlike scalar-tensor theories, there are also new interactions arising from intrinsic vector modes [23]. Most general vector-tensor theories with second-order equations of motion are dubbed generalized Proca (GP) theories [23–25]. The applications of GP theories to dark energy [26–29] and spherically symmetric objects [30–35] were extensively performed in the literature.

It is possible to unify Horndeski and GP theories in the form of scalar-vector-tensor (SVT) theories. In Ref. [36], the action of SVT theories with second-order equations of motion was constructed by keeping the  $U(1)$  gauge symmetry or by abandoning it. In the gauge-invariant setup the longitudinal component of a vector field  $A_\mu$  does not propagate, so a scalar field  $\phi$  is the only scalar propagating DOF besides two transverse vector modes and two tensor polarizations [36]. In this case, two of present authors found a new type of hairy black hole solutions in a static and spherically symmetric background [37] (see also Refs. [38, 39]), which are stable against odd-parity perturbations under certain bounds of coupling constants [40].

If we try to apply SVT theories to cosmology, the  $U(1)$  invariant theories do not allow the existence of a time-dependent vector field relevant to the dynamics on the FLRW background. In this case, the vector field needs to be promoted to a non-abelian gauge field with a broken  $SU(2)$  symmetry [41, 42], which we will not consider in this paper. In SVT theories with broken  $U(1)$  gauge invariance, the time-dependent temporal vector component  $A_0(t)$  can play a role for the background cosmology besides a scalar field  $\phi(t)$  [36]. It is of interest to apply such new theories to the dynamics of inflation and dark energy. In particular, there are six propagating DOFs (two scalars, two vectors, and two tensors) in SVT theories with broken  $U(1)$  symmetry, so we need to study whether any of the propagating DOFs are plagued by instability problems.

In this paper, we derive conditions for the absence of ghosts and Laplacian instabilities of linear cosmological perturbations in the presence of most general  $U(1)$  broken SVT interactions with second-order equations of motion and with parity invariance. In Sec. II, we revisit the action of  $U(1)$  broken SVT theories and obtain the background equations of motion on the flat FLRW spacetime. In Sec. III, we compute the second-order action of tensor perturbations and apply it to the speed of gravitational waves relevant to the late-time cosmic acceleration and to the calculation of the primordial tensor power spectrum generated during inflation. In Secs. IV and V, we obtain conditions for avoiding ghosts and Laplacian instabilities of vector and scalar perturbations by deriving their second-order actions. In Sec. VI, we construct a concrete inflationary model in the framework of SVT theories and show that all the stability conditions can be consistently satisfied during inflation and reheating. Sec. VII is devoted to conclusions. Throughout the paper, we use the natural unit in which the speed of light  $c$  is equivalent to 1.

## II. $U(1)$ BROKEN SVT THEORIES AND BACKGROUND EQUATIONS OF MOTION

In Ref. [36], the action of SVT theories with broken  $U(1)$  symmetry was constructed by unifying Horndeski theories with GP theories. In this paper, we focus on new interactions arising in SVT theories with second-order equations of motion and apply them to the cosmological dynamics on the flat FLRW background. The  $U(1)$  broken SVT theories consist of a vector field  $A_\mu$  and a scalar field  $\phi$ , both of which have direct couplings to gravity.

### A. SVT theories with broken $U(1)$ symmetry

We define a field strength  $F_{\mu\nu}$  of the vector field  $A_\mu$ , its dual  $\tilde{F}^{\mu\nu}$ , and a symmetric tensor  $S_{\mu\nu}$ , as

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu, \quad \tilde{F}^{\mu\nu} = \frac{1}{2} \mathcal{E}^{\mu\nu\alpha\beta} F_{\alpha\beta}, \quad S_{\mu\nu} = \nabla_\mu A_\nu + \nabla_\nu A_\mu, \quad (2.1)$$

where  $\nabla_\mu$  represents a covariant derivative operator, and  $\mathcal{E}^{\mu\nu\alpha\beta}$  is the anti-symmetric Levi-Civita tensor satisfying the normalization  $\mathcal{E}^{\mu\nu\alpha\beta} \mathcal{E}_{\mu\nu\alpha\beta} = -4!$ . While neither  $F_{\mu\nu}$  nor  $\tilde{F}^{\mu\nu}$  affects the cosmological background dynamics with a purely temporal component, this is not the case for  $S_{\mu\nu}$ . We introduce another tensor for convenience in form of an effective metric constructed from possible combinations of  $g_{\mu\nu}$ ,  $A_\mu$ , and  $\nabla_\mu \phi$ , i.e.,

$$\mathcal{G}_{\mu\nu}^{h_n} = h_{n1}(\phi, X_i) g_{\mu\nu} + h_{n2}(\phi, X_i) \nabla_\mu \phi \nabla_\nu \phi + h_{n3}(\phi, X_i) A_\mu A_\nu + h_{n4}(\phi, X_i) A_\mu \nabla_\nu \phi, \quad (2.2)$$

where  $g_{\mu\nu}$  is the four-dimensional spacetime metric, and

$$X_1 = -\frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi, \quad X_2 = -\frac{1}{2} A^\mu \nabla_\mu \phi, \quad X_3 = -\frac{1}{2} A_\mu A^\mu, \quad (2.3)$$

with  $i = 1, 2, 3$ . As we will see below,  $\mathcal{G}_{\mu\nu}^{h_n}$  appears in the fifth-order Lagrangian of SVT theories<sup>1</sup>, so that the subscript  $n$  represents  $n = 5$ . The new action arising in SVT theories with broken  $U(1)$  symmetry is given by [36]

$$\mathcal{S}_{\text{SVT}} = \int d^4x \sqrt{-g} \sum_{n=2}^6 \mathcal{L}_n, \quad (2.4)$$

with the Lagrangians

$$\begin{aligned} \mathcal{L}_2 &= f_2(\phi, X_1, X_2, X_3, F, Y_1, Y_2, Y_3), \\ \mathcal{L}_3 &= f_3(\phi, X_3) g^{\mu\nu} S_{\mu\nu} + \tilde{f}_3(\phi, X_3) A^\mu A^\nu S_{\mu\nu}, \\ \mathcal{L}_4 &= f_4(\phi, X_3) R + f_{4,X_3}(\phi, X_3) [(\nabla_\mu A^\mu)^2 - \nabla_\mu A_\nu \nabla^\nu A^\mu], \\ \mathcal{L}_5 &= f_5(\phi, X_3) G^{\mu\nu} \nabla_\mu A_\nu - \frac{1}{6} f_{5,X_3}(\phi, X_3) [(\nabla_\mu A^\mu)^3 - 3 \nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2 \nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma] \\ &\quad + \mathcal{M}_5^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \mathcal{N}_5^{\mu\nu} S_{\mu\nu}, \\ \mathcal{L}_6 &= f_6(\phi, X_1) L^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} + \mathcal{M}_6^{\mu\nu\alpha\beta} \nabla_\mu \nabla_\alpha \phi \nabla_\nu \nabla_\beta \phi + \tilde{f}_6(\phi, X_3) L^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} + \mathcal{N}_6^{\mu\nu\alpha\beta} S_{\mu\alpha} S_{\nu\beta}, \end{aligned} \quad (2.5)$$

<sup>1</sup> As was pointed out in Ref. [36], the explicit dependence on all the  $h_{nj}$  functions needs an additional caution, since an arbitrary dependence on a general background will introduce dynamics for the temporal component of the vector field. In order for this not to happen, the dependence of  $\mathcal{M}_5^{\mu\nu}$  would need to be restricted to  $X_1$  and similarly the dependence of  $\mathcal{N}_5^{\mu\nu}$  to  $X_3$  and so on. We leave them here as general functions since the background symmetries are not oblivious to this fact, but for a more general background this would need to be taken into account.

where  $R$  and  $G^{\mu\nu}$  are the Ricci scalar and the Einstein tensor, respectively, and  $f_{4,X_3} = \partial f_4 / \partial X_3$ ,  $f_{5,X_3} = \partial f_5 / \partial X_3$ . The double dual Riemann tensor  $L^{\mu\nu\alpha\beta}$  is defined by

$$L^{\mu\nu\alpha\beta} = \frac{1}{4} \mathcal{E}^{\mu\nu\rho\sigma} \mathcal{E}^{\alpha\beta\gamma\delta} R_{\rho\sigma\gamma\delta}, \quad (2.6)$$

where  $R_{\rho\sigma\gamma\delta}$  is the Riemann tensor. We also used the following notations:

$$F = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad Y_1 = \nabla_\mu \phi \nabla_\nu \phi F^{\mu\alpha} F^\nu{}_\alpha, \quad Y_2 = \nabla_\mu \phi A_\nu F^{\mu\alpha} F^\nu{}_\alpha, \quad Y_3 = A_\mu A_\nu F^{\mu\alpha} F^\nu{}_\alpha, \quad (2.7)$$

which correspond to the interactions arising from pure vector modes.

The 2-rank tensors  $\mathcal{M}_5^{\mu\nu}$  and  $\mathcal{N}_5^{\mu\nu}$  in  $\mathcal{L}_5$ , which encode intrinsic vector interactions, are given by

$$\mathcal{M}_5^{\mu\nu} = \mathcal{G}_{\rho\sigma}^{h_5} \tilde{F}^{\mu\rho} \tilde{F}^{\nu\sigma}, \quad \mathcal{N}_5^{\mu\nu} = \mathcal{G}_{\rho\sigma}^{\tilde{h}_5} \tilde{F}^{\mu\rho} \tilde{F}^{\nu\sigma}, \quad (2.8)$$

where the functions  $h_{5j}$  and  $\tilde{h}_{5j}$  ( $j = 1, 2, 3, 4$ ) appearing in  $\mathcal{G}_{\rho\sigma}^{h_5}$  and  $\mathcal{G}_{\rho\sigma}^{\tilde{h}_5}$  are functions of  $\phi$  and  $X_1, X_2, X_3$ . The Lagrangian  $\mathcal{L}_6$  also corresponds to the interactions of intrinsic vector modes. The 4-rank tensors  $\mathcal{M}_6^{\mu\nu\alpha\beta}$  and  $\mathcal{N}_6^{\mu\nu\alpha\beta}$  are defined by

$$\mathcal{M}_6^{\mu\nu\alpha\beta} = 2f_{6,X_1}(\phi, X_1) \tilde{F}^{\mu\nu} \tilde{F}^{\alpha\beta}, \quad \mathcal{N}_6^{\mu\nu\alpha\beta} = \frac{1}{2} \tilde{f}_{6,X_3}(\phi, X_3) \tilde{F}^{\mu\nu} \tilde{F}^{\alpha\beta}. \quad (2.9)$$

The functions  $f_3, \tilde{f}_3, f_4, f_5, \tilde{f}_6$  depend on  $\phi$  and  $X_3$ , whereas  $f_6$  has the dependence of  $\phi$  and  $X_1$ . The function  $f_2$  contains the dependence of  $\phi, X_1, X_2, X_3, F, Y_1, Y_2, Y_3$ . In  $f_2$ , we do not take into account the parity-violating term  $\tilde{F} = -F_{\mu\nu} \tilde{F}^{\mu\nu} / 4$  from [36].

The action of GP theories (which is given by Eqs. (2.2)-(2.6) of Ref. [28]) can be recovered by using the correspondence  $\phi \rightarrow 0, X_{1,2} \rightarrow 0, X_3 \rightarrow X, Y_{1,2} \rightarrow 0, Y_3 \rightarrow Y, f_2 \rightarrow G_2(X, F, Y), 2f_3 \rightarrow G_3(X), \tilde{f}_3 \rightarrow 0, f_4 \rightarrow G_4(X), f_5 \rightarrow G_5(X), h_{5j} \rightarrow 0, \tilde{h}_{51} \rightarrow -g_5(X)/2, \tilde{h}_{52}, \tilde{h}_{53}, \tilde{h}_{54} \rightarrow 0, f_6 \rightarrow 0$ , and  $4\tilde{f}_6 \rightarrow G_6(X)$  in the action (2.4).

We note that the full action of SVT theories with second-order equations of motion is given by  $\mathcal{S} = \mathcal{S}_{\text{SVT}} + \mathcal{S}_{\text{ST}}$ , where  $\mathcal{S}_{\text{ST}}$  is the action of scalar-tensor Horndeski theories with the Lagrangians (2.1)-(2.4) of Ref. [11]. Since we are interested in the effect of new interactions  $\mathcal{S}_{\text{SVT}}$  on the cosmological dynamics, we focus on  $U(1)$  broken SVT theories given by the action (2.4). In such theories, there are six propagating DOFs in total (two tensors, two vectors, and two scalars) on the flat FLRW background. In Secs. III-V, we study the propagation of tensor, vector, and scalar perturbations in turn. In Sec. VI, we apply our  $U(1)$  broken SVT theories to the inflationary cosmology.

## B. Background equations of motion

To derive the equations of motion on the flat FLRW background, we begin with the line element

$$ds^2 = -N^2(t) dt^2 + a^2(t) \delta_{ij} dx^i dx^j, \quad (2.10)$$

where  $N(t)$  is the lapse and  $a(t)$  is the scale factor. We also consider the configuration of a time-dependent scalar field  $\phi(t)$  and a vector field  $A_\mu(t)$  given by

$$A_\mu(t) = (A_0(t)N(t), 0, 0, 0), \quad (2.11)$$

where  $A_0(t)$  is a time-dependent temporal vector component. The quantities  $F, Y_1, Y_2, Y_3$  vanish on the spacetime metric (2.10), so they do not contribute to the background equations of motion. Moreover, the Lagrangian  $\mathcal{L}_6$  and the interactions proportional to  $\mathcal{M}_5^{\mu\nu}$  and  $\mathcal{N}_5^{\mu\nu}$  in  $\mathcal{L}_5$  do not affect the background cosmology either. The quantities  $X_1, X_2, X_3$  are given, respectively, by

$$X_1 = \frac{\dot{\phi}^2}{2N^2}, \quad X_2 = \frac{\dot{\phi} A_0}{2N}, \quad X_3 = \frac{A_0^2}{2}, \quad (2.12)$$

where a dot represents a derivative with respect to  $t$ . We compute the action (2.4) on the spacetime metric (2.10) and vary it with respect to  $N, a, \phi$ , and  $A_0$ . Setting  $N = 1$  at the end, we obtain the following equations of motion

on the flat FLRW background:

$$6f_4H^2 + f_2 - \dot{\phi}^2 f_{2,X_1} - \frac{1}{2}\dot{\phi}A_0f_{2,X_2} + 6H\left(\dot{\phi}f_{4,\phi} - HA_0^2f_{4,X_3}\right) + 2A_0H^2\left(3\dot{\phi}f_{5,\phi} - A_0^2Hf_{5,X_3}\right) = 0, \quad (2.13)$$

$$\begin{aligned} & 2f_4\left(2\dot{H} + 3H^2\right) + f_2 + 2\dot{A}_0A_0^2\left(f_{3,X_3} + \tilde{f}_3\right) + 2\dot{\phi}A_0f_{3,\phi} + 2\left(\ddot{\phi} + 2H\dot{\phi}\right)f_{4,\phi} - 2A_0\left[A_0(2\dot{H} + 3H^2) + 2\dot{A}_0H\right]f_{4,X_3} \\ & + 2\dot{\phi}\dot{A}_0A_0f_{4,X_3\phi} + 2\dot{\phi}^2f_{4,\phi\phi} - 4HA_0^2\left(\dot{A}_0A_0f_{4,X_3X_3} + \dot{\phi}f_{4,X_3\phi}\right) + \left[2A_0\left(H\ddot{\phi} + \dot{H}\dot{\phi}\right) + \dot{\phi}\left(2H\dot{A}_0 + 3H^2A_0\right)\right]f_{5,\phi} \\ & - HA_0^2\left[2A_0\left(\dot{H} + H^2\right) + 3\dot{A}_0H\right]f_{5,X_3} + H\dot{\phi}A_0^2\left(2\dot{A}_0 - HA_0\right)f_{5,X_3\phi} + HA_0\left(2\dot{\phi}^2f_{5,\phi\phi} - \dot{A}_0A_0^3Hf_{5,X_3X_3}\right) = 0, \quad (2.14) \\ & \left(f_{2,X_1} + \dot{\phi}^2f_{2,X_1X_1} + \dot{\phi}A_0f_{2,X_1X_2} + \frac{1}{4}A_0^2f_{2,X_2X_2}\right)\ddot{\phi} + 3Hf_{2,X_1}\dot{\phi} - f_{2,\phi} + \dot{\phi}^2f_{2,X_1\phi} - 6\left(\dot{H} + 2H^2\right)f_{4,\phi} \\ & + \left[\frac{1}{2}f_{2,X_2} + \frac{1}{2}\dot{\phi}^2f_{2,X_1X_2} + 2f_{3,\phi} - 3H^2f_{5,\phi} + A_0\left(\dot{\phi}f_{2,X_1X_3} + \frac{1}{4}\dot{\phi}f_{2,X_2X_2} - 6Hf_{4,X_3\phi}\right)\right. \\ & \left.+ \frac{A_0^2}{2}\left(f_{2,X_2X_3} - 4\tilde{f}_{3,\phi} - 6H^2f_{5,X_3\phi}\right)\right]\dot{A}_0 + \left[\frac{1}{2}\dot{\phi}f_{2,X_2\phi} + \frac{3}{2}Hf_{2,X_2} + 6Hf_{3,\phi} - 6A_0H^2f_{4,X_3\phi}\right. \\ & \left.- 3H\left(2\dot{H} + 3H^2\right)f_{5,\phi} - A_0^2H^3f_{5,X_3\phi}\right]A_0 = 0, \quad (2.15) \end{aligned}$$

$$\begin{aligned} & 2\left(f_{2,X_3} + 6H^2f_{4,X_3} - 6H\dot{\phi}f_{4,X_3\phi}\right)A_0 - 2\left(6Hf_{3,X_3} + 6H\tilde{f}_3 + 2\dot{\phi}\tilde{f}_{3,\phi} - 3H^3f_{5,X_3} + 3H^2\dot{\phi}f_{5,X_3\phi}\right)A_0^2 \\ & + 12H^2f_{4,X_3X_3}A_0^3 + 2H^3f_{5,X_3X_3}A_0^4 + \left(f_{2,X_2} + 4f_{3,\phi} - 6H^2f_{5,\phi}\right)\dot{\phi} = 0, \quad (2.16) \end{aligned}$$

where  $H = \dot{a}/a$  is the Hubble expansion rate. As we observe in Eqs. (2.15) and (2.16), the scalar field  $\phi$  and the temporal vector component  $A_0$  are coupled to each other in a non-trivial way. From Eq. (2.16), we find that  $A_0$  depends not only on  $H$  but also on  $\phi$  and  $\dot{\phi}$ . In GP theories,  $A_0$  depends solely on  $H$  and hence there exists a de Sitter solution characterized by constant  $A_0$  and  $H$  [27, 28]. In SVT theories, this structure is broken by the interaction between  $\phi$  and  $A_0$ , which we need to take into account.

If  $A_0$  is the dominant source for the background dynamics relevant to cosmic acceleration, the nonvanishing time derivative  $\dot{\phi}$  leads to the deviation from de Sitter solutions characterized by constant  $A_0$ . On the other hand, if the energy density of  $\phi$  dominates over that of  $A_0$ , the cosmological dynamics of  $\phi$  is subject to modifications by the existence of  $A_0$ . If we apply this scenario to the early Universe, the modification induced by  $A_0$  affects the dynamics of inflation and primordial power spectra of perturbations generated during inflation. If the energy densities of  $\phi$  and  $A_0$  are comparable to each other, there is the possibility for realizing “multi-field” inflation driven by the two fields, even though one of them will play the role of an auxiliary field. It would be also possible to apply the above scenario to the dynamics of dark energy and possibly to dark matter [36].

### III. TENSOR PERTURBATIONS

We derive the second-order action of tensor perturbations for the SVT theories given by the action (2.4). Let us consider the linearly perturbed line element of intrinsic tensor modes:

$$ds_t^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j, \quad (3.1)$$

where the tensor perturbation  $h_{ij}$  obeys the transverse and traceless conditions  $\nabla^j h_{ij} = 0$  and  $h_i^i = 0$ .

#### A. Second-order action

Expanding the action (2.4) up to quadratic order in  $h_{ij}$  and integrating it by parts, the second-order action of tensor perturbations yields

$$\mathcal{S}_t^{(2)} = \int dt d^3x \frac{a^3 q_t}{8} \delta^{ik} \delta^{jl} \left[ \dot{h}_{ij} \dot{h}_{kl} - \frac{c_t^2}{a^2} (\partial h_{ij})(\partial h_{kl}) \right], \quad (3.2)$$

where the symbol  $\partial$  represents the spatial partial derivative, and

$$q_t = 2f_4 - 2A_0^2f_{4,X_3} + A_0\dot{\phi}f_{5,\phi} - HA_0^3f_{5,X_3}, \quad (3.3)$$

$$c_t^2 = \frac{2f_4 - A_0\dot{\phi}f_{5,\phi} - \dot{A}_0A_0^2f_{5,X_3}}{2f_4 - 2A_0^2f_{4,X_3} + A_0\dot{\phi}f_{5,\phi} - HA_0^3f_{5,X_3}}. \quad (3.4)$$

The terms associated with the tensor mass like  $\delta^{ik}\delta^{jl}h_{ij}h_{kl}$  vanish on account of the background Eq. (2.14). The quantity  $c_t$  corresponds to the propagation speed of gravitational waves on the FLRW background. As we will see in Sec. III C, there are two polarized states for tensor perturbations, both of which have the same propagation speed  $c_t$ . The existence of additional matter minimally coupled to gravity does not affect the value of  $c_t^2$  given above. The values of  $q_t$  and  $c_t^2$  derived in generalized Proca theories [27, 28] can be recovered by using the correspondence  $f_4 \rightarrow G_4$ ,  $f_5 \rightarrow G_5$ ,  $A_0 \rightarrow -\phi$ ,  $X_3 \rightarrow X$ , and  $\dot{\phi} \rightarrow 0$ . Under the two conditions

$$q_t > 0, \quad c_t^2 > 0, \quad (3.5)$$

there are neither ghost nor Laplacian instabilities in the tensor sector.

### B. Application to the speed of gravity in late-time cosmology

If we apply SVT theories to the late-time cosmology, there is a tight bound  $-3 \times 10^{-15} \leq c_t - 1 \leq 7 \times 10^{-16}$  constrained from the gravitational wave event GW170817 [43] together with the gamma-ray burst GRB 170817A [44]. From Eq. (3.4), the SVT theories realizing the exact value  $c_t = 1$  need to satisfy the following conditions:

$$f_4(\phi, X_3) = f_4(\phi), \quad f_5(\phi, X_3) = \text{constant}. \quad (3.6)$$

This mean that  $f_4$  does not contain the  $X_3$  dependence and that  $f_5$  depends on neither  $\phi$  nor  $X_3$ . This property is similar to what happens in scalar-tensor theories with the replacement  $X_3 \rightarrow -\nabla_\mu \phi \nabla^\mu \phi / 2$  [45]. As expected, the couplings  $f_2, f_3, \tilde{f}_3, f_6, \tilde{f}_6$  and  $h_{5j}, \tilde{h}_{5j}$  do not modify the tensor propagation speed. If one is willing to apply SVT theories to the late-time cosmology, one has to bear in mind the restriction of (3.6) and include the presence of matter fields. Otherwise, for applications to inflation, this restriction can be lifted. We will mostly consider the second option here and do not take into account matter fields.

### C. Tensor power spectrum generated during inflation

If we apply SVT theories to inflation, we do not need to impose the conditions (3.6). The structure of the second-order action (3.2) is of the same form as that derived in Refs. [14, 15] for Horndeski theories, so it is straightforward to compute the primordial tensor power spectrum generated during inflation. We express  $h_{ij}$  in terms of the Fourier series, as

$$h_{ij}(\mathbf{x}, \tau) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k} \cdot \mathbf{x}} \sum_{\lambda=+, \times} \left[ h_\lambda(k, \tau) a_\lambda(\mathbf{k}) + h_\lambda^*(k, \tau) a_\lambda^\dagger(-\mathbf{k}) \right] e_{ij}^{(\lambda)}(\mathbf{k}), \quad (3.7)$$

where  $\tau = \int a^{-1} dt$  is the conformal time,  $\mathbf{k}$  is the coming wavenumber, and  $\lambda = +, \times$  denote the two polarization states. The polarization tensors  $e_{ij}^{(\lambda)}(\mathbf{k})$  obey transverse and traceless conditions  $k^j e_{ij}^{(\lambda)} = \delta^{ij} e_{ij}^{(\lambda)} = 0$  together with the normalization  $\delta^{ik}\delta^{jl} e_{ij}^{(\lambda)}(\mathbf{k}) e_{kl}^{*(\lambda')}(\mathbf{k}) = \delta_{\lambda\lambda'}$ . The annihilation and creation operators  $a_\lambda(\mathbf{k})$  and  $a_\lambda^\dagger(\mathbf{k}')$  satisfy the commutation relation  $[a_\lambda(\mathbf{k}), a_{\lambda'}^\dagger(\mathbf{k}')] = \delta_{\lambda\lambda'} \delta^{(3)}(\mathbf{k} - \mathbf{k}')$ . The primordial power spectrum per unit logarithmic wavenumber interval is given by

$$\mathcal{P}_h(k, \tau) = \frac{k^3}{2\pi^2} \left( |h_+(k, \tau)|^2 + |h_\times(k, \tau)|^2 \right). \quad (3.8)$$

We introduce a canonically normalized field  $v_\lambda(k, \tau)$ , as

$$v_\lambda(k, \tau) = z h_\lambda(k, \tau), \quad z = \frac{a}{2\sqrt{q_t}}. \quad (3.9)$$

Varying the action (3.2) with respect to  $h_{ij}$ , each Fourier component obeys

$$v_\lambda'' + \left( c_t^2 k^2 - \frac{z''}{z} \right) v_\lambda = 0, \quad (3.10)$$

where a prime represents a derivative with respect to  $\tau$ . We consider a quasi de Sitter background on which the variations of  $H, q_t, c_t$  are small such that  $|\dot{H}/H^2| \ll 1$ ,  $|\dot{q}_t/(Hq_t)| \ll 1$ , and  $|\dot{c}_t/(Hc_t)| \ll 1$ , with the relation

$\tau \simeq -(aH)^{-1}$ . Then, the leading-order contribution to  $z''/z$  is given by  $2(aH)^2$ . For the modes deep inside the tensor sound horizon ( $c_t^2 k^2 \gg a^2 H^2$ ), the solution corresponding to the Bunch-Davies vacuum is given by  $v_\lambda = e^{-ic_t k \tau} / \sqrt{2c_t k}$ . On using the de Sitter approximation  $z''/z \simeq 2\tau^{-2}$ , the solution to Eq. (3.10) recovering  $v_\lambda = e^{-ic_t k \tau} / \sqrt{2c_t k}$  in the asymptotic past is

$$v_\lambda(k, \tau) = \frac{i + c_t k |\tau|}{\sqrt{2}(c_t k)^{3/2} |\tau|} e^{-ic_t k \tau}. \quad (3.11)$$

Then, the solution to  $h_\lambda$  long after the tensor sound horizon crossing reduces to  $h_\lambda(k, 0) = i\sqrt{2/q_t} H/(c_t k)^{3/2}$ . From Eq. (3.8), the leading-order primordial power spectrum  $\mathcal{P}_t(k) \equiv \mathcal{P}_h(k, 0)$  yields

$$\mathcal{P}_t(k) = \frac{2H^2}{\pi^2 q_t c_t^3}. \quad (3.12)$$

Since the perturbations  $h_\lambda$  are frozen right after the tensor sound horizon crossing, it is sufficient to evaluate the value (3.12) at the moment  $c_t k = aH$ . In GR, we have  $q_t = M_{\text{pl}}^2$  and  $c_t^2 = 1$ , where  $M_{\text{pl}}$  is the reduced Planck mass, so the tensor power spectrum (3.12) reduces to  $\mathcal{P}_t(k) = 2H^2/(\pi^2 M_{\text{pl}}^2)$ . This is modified in SVT theories due to the changes of  $q_t$  and  $c_t^2$ . We note that the next-to-leading order tensor power spectrum can be also computed along the line of Refs. [46–48].

#### IV. VECTOR PERTURBATIONS

For perturbations in the vector sector, we take the perturbed line element in the flat gauge:

$$ds_v^2 = -dt^2 + 2V_i dt dx^i + a^2(t) \delta_{ij} dx^i dx^j, \quad (4.1)$$

where  $V_i$  is the vector perturbation obeying the transverse condition  $\nabla^i V_i = 0$ . At linear order in perturbations, the transverse condition translates to  $\partial^i V_i = 0$ , where  $\partial^i \equiv \partial/\partial x_i$ . The temporal and spatial components of  $A^\mu$  associated with the intrinsic vector sector are expressed in the form

$$A_0 = A_0(t), \quad A_i = Z_i(t, x^i), \quad (4.2)$$

where  $Z_i$  is the intrinsic vector perturbation satisfying  $\partial^i Z_i = 0$ .

For the practical computation, we will consider the vector components  $V_i = (V_1(t, z), V_2(t, z), 0)$  and  $Z_i = (Z_1(t, z), Z_2(t, z), 0)$ , which automatically satisfy the transverse conditions mentioned above. Expanding Eq. (2.4) up to quadratic order in perturbations and using the background Eqs. (2.13) and (2.16), the resulting second-order action in the vector sector yields

$$\mathcal{S}_v^{(2)} = \int dt d^3x \sum_{i=1}^2 \left[ \frac{aq_v}{2} \dot{Z}_i^2 - \frac{1}{2a} \alpha_1 (\partial Z_i)^2 - \frac{a}{2} \alpha_2 Z_i^2 + \frac{1}{2a} \alpha_3 (\partial V_i)(\partial Z_i) + \frac{q_t}{4a} (\partial V_i)^2 \right], \quad (4.3)$$

where

$$q_v = f_{2,F} + 2\dot{\phi}^2 f_{2,Y_1} + 2\dot{\phi} A_0 f_{2,Y_2} + 2A_0^2 f_{2,Y_3} - 4H \left( \dot{\phi} h_{51} + 2A_0 \tilde{h}_{51} \right) + 8H^2 \left( f_6 + \tilde{f}_6 + \dot{\phi}^2 f_{6,X_1} + A_0^2 \tilde{f}_{6,X_3} \right), \quad (4.4)$$

$$\begin{aligned} \alpha_1 = & f_{2,F} - 4\dot{A}_0 \tilde{h}_{51} + 8 \left( H^2 + \dot{H} \right) \left( f_6 + \tilde{f}_6 \right) - 2\ddot{\phi} h_{51} + H \left[ 2\dot{\phi} \left( \dot{\phi}^2 h_{52} - h_{51} + 4\ddot{\phi} f_{6,X_1} \right) \right. \\ & \left. - A_0 \left\{ 4\tilde{h}_{51} - 2\dot{\phi}^2 \left( h_{54} + 2\tilde{h}_{52} \right) - 8\dot{A}_0 \tilde{f}_{6,X_3} \right\} + 2\dot{\phi} A_0^2 (h_{53} + 2\tilde{h}_{54}) + 4A_0^3 \tilde{h}_{53} \right], \end{aligned} \quad (4.5)$$

$$\begin{aligned} \alpha_2 = & f_{2,X_3} + 4\dot{H} f_{4,X_3} - 2 \left( \dot{A}_0 + 3HA_0 \right) \left( \tilde{f}_3 + f_{3,X_3} \right) - 2\dot{\phi} A_0 \tilde{f}_{3,\phi} + 2H(3Hf_{4,X_3} + 3HA_0^2 f_{4,X_3 X_3} + 2A_0 \dot{A}_0 f_{4,X_3 X_3} \\ & - \dot{\phi} f_{4,X_3 \phi}) + H \left( H\dot{A}_0 + 2\dot{H}A_0 + 3H^2 A_0 \right) f_{5,X_3} + H^2 A_0 \left( HA_0^2 f_{5,X_3 X_3} + A_0 \dot{A}_0 f_{5,X_3 X_3} - 2\dot{\phi} f_{5,X_3 \phi} \right), \end{aligned} \quad (4.6)$$

$$\alpha_3 = -2A_0 f_{4,X_3} - HA_0^2 f_{5,X_3} + \dot{\phi} f_{5,\phi}, \quad (4.7)$$

where  $q_t$  is defined by Eq. (3.3).

Varying the second-order action (4.3) with respect to  $V_i$ , it follows that

$$\partial^2 (\alpha_3 Z_i + q_t V_i) = 0. \quad (4.8)$$



This is integrated to give  $V_i = -\alpha_3 Z_i / q_t$ , where we set the integration constant 0. Substituting this relation into Eq. (4.3), we obtain

$$\mathcal{S}_v^{(2)} = \int dt d^3x \sum_{i=1}^2 \frac{aq_v}{2} \left[ \dot{Z}_i^2 - \frac{c_v^2}{a^2} (\partial Z_i)^2 - \frac{\alpha_2}{q_v} Z_i^2 \right], \quad (4.9)$$

where

$$c_v^2 = \frac{2\alpha_1 q_t + \alpha_3^2}{2q_t q_v}. \quad (4.10)$$

This result shows that there are two dynamical fields  $Z_1$  and  $Z_2$  in the vector sector with the same propagation speed  $c_v$ . Varying the action (4.9) with respect to  $Z_i$ , the equation of motion of vector perturbations in Fourier space is given by

$$\ddot{Z}_i + \left( H + \frac{\dot{q}_v}{q_v} \right) \dot{Z}_i + \left( c_v^2 \frac{k^2}{a^2} + \frac{\alpha_2}{q_v} \right) Z_i = 0, \quad (4.11)$$

where the term  $m_v^2 = \alpha_2 / q_v$  corresponds to the vector mass squared. In the limit that  $c_v^2 k^2 / a^2 \gg m_v^2$ , the mass term is irrelevant to the dynamics of vector perturbations, so there are neither ghost nor Laplacian instabilities under the conditions

$$q_v > 0, \quad c_v^2 > 0. \quad (4.12)$$

From Eq. (4.4), we find that the functional dependence  $f_2(F, Y_1, Y_2, Y_3)$  as well as the functions  $h_{51}, \tilde{h}_{51}, f_6, \tilde{f}_6$  themselves affect the no-ghost condition of vector perturbations. The value of  $q_v$  derived in GP theories [28] can be recovered by using the correspondence  $\dot{\phi} \rightarrow 0$ ,  $X_3 \rightarrow X$ ,  $Y_3 \rightarrow Y$ ,  $A_0 \rightarrow -\phi$ ,  $f_2 \rightarrow G_2$ ,  $\tilde{h}_{51} \rightarrow -g_5(X)/2$ ,  $f_6 \rightarrow 0$ , and  $4\tilde{f}_6 \rightarrow G_6(X)$  in Eq. (4.4). Since Eq. (4.10) contains  $q_t, \alpha_1, \alpha_3$ , the vector propagation speed is also affected by the dependence of  $f_4(X_3)$ ,  $f_5(\phi, X_3)$  and the functions  $h_{52}, \tilde{h}_{52}, h_{53}, \tilde{h}_{53}, h_{54}, \tilde{h}_{54}$ .

If we apply SVT theories to inflation, the evolution of  $Z_i$  is different depending on the mass term  $m_v^2$ . If the condition  $m_v^2 \gg H^2$  is satisfied during inflation, the vector perturbation is subject to strong suppression for the modes  $c_v^2 k^2 / a^2 < m_v^2$  (analogous to what happens for tensor perturbations in a Lorentz-violating massive gravity scenario studied in Ref. [49]). If  $m_v^2 \ll H^2$ , then the vector perturbation is not suppressed even after the vector horizon crossing ( $c_v^2 k^2 / a^2 < H^2$ ). We leave the detailed analysis for the computation of the primordial vector power spectrum generated during inflation as a future work.

## V. SCALAR PERTURBATIONS

For scalar perturbations, we consider the linearly perturbed line-element in the flat gauge:

$$ds_s^2 = -(1 + 2\alpha) dt^2 + 2\partial_i \chi dt dx^i + a^2(t) \delta_{ij} dx^i dx^j, \quad (5.1)$$

where  $\alpha$  and  $\chi$  are scalar metric perturbations. We write the components of the vector field in the form

$$A^0 = -A_0(t) + \delta A, \quad A_i = \partial_i \psi, \quad (5.2)$$

where  $\delta A$  is the perturbation of the temporal vector component  $A^0$ , and  $\psi$  is the longitudinal scalar perturbation. The scalar field  $\phi$  is decomposed into the background and perturbed parts, as

$$\phi = \phi_0(t) + \delta\phi, \quad (5.3)$$

where, in the following, we omit the subscript “0” from the background value of  $\phi$ .

We expand the action (2.4) up to quadratic order in scalar perturbations  $\alpha, \chi, \delta A, \psi, \delta\phi$ . In doing so, we use the background Eqs. (2.13) and (2.16) to eliminate the terms  $f_2$  and  $f_{3,\phi}$ . Then, the second-order action for scalar perturbations can be expressed as

$$\mathcal{S}_s^{(2)} = \int dt d^3x (\mathcal{L}_s^\phi + \mathcal{L}_s^{\text{GP}}), \quad (5.4)$$

where

$$\begin{aligned} \mathcal{L}_s^\phi = a^3 \left[ D_1 \dot{\phi}^2 + D_2 \frac{(\partial\delta\phi)^2}{a^2} + D_3 \delta\phi^2 + \left( D_4 \dot{\phi} + D_5 \delta\phi + D_6 \frac{\partial^2 \delta\phi}{a^2} \right) \alpha - \left( D_6 \dot{\phi} - D_7 \delta\phi \right) \frac{\partial^2 \chi}{a^2} \right. \\ \left. + \left( D_8 \dot{\phi} + D_9 \delta\phi \right) \delta A + D_{10} \delta\phi \frac{\partial^2 \psi}{a^2} \right], \end{aligned} \quad (5.5)$$

and

$$\begin{aligned} \mathcal{L}_s^{\text{GP}} = a^3 \left[ \left( w_1 \alpha - w_2 \frac{\delta A}{A_0} \right) \frac{\partial^2 \chi}{a^2} - w_3 \frac{(\partial\alpha)^2}{a^2} + w_4 \alpha^2 - \left( w_3 \frac{\partial^2 \delta A}{a^2 A_0} - w_8 \frac{\delta A}{A_0} + w_3 \frac{\partial^2 \dot{\psi}}{a^2 A_0} + w_6 \frac{\partial^2 \psi}{a^2} \right) \alpha \right. \\ \left. - w_3 \frac{(\partial\delta A)^2}{4a^2 A_0^2} + w_5 \frac{\delta A^2}{A_0^2} + \left\{ w_3 \dot{\psi} - (w_2 - A_0 w_6) \psi \right\} \frac{\partial^2 \delta A}{2a^2 A_0^2} - w_3 \frac{(\partial\dot{\psi})^2}{4a^2 A_0^2} + w_7 \frac{(\partial\psi)^2}{2a^2} \right]. \end{aligned} \quad (5.6)$$

The coefficients  $D_1, \dots, D_{10}$  and  $w_1, \dots, w_8$  are given in Appendix. The Lagrangian  $\mathcal{L}_s^\phi$  arises from the scalar perturbation  $\delta\phi$ . The other Lagrangian  $\mathcal{L}_s^{\text{GP}}$  has the similar structure to that in GP theories [27]. In GP theories, the coefficient  $w_8$  is related to  $w_1$  and  $w_4$ , as  $w_8 = 3Hw_1 - 2w_4$  [27], while, in the present case, this relation is modified to  $w_8 = 3Hw_1 - 2w_4 - \dot{\phi}D_4$  by the presence of the scalar field  $\phi$ . The effect of intrinsic vector modes on scalar perturbations appears only through  $w_3 = -2A_0^2 q_v$ , where  $q_v$  is given by Eq. (4.4).

Since there are no time derivatives of  $\alpha, \chi, \delta A$  in Eqs. (5.5) and (5.6), these fields are non-dynamical. On the other hand, the perturbations  $\psi$  and  $\delta\phi$  correspond to the dynamical fields in the scalar sector. The field  $\psi$  is the longitudinal scalar component of vector field associated with the breaking of  $U(1)$  gauge symmetry, whereas the perturbation  $\delta\phi$  arises from the scalar field  $\phi$ . Varying the action (5.4) with respect to  $\alpha, \chi, \delta A$ , we obtain the three constraint equations in Fourier space:

$$D_4 \dot{\phi} + D_5 \delta\phi + 2w_4 \alpha + w_8 \frac{\delta A}{A_0} + \frac{k^2}{a^2} \left( w_3 \frac{\dot{\psi}}{A_0} + w_6 \psi - D_6 \delta\phi - 2w_3 \alpha - w_1 \chi + w_3 \frac{\delta A}{A_0} \right) = 0, \quad (5.7)$$

$$D_6 \dot{\phi} - D_7 \delta\phi - w_1 \alpha + w_2 \frac{\delta A}{A_0} = 0, \quad (5.8)$$

$$D_8 \dot{\phi} + D_9 \delta\phi + w_8 \frac{\alpha}{A_0} + 2w_5 \frac{\delta A}{A_0^2} - \frac{k^2}{a^2} \frac{1}{A_0} \left( \frac{w_3}{2} \frac{\dot{\psi}}{A_0} + \frac{A_0 w_6 - w_2}{2} \frac{\psi}{A_0} - w_3 \alpha - w_2 \chi + \frac{w_3}{2} \frac{\delta A}{A_0} \right) = 0. \quad (5.9)$$

We solve Eqs. (5.7)-(5.9) for  $\alpha, \chi, \delta A$  and eliminate these variables from the action (5.4). Then, the second-order action of scalar perturbations can be expressed in the form

$$\mathcal{S}_s^{(2)} = \int dt d^3x a^3 \left( \dot{\vec{\chi}}^t \mathbf{K} \vec{\chi} - \frac{k^2}{a^2} \vec{\chi}^t \mathbf{G} \vec{\chi} - \vec{\chi}^t \mathbf{M} \vec{\chi} - \vec{\chi}^t \mathbf{B} \dot{\vec{\chi}} \right), \quad (5.10)$$

where  $\mathbf{K}, \mathbf{G}, \mathbf{M}, \mathbf{B}$  are  $2 \times 2$  matrices, and  $\vec{\chi}$  is defined by

$$\vec{\chi}^t = (\psi, \delta\phi). \quad (5.11)$$

In the small-scale limit, the leading-order contributions to the matrix  $\mathbf{M}$  do not contain the  $k^2$  terms. We shift the  $k^2$  terms appearing in  $\mathbf{B}$  to the matrix components of  $\mathbf{G}$  after integrating them by parts. Then, in the  $k \rightarrow \infty$  limit, the components of  $\mathbf{K}$  and  $\mathbf{G}$  are given, respectively, by

$$\begin{aligned} K_{11} &= \frac{w_1^2 w_5 + w_2^2 w_4 + w_1 w_2 w_8}{A_0^2 (w_1 - 2w_2)^2}, \\ K_{22} &= D_1 + \frac{D_6}{w_1 - 2w_2} \left( D_4 + \frac{w_4 + 4w_5 + 2w_8}{w_1 - 2w_2} D_6 + 2A_0 D_8 \right), \\ K_{12} = K_{21} &= -\frac{1}{2A_0 (w_1 - 2w_2)} \left[ w_2 D_4 + \frac{w_1 (4w_5 + w_8) + 2w_2 (w_4 + w_8)}{w_1 - 2w_2} D_6 + A_0 w_1 D_8 \right], \end{aligned} \quad (5.12)$$



and

$$\begin{aligned}
G_{11} &= \dot{E}_1 + H E_1 - \frac{4A_0^2}{w_3} E_1^2 - \frac{w_7}{2}, \\
G_{22} &= \dot{E}_2 + H E_2 - \frac{2A_0}{w_2} D_7 E_3 - \frac{4A_0^2}{w_3} E_3^2 - D_2, \\
G_{12} = G_{21} &= \dot{E}_3 + H E_3 - \frac{4A_0^2}{w_3} E_1 E_3 + \frac{w_2}{2A_0(w_1 - 2w_2)} D_7 + \frac{D_{10}}{2},
\end{aligned} \tag{5.13}$$

where we introduced

$$E_1 = \frac{w_6}{4A_0} - \frac{w_1 w_2}{4A_0^2(w_1 - 2w_2)}, \quad E_2 = -\frac{D_6^2}{2(w_1 - 2w_2)}, \quad E_3 = \frac{w_2 D_6}{2A_0(w_1 - 2w_2)}. \tag{5.14}$$

In order to ensure the absence of scalar ghosts, the kinetic matrix  $\mathbf{K}$  must be positive definite. In other words, the determinants of principal sub-matrices of  $\mathbf{K}$  need to be positive. Thus, we require the following two no-ghost conditions:

$$K_{11} > 0 \quad \text{or} \quad K_{22} > 0, \tag{5.15}$$

$$q_s \equiv K_{11} K_{22} - K_{12}^2 > 0. \tag{5.16}$$

In the small-scale limit, the dispersion relation following from the action (5.10) with frequency  $\omega$  is given by

$$\det \left( \omega^2 \mathbf{K} - \frac{k^2}{a^2} \mathbf{G} \right) = 0. \tag{5.17}$$

Introducing the scalar sound speed  $c_s$  as  $\omega^2 = c_s^2 k^2 / a^2$ , the above dispersion relation leads to the two scalar propagation speed squares:

$$c_{s1}^2 = \frac{K_{11} G_{22} + K_{22} G_{11} - 2K_{12} G_{12} + \sqrt{(K_{11} G_{22} + K_{22} G_{11} - 2K_{12} G_{12})^2 - 4(K_{11} K_{22} - K_{12}^2)(G_{11} G_{22} - G_{12}^2)}}{2(K_{11} K_{22} - K_{12}^2)}, \tag{5.18}$$

$$c_{s2}^2 = \frac{K_{11} G_{22} + K_{22} G_{11} - 2K_{12} G_{12} - \sqrt{(K_{11} G_{22} + K_{22} G_{11} - 2K_{12} G_{12})^2 - 4(K_{11} K_{22} - K_{12}^2)(G_{11} G_{22} - G_{12}^2)}}{2(K_{11} K_{22} - K_{12}^2)}. \tag{5.19}$$

For the absence of Laplacian instabilities, we require that

$$c_{s1}^2 > 0, \quad c_{s2}^2 > 0. \tag{5.20}$$

From Eqs. (5.15)-(5.16), the functions  $f_3, \tilde{f}_3, f_4, f_5$  as well as the functional dependence of  $X_1, X_2, X_3$  in  $f_2$  affect no-ghost conditions of scalar perturbations. Since the matrix components  $G_{11}, G_{22}, G_{12}$  contain the term  $w_3 = -2A_0^2 q_v$ , the propagation speeds  $c_{s1}$  and  $c_{s2}$  are affected by intrinsic vector modes.

If we apply SVT theories to “multi-field” inflation driven by the background field  $\phi(t)$  and the auxiliary field  $A_0(t)$ , both  $\delta\phi$  and  $\psi$  contribute to the curvature perturbation  $\mathcal{R}$  [50]. In such cases, the separation between adiabatic and isocurvature perturbations is useful for the computation of primordial scalar power spectrum generated during inflation [51]. The resulting power spectra of curvature and isocurvature perturbations as well as their correlations depend on the models of inflation [52], so the observational prediction of SVT theories relevant to CMB temperature anisotropies deserves for separate detailed analysis in future.

## VI. APPLICATION TO A CONCRETE MODEL OF INFLATION

Let us apply the  $U(1)$  broken SVT theories to the background dynamics of inflation and reheating as well as to the stability conditions in these epochs. We consider the following model

$$\begin{aligned}
f_2 &= F + X_1 - V(\phi) + \beta_m M X_2 + \beta_A M^2 X_3, \\
f_4 &= \frac{M_{\text{pl}}^2}{2} + \beta_G X_3,
\end{aligned} \tag{6.1}$$

where  $V(\phi)$  is a scalar potential,  $M$  is a constant having a dimension of mass (of order the Hubble expansion rate during inflation), and  $\beta_m, \beta_A, \beta_G$  are dimensionless coupling constants. The other functions  $f_3, \tilde{f}_3, f_5, h_{5j}, \tilde{h}_{5j}, f_6, \tilde{f}_6$  in the action (2.4) are taken to be 0.

### A. Background dynamics

For the above model, the background Eqs. (2.13)-(2.16) reduce, respectively, to

$$3M_{\text{pl}}^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V - \frac{1}{2}\beta_A M^2 A_0^2 + 3\beta_G H^2 A_0^2, \quad (6.2)$$

$$(2\dot{H} + 3H^2)(M_{\text{pl}}^2 - \beta_G A_0^2) = -\frac{1}{2}\dot{\phi}^2 + V - \frac{1}{2}\beta_m M \dot{\phi} A_0 - \frac{1}{2}\beta_A M^2 A_0^2 + 4\beta_G H \dot{A}_0 A_0, \quad (6.3)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + \frac{1}{2}M\beta_m(\dot{A}_0 + 3HA_0) = 0, \quad (6.4)$$

$$A_0 = -\frac{\beta_m M}{2(\beta_A M^2 + 6\beta_G H^2)}\dot{\phi}. \quad (6.5)$$

As we see in Eqs. (6.4) and (6.5), the nonvanishing coupling  $\beta_m$  induces a mixing between the scalar derivative  $\dot{\phi}$  and the temporal vector component  $A_0$ . If  $\beta_m = 0$ , then the system reduces to the single-field slow-roll inflation with  $A_0 = 0$ . During the inflationary stage in which  $H$  is nearly constant, the ratio  $A_0/\dot{\phi}$  stays nearly constant. This property also holds for the case in which the condition  $|\beta_A M^2| \gg |6\beta_G H^2|$  is satisfied.

To study the dynamics of inflation, it is convenient to define the following slow-roll parameters [50]:

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \epsilon_V \equiv \frac{M_{\text{pl}}^2}{2} \left( \frac{V_{,\phi}}{V} \right)^2, \quad \eta \equiv \frac{\ddot{\phi}}{H\dot{\phi}}, \quad (6.6)$$

whose orders are less than 1 during inflation. Substituting Eq. (6.5) and its time derivative into Eq. (6.3) and using Eq. (6.2) to eliminate  $V$ , we find that  $\epsilon$  can be expressed as

$$\epsilon = \frac{\dot{\phi}^2(\beta_A M^2 + 6\beta_G H^2)[144\beta_G^2 H^4 + 2\beta_G H^2 M^2\{24\beta_A - (3 + 2\eta)\beta_m^2\} + \beta_A M^4(4\beta_A - \beta_m^2)]}{2H^2[4M_{\text{pl}}^2(\beta_A M^2 + 6\beta_G H^2)^3 + \beta_m^2 \beta_G M^2(18\beta_G H^2 - \beta_A M^2)\dot{\phi}^2]}. \quad (6.7)$$

We employ the approximation that the three slow-roll parameters defined in Eq. (6.6) are smaller than the order 1. From Eqs. (6.4) and (6.5), it follows that

$$\dot{\phi} \simeq -\frac{4V_{,\phi}(\beta_A M^2 + 6\beta_G H^2)}{3H(4\beta_A M^2 + 24\beta_G H^2 - \beta_m^2 M^2)}. \quad (6.8)$$

Substituting Eq. (6.5) into Eq. (6.2), the ratio between the vector kinetic energy  $K_A = -\beta_A M^2 A_0^2/2 + 3\beta_G H^2 A_0^2$  and the scalar kinetic energy  $K_\phi = \dot{\phi}^2/2$  is given by

$$\frac{K_A}{K_\phi} = \frac{M^2(6\beta_G H^2 - \beta_A M^2)}{4(6\beta_G H^2 + \beta_A M^2)^2} \beta_m^2. \quad (6.9)$$

In the following, we assume that  $\beta_m^2$  is at most of the orders  $|\beta_G|$  and  $|\beta_A|$ , in which case  $K_A$  does not exceed  $K_\phi$ . Then, the scalar potential  $V$  dominates over the other terms on the r.h.s. of Eq. (6.2) during slow-roll inflation, so the Hubble expansion rate can be estimated as  $H \simeq \sqrt{V/3}/M_{\text{pl}}$ . We substitute this relation and Eq. (6.8) into Eq. (6.7) and neglect the slow-roll parameters  $\eta$  and  $\epsilon_V$  relative to 1 in the end. This process leads to

$$\epsilon \simeq \epsilon_V \left[ 1 + \frac{\beta_m^2 M^2 M_{\text{pl}}^2}{4(\beta_A M^2 M_{\text{pl}}^2 + 2\beta_G V)} \right], \quad (6.10)$$

where we employed the approximation that  $|\beta_m| \ll 1$  and picked up the leading-order contribution of  $\beta_m$  to  $\epsilon$ . For  $\beta_m = 0$ , we have  $\epsilon \simeq \epsilon_V$  as in standard slow-roll inflation, but the presence of the coupling  $\beta_m$  leads to  $\epsilon \neq \epsilon_V$ . If  $\beta_A M^2 M_{\text{pl}}^2 + 2\beta_G V > 0$ , then  $\epsilon$  is larger than  $\epsilon_V$ . In this case, the mixing between  $\dot{\phi}$  and  $A_0$  effectively leads to the faster evolution of inflaton, so inflation tends to be less efficient for a given potential  $V(\phi)$ .

To confirm the above analytic estimation, we consider the  $\alpha$ -attractor model given by the potential [53]

$$V(\phi) = \frac{M^2 M_{\text{pl}}^2}{2\alpha_c^2} \left( 1 - e^{-\alpha_c \phi/M_{\text{pl}}} \right)^2, \quad (6.11)$$

where  $\alpha_c$  is a positive constant. The Starobinsky inflation [54] corresponds to  $\alpha_c = \sqrt{6}/3$  in the Einstein frame [55]. Inflation occurs for  $\alpha_c \phi/M_{\text{pl}} \gg 1$ , in which regime the potential is nearly constant:  $V(\phi) \simeq M^2 M_{\text{pl}}^2/(2\alpha_c^2)$ . Then,

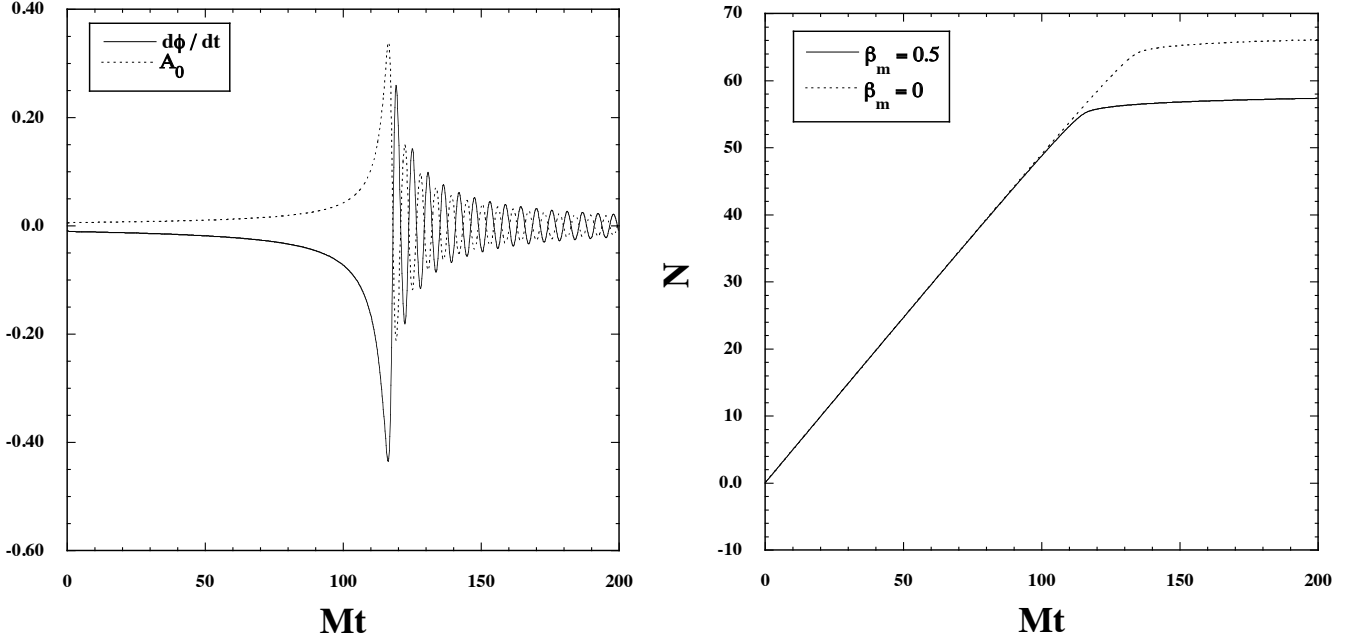


FIG. 1. (Left) Evolution of  $\dot{\phi}$  and  $A_0$  (normalized by  $MM_{\text{Pl}}$  and  $M_{\text{Pl}}$  respectively) during inflation and reheating for the potential (6.11) with  $\alpha_c = \sqrt{6}/3$ . We choose the coupling constants  $\beta_m = 0.5$ ,  $\beta_A = 0.3$ , and  $\beta_G = 0.1$  with the initial conditions  $\phi = 5.5M_{\text{Pl}}$  and  $\dot{\phi} = -9.0 \times 10^{-3}MM_{\text{Pl}}$ . For the consistency with Eqs. (6.2) and (6.5), the initial values of  $H$  and  $A_0$  are chosen to be  $H = 0.4944M$  and  $A_0 = 5.0374 \times 10^{-3}M_{\text{Pl}}$ . (Right) Evolution of the number of e-foldings  $N$  for the case shown in the left panel (plotted as a solid line). The dotted line corresponds to the case  $\beta_m = 0$  and  $\alpha_c = \sqrt{6}/3$  (i.e., standard Starobinsky inflation) with the same initial conditions of  $\phi$  and  $\dot{\phi}$  as those used in the left panel.

the Hubble expansion rate during inflation  $H_{\text{inf}}$  is related to the mass  $M$ , as  $H_{\text{inf}} \simeq M/(\sqrt{6}\alpha_c)$ . For  $\alpha_c = \mathcal{O}(1)$ ,  $H_{\text{inf}}$  is of the same order as  $M$ . The system enters the reheating stage for  $|\alpha_c\phi/M_{\text{Pl}}| \lesssim 1$ , during which the potential is approximately given by  $V(\phi) \simeq M^2\phi^2/2$ .

In the left panel of Fig. 1, we show the numerical solutions to  $\dot{\phi}$  and  $A_0$  versus  $Mt$  for the Starobinsky potential ( $\alpha_c = \sqrt{6}/3$ ) with the couplings  $\beta_m = 0.5$ ,  $\beta_A = 0.3$ , and  $\beta_G = 0.1$ . The initial conditions of  $\phi$ ,  $\dot{\phi}$ , and  $A_0$  are chosen to be consistent with Eqs. (6.2) and (6.5). As estimated from Eq. (6.5), the ratio between  $A_0$  and  $\dot{\phi}$  stays nearly constant ( $A_0/M_{\text{Pl}} \simeq -0.56\dot{\phi}/(MM_{\text{Pl}})$ ) during inflation. In the reheating stage, the term  $6\beta_G H^2$  starts to be negligible relative to  $\beta_A M^2$  due to the decrease of  $H$ . Then, the amplitude of  $A_0$  decreases in the same way as that of  $\dot{\phi}$  according to the relation  $A_0/M_{\text{Pl}} = -[\beta_m/(2\beta_A)]\dot{\phi}/(MM_{\text{Pl}}) = -0.83\dot{\phi}/(MM_{\text{Pl}})$ . In the left panel of Fig. 1, we can confirm that  $\dot{\phi}$  and  $A_0$  slowly evolve during inflation with the relation mentioned above and that they oscillate during reheating with the asymptotic behavior  $A_0/\dot{\phi} = \text{constant}$ .

In the right panel of Fig. 1, the number of e-foldings  $N = \ln a$  from the onset of inflation is plotted for the model parameters and initial conditions same as those used in the left panel. We also show the evolution of  $N$  for  $\beta_m = 0$ , i.e., Starobinsky inflation with  $A_0 = 0$ . For  $\beta_m = 0.5$ , the value of  $N$  reached at the end of inflation is smaller than that for  $\beta_m = 0$  about by 14 %. This is consistent with the fact that the slow-roll parameter (6.10) for  $\beta_m = 0.5$  can be estimated as  $\epsilon \simeq 1.14\epsilon_V$ . Thus, the nonvanishing coupling  $\beta_m$  leads to the smaller amount of inflation due to the additional evolution of  $A_0$  besides  $\dot{\phi}$ .

## B. Stability conditions

For the model (6.1), the quantities  $q_t$  and  $c_t^2$  are given, respectively, by

$$q_t = M_{\text{Pl}}^2 - \frac{\beta_m^2 \beta_G M^2 \dot{\phi}^2}{4(\beta_A M^2 + 6\beta_G H^2)^2}, \quad c_t^2 = 1 + \frac{2\beta_m^2 \beta_G M^2 \dot{\phi}^2}{4(\beta_A M^2 + 6\beta_G H^2)^2 M_{\text{Pl}}^2 - \beta_m^2 \beta_G M^2 \dot{\phi}^2}, \quad (6.12)$$

where we used Eq. (6.5) to express  $A_0$  in terms of  $\dot{\phi}$ . Then, the stability conditions (3.5) of tensor perturbations translate to

$$-4(\beta_A M^2 + 6\beta_G H^2)^2 M_{\text{pl}}^2 \leq \beta_m^2 \beta_G M^2 \dot{\phi}^2 < 4(\beta_A M^2 + 6\beta_G H^2)^2 M_{\text{pl}}^2. \quad (6.13)$$

For vector perturbations, we have

$$q_v = 1, \quad c_v^2 = 1 + \frac{2\beta_m^2 \beta_G^2 M^2 \dot{\phi}^2}{4(\beta_A M^2 + 6\beta_G H^2)^2 M_{\text{pl}}^2 - \beta_m^2 \beta_G M^2 \dot{\phi}^2}, \quad (6.14)$$

so the no-ghost condition is automatically satisfied. Under the conditions (6.13), there are no Laplacian instabilities of vector perturbations.

For scalar perturbations, the quantity  $K_{22}$  defined in Eq. (5.12) is given by

$$K_{22} = \frac{1}{2}, \quad (6.15)$$

and hence the latter condition of Eq. (5.15) is always satisfied. The other no-ghost condition translates to

$$q_s = \left[ \beta_m^2 \beta_G M^2 \{(\beta_m^2 - 4\beta_A)M^2 + 72\beta_G H^2\} \dot{\phi}^2 - 4M_{\text{pl}}^2 (\beta_A M^2 + 6\beta_G H^2)^2 \{(\beta_m^2 - 4\beta_A)M^2 - 24\beta_G H^2\} \right] \\ \times \left[ 4(6\beta_G H^2 + \beta_A M^2)^2 M_{\text{pl}}^2 - \beta_m^2 \beta_G M^2 \dot{\phi}^2 \right] / \left[ 16 \left\{ 4(6\beta_G H^2 + \beta_A M^2)^2 M_{\text{pl}}^2 - 3\beta_m^2 \beta_G M^2 \dot{\phi}^2 \right\}^2 \right] > 0. \quad (6.16)$$

If  $\beta_G = 0$ , then the condition (6.16) translates to  $\beta_A > \beta_m^2/4$ . To satisfy this inequality, it is necessary to have  $\beta_A > 0$ , which means that the mass squared of the vector field  $A_\mu$  is positive.

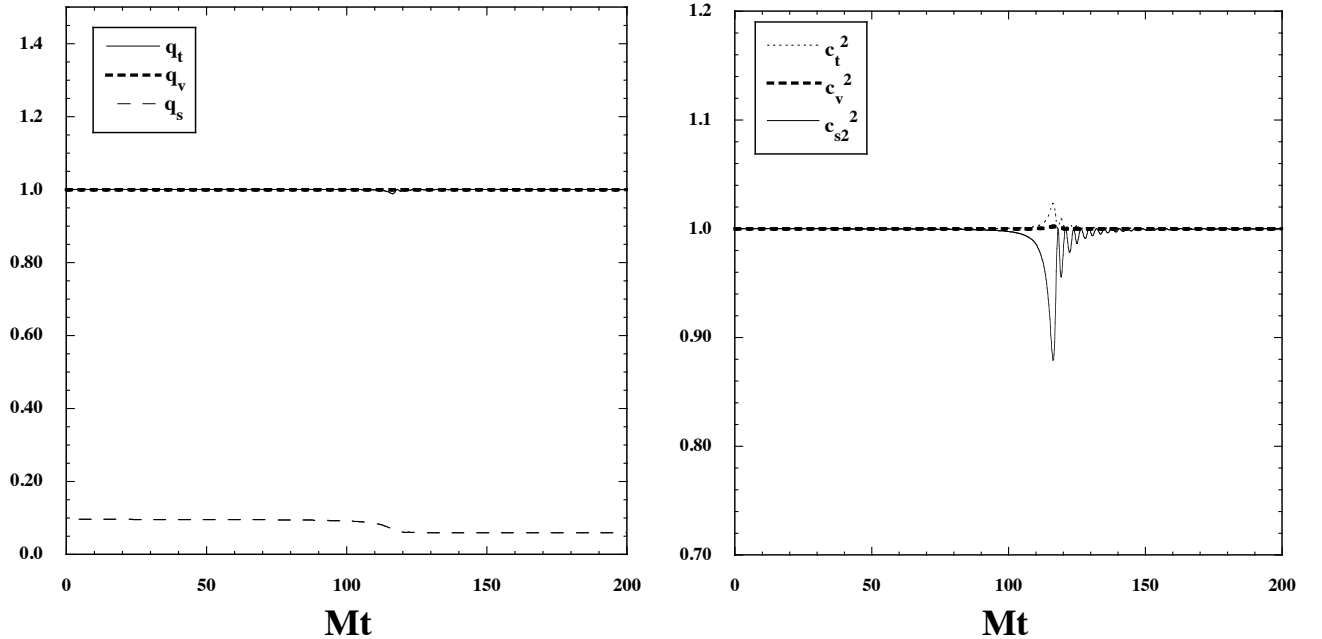


FIG. 2. Evolution of  $q_t, q_v, q_s$  (left) and  $c_t^2, c_v^2, c_{s2}^2$  (right) for the same model parameters and initial conditions as those used in the left panel of Fig. 1. Note that  $q_t$  and  $q_s$  are normalized by  $M_{\text{pl}}^2$  and  $M^2$ , respectively.

In addition to Eq. (6.15), we also have  $G_{22} = 1/2$  and  $K_{12} = G_{12}$  for the model (6.1). Substituting them into Eqs. (5.18)-(5.19), it follows that one of the scalar propagation speed squares reduces to

$$c_{s1}^2 = 1, \quad (6.17)$$

while the other is given by  $c_{s2}^2 = (G_{11} - 2K_{12}^2)/(K_{11} - 2K_{12}^2) = 1 + (G_{11} - K_{11})/(2q_s)$ . More explicitly, the latter is expressed as

$$c_{s2}^2 = 1 + \frac{1}{2q_s(M_{\text{pl}}^2 - 3\beta_G A_0^2)^2} \left[ 2\beta_G(M_{\text{pl}}^2 - 3\beta_G A_0^2)(M_{\text{pl}}^2 - \beta_G A_0^2)\dot{H} + 8\beta_G^2 M_{\text{pl}}^2 H A_0 \dot{A}_0 \right. \\ \left. + 32\beta_G^4 H^2 A_0^4 - \beta_m \beta_G^2 M A_0^3 \dot{\phi} - 2\beta_G^2 (4M_{\text{pl}}^2 H^2 + \dot{\phi}^2) A_0^2 \right]. \quad (6.18)$$

Let us derive an approximate expression of  $c_{s2}^2$  for the coupling  $\beta_m^2$  smaller than the order 1. Eliminating the terms  $\dot{H}$ ,  $\dot{A}_0$ ,  $A_0$  in Eq. (6.18) by using the background equations of motion, we find

$$c_{s2}^2 = 1 - \frac{2\beta_G \dot{\phi}^2}{(\beta_A M^2 + 6\beta_G H^2) M_{\text{pl}}^2} + \mathcal{O}(\beta_m^2). \quad (6.19)$$

Since  $\dot{\phi}^2$  is smaller than the orders  $H^2 M_{\text{pl}}^2$  and  $M^2 M_{\text{pl}}^2$  during inflation, the condition  $c_{s2}^2 > 0$  can be satisfied for  $|\beta_G| \lesssim |\beta_A|$ .

In Fig. 2, we show the evolution of  $q_t, q_v, q_s$  and  $c_t^2, c_v^2, c_{s2}^2$  for the same model parameters and initial conditions as those used in the left panel of Fig. 1. As estimated from Eq. (6.12), the quantity  $q_t$  is close to  $M_{\text{pl}}^2$  during inflation and reheating with a small deviation induced by the time variation of  $\phi$ . In the numerical simulation of Fig. 2, the quantity  $q_s$  also remains positive with  $q_v = 1$ . Hence there are no ghosts of tensor, vector, and scalar perturbations.

As we observe in the right panel of Fig. 2, the deviations of  $c_t^2$  and  $c_v^2$  from 1 are smaller than the order 0.1, so there are no Laplacian instabilities of tensor and vector perturbations. The scalar propagation speed squared  $c_{s2}^2$  deviates from 1 in the transient regime from inflation to reheating. This comes from the fact that  $\dot{\phi}^2$  reaches a maximum around the end of inflation. In the numerical simulation of Fig. 2, the peak value of  $|\dot{\phi}|$  is about  $0.435 M M_{\text{pl}}$  with  $\phi \simeq 0.255 M_{\text{pl}}$  and the Hubble expansion rate  $H \simeq 0.187 M$  around  $Mt \simeq 116$ , in which case the analytic estimation (6.19) gives  $c_{s2}^2 \simeq 0.88$ . This exhibits good agreement with the minimum value of  $c_{s2}^2$  seen in Fig. 2. After the onset of reheating, the term  $6\beta_G H^2$  becomes negligible relative to  $\beta_A M^2$ , and hence  $c_{s2}^2$  approaches to 1 according to the relation  $c_{s2}^2 \simeq 1 - 2\beta_G \dot{\phi}^2 / (\beta_A M^2 M_{\text{pl}}^2)$  with the damped oscillation of  $\dot{\phi}$ . Thus, the scalar perturbations are free from Laplacian instabilities for the model parameters used in Fig. 2.

## VII. CONCLUSIONS

In this paper, we have studied cosmological implications of SVT theories with nonlinear derivative scalar and vector-field interactions and nonminimal couplings to gravity [36]. In Sec. II, we obtained the background equations of motion on the flat FLRW spacetime for  $U(1)$  broken SVT theories given by the action (2.4). The time-dependent scalar field  $\phi(t)$  as well as the temporal vector component  $A_0(t)$  contribute to the background cosmological dynamics relevant to the physics of cosmic acceleration. The  $U(1)$  broken SVT theories contain six propagating DOFs—two scalar modes, two transverse vector modes, and two tensor polarizations.

In Sec. III, we derived conditions for the absence of ghosts and Laplacian instabilities in the tensor sector. The two polarized states of tensor perturbations have the propagation speed  $c_t$  given by Eq. (3.4). Applying these results to the late-time cosmology, we showed that the functions  $f_4$  and  $f_5$  are restricted to be of the form (3.6) for the realization of  $c_t$  equivalent to that of light. We also computed the leading-order primordial power spectrum of tensor perturbations generated during inflation, see Eq. (3.12).

In Sec. IV, we considered the vector perturbations  $Z_i$  and  $V_i$  arising from the spatial part of  $A_\mu$  and the shift  $V_i$  in the metric, respectively, and obtained their second-order actions of the form (4.3). On using the equation of motion for the non-dynamical field  $V_i$ , the final action (4.9) of vector perturbations contains two propagating fields  $Z_1$  and  $Z_2$  with the same propagation speed squared  $c_v^2$  given by Eq. (4.10). In the small-scale limit where the vector-field mass squared is irrelevant to the dynamics of perturbations, there are neither ghost nor Laplacian instabilities under the conditions (4.12).

In Sec. V, we derived the quadratic action of scalar perturbations by considering metric perturbations  $\alpha, \chi$  in the flat gauge, scalar perturbations  $\delta A, \psi$  arising from the temporal and spatial components of  $A_\mu$  respectively, and the scalar-field perturbation  $\delta\phi$ . After integrating out the non-dynamical fields  $\alpha, \chi, \delta A$ , the action of dynamical perturbations  $\psi$  and  $\delta\phi$  is expressed in the form (5.10). In the small-scale limit, the no-ghost conditions of scalar perturbations are given by Eqs. (5.15) and (5.16). We also derived the two different propagation speed squares  $c_{s1}^2$  and  $c_{s2}^2$ , both of which need to be positive to avoid small-scale Laplacian instabilities. We found that intrinsic vector modes do not affect no-ghost conditions of scalar perturbations, but they can modify the values of  $c_{s1}^2$  and  $c_{s2}^2$ .

In Sec. VI, we constructed a concrete model of inflation in the framework of SVT theories characterized by the functions (6.1). The nonvanishing coupling  $\beta_m$  gives rise to a kinetic mixing between  $\phi$  and  $A_0$ , so that the amount

of inflation is modified due to the change of the slow-roll parameter  $\epsilon = -\dot{H}/H^2$ . This property was numerically confirmed for the inflaton potential of the  $\alpha$ -attractor model, see Fig. 1. We also showed that, under certain bounds of the coupling constants  $\beta_m, \beta_G, \beta_A$ , this model can satisfy all the no-ghost and stability conditions of tensor, vector, and scalar perturbations during inflation and reheating.

It will be of interest to compute inflationary observables relevant to CMB temperature anisotropies for the model proposed in this paper. In particular, the contribution of vector perturbations to CMB temperature anisotropies is one of distinguished features of SVT theories. The coupling between the scalar and vector fields can also modify observational predictions of standard slow-roll inflation, e.g., the scalar spectral index and the tensor-to-scalar ratio. Moreover, it will be interesting to apply SVT theories to dark energy and estimate observables associated with the background and perturbations. These issues are left for future works.

## ACKNOWLEDGEMENTS

LH thanks financial support from Dr. Max Rössler, the Walter Haefner Foundation and the ETH Zurich Foundation. RK is supported by the Grant-in-Aid for Young Scientists B of the JSPS No. 17K14297. ST is supported by the Grant-in-Aid for Scientific Research Fund of the JSPS No. 16K05359 and MEXT KAKENHI Grant-in-Aid for Scientific Research on Innovative Areas ‘‘Cosmic Acceleration’’ (No.15H05890). ST thanks warm hospitality to ETH-ITS Zurich in which a part of this work was done.

## Appendix A: Coefficients in the second-order action of scalar perturbations

The coefficients  $D_{1,\dots,10}$  and  $w_{1,\dots,8}$  appearing in Eqs. (5.5)-(5.6) are given by

$$\begin{aligned}
D_1 &= \frac{1}{2} \left( f_{2,X_1} + \dot{\phi}^2 f_{2,X_1 X_1} + \dot{\phi} A_0 f_{2,X_1 X_2} + \frac{A_0^2}{4} f_{2,X_2 X_2} \right), \\
D_2 &= -\frac{1}{2} f_{2,X_1}, \\
D_3 &= (3f_{4,\phi\phi} + 3HA_0 f_{5,\phi\phi}) \ddot{H} - \frac{1}{2} \left( f_{2,X_1\phi} + \dot{\phi}^2 f_{2,X_1 X_1\phi} + \dot{\phi} A_0 f_{2,X_1 X_2\phi} + \frac{A_0^2}{4} f_{2,X_2 X_2\phi} \right) \ddot{\phi} \\
&\quad + \frac{H^3 A_0 (9f_{5,\phi\phi} + A_0^2 f_{5,X_3\phi\phi})}{2} + 3H^2 \left[ 2f_{4,\phi\phi} + A_0^2 f_{4,X_3\phi\phi} + \frac{\dot{A}_0 (f_{5,\phi\phi} + A_0^2 f_{5,X_3\phi\phi})}{2} \right] \\
&\quad - 3H \left[ \frac{\dot{\phi}}{2} f_{2,X_1\phi} + \frac{A_0 (f_{2,X_2\phi} + 4f_{3,\phi\phi})}{4} - A_0 \dot{A}_0 f_{4,X_3\phi\phi} \right] - \frac{\dot{\phi}^2 (2f_{2,X_1\phi\phi} + \dot{A}_0 f_{2,X_1 X_2\phi})}{4} \\
&\quad - \frac{\dot{\phi} A_0}{4} \left[ f_{2,X_2\phi\phi} + \frac{\dot{A}_0 (4f_{2,X_1 X_3\phi} + f_{2,X_2 X_2\phi})}{2} \right] - \dot{A}_0 \left( f_{3,\phi\phi} - A_0^2 \tilde{f}_{3,\phi\phi} + \frac{f_{2,X_2\phi} + A_0^2 f_{2,X_2 X_3\phi}}{4} \right) + \frac{f_{2,\phi\phi}}{2}, \\
D_4 &= -\dot{\phi}^3 f_{2,X_1 X_1} - \frac{\dot{\phi}^2 A_0 f_{2,X_1 X_2}}{2} - \dot{\phi} (f_{2,X_1} - A_0^2 f_{2,X_1 X_3}) + 3H^2 A_0 (f_{5,\phi} - A_0^2 f_{5,X_3\phi}) + 6H (f_{4,\phi} - A_0^2 f_{4,X_3\phi}) \\
&\quad + \frac{A_0 (f_{2,X_2} + A_0^2 f_{2,X_2 X_3} + 4f_{3,\phi} - 4A_0^2 \tilde{f}_{3,\phi})}{2}, \\
D_5 &= H^3 A_0^3 (f_{5,X_3\phi} + A_0^2 f_{5,X_3 X_3\phi}) + 3H^2 \left[ \dot{\phi} A_0 (f_{5,\phi\phi} - A_0^2 f_{5,X_3\phi\phi}) + 2 (f_{4,\phi} + A_0^4 f_{4,X_3 X_3\phi}) \right] \\
&\quad + 6H \left[ \dot{\phi} (f_{4,\phi\phi} - A_0^2 f_{4,X_3\phi\phi}) - A_0^3 (\tilde{f}_{3,\phi} + f_{3,X_3\phi}) \right] + 2\dot{\phi} A_0 (f_{3,\phi\phi} - A_0^2 \tilde{f}_{3,\phi\phi}) - \dot{\phi}^2 f_{2,X_1\phi} + A_0^2 f_{2,X_3\phi} + f_{2,\phi}, \\
D_6 &= -2 (f_{4,\phi} + HA_0 f_{5,\phi}), \\
D_7 &= -H^2 A_0 (3f_{5,\phi} + A_0^2 f_{5,X_3\phi}) - 2H (f_{4,\phi} + 2A_0^2 f_{4,X_3\phi} - \dot{\phi} A_0 f_{5,\phi\phi}) + \dot{\phi} (f_{2,X_1} + 2f_{4,\phi\phi}) + \frac{A_0 (f_{2,X_2} + 4f_{3,\phi})}{2}, \\
D_8 &= -\frac{2\dot{\phi} D_1 + D_4 + 3H D_6}{A_0}, \\
D_9 &= -\frac{D_5}{A_0} - 2H^3 A_0^2 f_{5,X_3\phi} + 6H^2 \left( \dot{\phi} f_{5,\phi\phi} + \frac{f_{4,\phi} - A_0^2 f_{4,X_3\phi}}{A_0} \right) + \frac{6H \dot{\phi} f_{4,\phi\phi}}{A_0} + \frac{2f_{2,\phi} - 2\dot{\phi}^2 f_{2,X_1\phi} - \dot{\phi} A_0 f_{2,X_2\phi}}{2A_0},
\end{aligned}$$



$$D_{10} = -2\dot{H}f_{5,\phi} - H^2(3f_{5,\phi} + A_0^2f_{5,X_3\phi}) - 2HA_0(2f_{4,X_3\phi} + \dot{A}_0f_{5,X_3\phi}) - 2\dot{A}_0f_{4,X_3\phi} + 2f_{3,\phi} + \frac{f_{2,X_2}}{2}, \quad (A1)$$

and

$$\begin{aligned} w_1 &= -H^2A_0^3(f_{5,X_3} + A_0^2f_{5,X_3X_3}) - 2H\left[\dot{\phi}A_0(f_{5,\phi} - A_0^2f_{5,X_3\phi}) + 2f_4 + 2A_0^4f_{4,X_3X_3}\right], \\ &\quad -2\dot{\phi}(f_{4,\phi} - A_0^2f_{4,X_3\phi}) + 2A_0^3(\tilde{f}_3 + f_{3,X_3}), \\ w_2 &= w_1 + 2Hq_t - \dot{\phi}D_6, \\ w_3 &= -2A_0^2q_v, \\ w_4 &= -\frac{H^3A_0^3(9f_{5,X_3} - A_0^4f_{5,X_3X_3X_3})}{2} - 3H^2(2f_4 + 2A_0^2f_{4,X_3} + A_0^4f_{4,X_3X_3} - A_0^6f_{4,X_3X_3X_3}) \\ &\quad + \frac{3H^2\dot{\phi}A_0(f_{5,\phi} + 4A_0^2f_{5,X_3\phi} - A_0^4f_{5,X_3X_3\phi})}{2} - 3H\dot{\phi}(2f_{4,\phi} - 4A_0^2f_{4,X_3\phi} + A_0^4f_{4,X_3X_3\phi}) \\ &\quad + 3HA_0^3\left[\tilde{f}_3 + f_{3,X_3} - A_0^2(\tilde{f}_{3,X_3} + f_{3,X_3X_3})\right] - \dot{\phi}A_0\left[3f_{3,\phi} - A_0^2(\tilde{f}_{3,\phi} + f_{3,X_3\phi}) + A_0^4\tilde{f}_{3,X_3\phi}\right] \\ &\quad + \frac{1}{2}\left[\dot{\phi}^4f_{2,X_1X_1} + \dot{\phi}^2(f_{2,X_1} - 2A_0^2f_{2,X_1X_3}) + A_0^4f_{2,X_3X_3} - \frac{3\dot{\phi}A_0f_{2,X_2}}{2}\right], \\ w_5 &= w_4 - \frac{3H(w_1 + w_2)}{2} - 3H^2\dot{\phi}A_0^3f_{5,X_3\phi} + 3H\dot{\phi}(f_{4,\phi} - 2A_0^2f_{4,X_3\phi}) + 2\dot{\phi}A_0(f_{3,\phi} - A_0^2\tilde{f}_{3,\phi}) \\ &\quad + \frac{\dot{\phi}}{2}\left[\dot{\phi}A_0^2\left(2f_{2,X_1X_3} + \frac{1}{4}f_{2,X_2X_2}\right) - \dot{\phi}^3f_{2,X_1X_1} + A_0^3f_{2,X_2X_3} - \dot{\phi}f_{2,X_1} + A_0f_{2,X_2}\right], \\ w_6 &= -\frac{2w_1 - w_2 - 2\dot{\phi}D_6 + 8Hf_4}{A_0}, \\ w_7 &= -2\dot{H}(2f_{4,X_3} + HA_0f_{5,X_3}) - H^2\left[\frac{\dot{\phi}(3f_{5,\phi} + A_0^2f_{5,X_3\phi})}{A_0} + \dot{A}_0(f_{5,X_3} + A_0^2f_{5,X_3X_3})\right] \\ &\quad - 4H(\dot{\phi}f_{4,X_3\phi} + A_0\dot{A}_0f_{4,X_3X_3}) + 2\dot{A}_0(\tilde{f}_3 + f_{3,X_3}) + \frac{\dot{\phi}(4f_{3,\phi} + f_{2,X_2})}{2A_0}, \\ w_8 &= 3Hw_1 - 2w_4 - \dot{\phi}D_4. \end{aligned} \quad (A2)$$

- 
- [1] D. N. Spergel *et al.*, *Astrophys. J. Suppl.* **148**, 175 (2003) [astro-ph/0302209]; P. A. R. Ade *et al.*, *Astron. Astrophys.* **571**, A16 (2014) [arXiv:1303.5076 [astro-ph.CO]].
- [2] A. G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998) [astro-ph/9805201]; S. Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999) [astro-ph/9812133]; M. Betoule *et al.*, *Astron. Astrophys.* **568**, A22 (2014) [arXiv:1401.4064 [astro-ph.CO]].
- [3] M. Tegmark *et al.* [SDSS Collaboration], *Phys. Rev. D* **69**, 103501 (2004) [astro-ph/0310723]; M. Tegmark *et al.* [SDSS Collaboration], *Phys. Rev. D* **74**, 123507 (2006) [astro-ph/0608632].
- [4] E. J. Copeland, M. Sami and S. Tsujikawa, *Int. J. Mod. Phys. D* **15**, 1753 (2006) [hep-th/0603057]; T. Clifton, P. G. Ferreira, A. Padilla and C. Skordis, *Phys. Rept.* **513**, 1 (2012) [arXiv:1106.2476 [astro-ph.CO]]; A. Joyce, B. Jain, J. Khoury and M. Trodden, *Phys. Rept.* **568**, 1 (2015) [arXiv:1407.0059 [astro-ph.CO]]. P. Bull *et al.*, *Phys. Dark Univ.* **12**, 56 (2016) [arXiv:1512.05356 [astro-ph.CO]]; L. Amendola *et al.* [Euclid Theory Working Group], *Living Rev. Rel.* **16**, 6 (2013) [arXiv:1206.1225 [astro-ph.CO]]; L. Amendola *et al.*, *Living Rev. Rel.* **21**, 2 (2018) [arXiv:1606.00180 [astro-ph.CO]].
- [5] M. Gasperini and G. Veneziano, *Phys. Rept.* **373**, 1 (2003) [hep-th/0207130].
- [6] L. Amendola, *Phys. Lett. B* **301**, 175 (1993) [gr-qc/9302010].
- [7] M. V. Ostrogradsky, *Mem. Acad. St. Petersburg* **VI** **4**, 385 (1850).
- [8] C. Deffayet, G. Esposito-Farese and A. Vikman, *Phys. Rev. D* **79**, 084003 (2009) [arXiv:0901.1314 [hep-th]].
- [9] G. W. Horndeski, *Int. J. Theor. Phys.* **10**, 363 (1974).
- [10] C. Deffayet, X. Gao, D. A. Steer and G. Zahariade, *Phys. Rev. D* **84**, 064039 (2011) [arXiv:1103.3260 [hep-th]]; C. Charmousis, E. J. Copeland, A. Padilla and P. M. Saffin, *Phys. Rev. Lett.* **108**, 051101 (2012) [arXiv:1106.2000 [hep-th]].
- [11] T. Kobayashi, M. Yamaguchi and J. 'i. Yokoyama, *Prog. Theor. Phys.* **126**, 511 (2011) [arXiv:1105.5723 [hep-th]].
- [12] T. Kobayashi, M. Yamaguchi and J. Yokoyama, *Phys. Rev. Lett.* **105**, 231302 (2010) [arXiv:1008.0603 [hep-th]].
- [13] C. Burrage, C. de Rham, D. Seery and A. J. Tolley, *JCAP* **1101**, 014 (2011) [arXiv:1009.2497 [hep-th]].
- [14] T. Kobayashi, M. Yamaguchi and J. Yokoyama, *Prog. Theor. Phys.* **126**, 511 (2011) [arXiv:1105.5723 [hep-th]].

- [15] A. De Felice and S. Tsujikawa, Phys. Rev. D **84**, 083504 (2011) [arXiv:1107.3917 [gr-qc]].
- [16] R. Gannouji and M. Sami, Phys. Rev. D **82**, 024011 (2010) [arXiv:1004.2808 [gr-qc]].
- [17] A. De Felice and S. Tsujikawa, Phys. Rev. Lett. **105**, 111301 (2010) [arXiv:1007.2700 [astro-ph.CO]]; A. De Felice and S. Tsujikawa, Phys. Rev. D **84**, 124029 (2011) [arXiv:1008.4236 [hep-th]].
- [18] A. De Felice, T. Kobayashi and S. Tsujikawa, Phys. Lett. B **706**, 123 (2011) [arXiv:1108.4242 [gr-qc]]; A. De Felice and S. Tsujikawa, JCAP **1202**, 007 (2012) [arXiv:1110.3878 [gr-qc]].
- [19] C. Charmousis, E. J. Copeland, A. Padilla and P. M. Saffin, Phys. Rev. D **85**, 104040 (2012) [arXiv:1112.4866 [hep-th]].
- [20] E. Bellini and I. Sawicki, JCAP **1407**, 050 (2014) [arXiv:1404.3713 [astro-ph.CO]]; M. Zumalacarregui, E. Bellini, I. Sawicki, J. Lesgourgues and P. G. Ferreira, JCAP **1708**, 019 (2017) [arXiv:1605.06102 [astro-ph.CO]].
- [21] C. de Rham and L. Heisenberg, Phys. Rev. D **84**, 043503 (2011) [arXiv:1106.3312 [hep-th]]; L. Heisenberg, R. Kimura and K. Yamamoto, Phys. Rev. D **89**, 103008 (2014) [arXiv:1403.2049 [hep-th]].
- [22] C. Deffayet, A. E. Gumrukcuoglu, S. Mukohyama and Y. Wang, JHEP **1404**, 082 (2014). [arXiv:1312.6690 [hep-th]]; C. Deffayet, S. Mukohyama and V. Sivanesan, Phys. Rev. D **93**, 085027 (2016) [arXiv:1601.01287 [hep-th]].
- [23] L. Heisenberg, JCAP **1405**, 015 (2014) [arXiv:1402.7026 [hep-th]].
- [24] E. Allys, P. Peter and Y. Rodriguez, JCAP **1602**, 004 (2016) [arXiv:1511.03101 [hep-th]].
- [25] J. Beltran Jimenez and L. Heisenberg, Phys. Lett. B **757**, 405 (2016) [arXiv:1602.03410 [hep-th]].
- [26] G. Tasinato, JHEP **1404**, 067 (2014) [arXiv:1402.6450 [hep-th]]; G. Tasinato, Class. Quant. Grav. **31**, 225004 (2014) [arXiv:1404.4883 [hep-th]].
- [27] A. De Felice, L. Heisenberg, R. Kase, S. Mukohyama, S. Tsujikawa and Y. I. Zhang, JCAP **1606**, 048 (2016) [arXiv:1603.05806 [gr-qc]].
- [28] A. De Felice, L. Heisenberg, R. Kase, S. Mukohyama, S. Tsujikawa and Y. I. Zhang, Phys. Rev. D **94**, 044024 (2016) [arXiv:1605.05066 [gr-qc]].
- [29] A. De Felice, L. Heisenberg and S. Tsujikawa, Phys. Rev. D **95**, 123540 (2017) [arXiv:1703.09573 [astro-ph.CO]].
- [30] A. De Felice, L. Heisenberg, R. Kase, S. Tsujikawa, Y. I. Zhang and G. B. Zhao, Phys. Rev. D **93**, 104016 (2016) [arXiv:1602.00371 [gr-qc]]; S. Nakamura, R. Kase and S. Tsujikawa, Phys. Rev. D **96**, no. 8, 084005 (2017) [arXiv:1707.09194 [gr-qc]].
- [31] J. Chagoya, G. Niz and G. Tasinato, Class. Quant. Grav. **33**, 175007 (2016) [arXiv:1602.08697 [hep-th]]; J. Chagoya, G. Niz and G. Tasinato, Class. Quant. Grav. **34**, 165002 (2017) [arXiv:1703.09555 [gr-qc]]; J. Chagoya and G. Tasinato, JCAP **1801**, 046 (2018) [arXiv:1707.07951 [hep-th]].
- [32] Z. Y. Fan, JHEP **1609**, 039 (2016) [arXiv:1606.00684 [hep-th]].
- [33] M. Minamitsuji, Phys. Rev. D **94**, 084039 (2016) [arXiv:1607.06278 [gr-qc]].
- [34] A. Cisterna, M. Hassaine, J. Oliva and M. Rinaldi, Phys. Rev. D **94**, 104039 (2016). [arXiv:1609.03430 [gr-qc]].
- [35] L. Heisenberg, R. Kase, M. Minamitsuji and S. Tsujikawa, Phys. Rev. D **96**, 084049 (2017) [arXiv:1705.09662 [gr-qc]]; L. Heisenberg, R. Kase, M. Minamitsuji and S. Tsujikawa, JCAP **1708**, 024 (2017) [arXiv:1706.05115 [gr-qc]]; R. Kase, M. Minamitsuji, S. Tsujikawa and Y. L. Zhang, JCAP **1802**, 048 (2018) [arXiv:1801.01787 [gr-qc]].
- [36] L. Heisenberg, arXiv:1801.01523 [gr-qc].
- [37] L. Heisenberg and S. Tsujikawa, Phys. Lett. B **780**, 638 (2018) [arXiv:1802.07035 [gr-qc]].
- [38] P. Channuie and D. Momeni, arXiv:1802.03672 [gr-qc].
- [39] O. J. Tattersall, P. G. Ferreira and M. Lagos, Phys. Rev. D **97**, n084005 (2018) [arXiv:1802.08606 [gr-qc]].
- [40] L. Heisenberg, R. Kase and S. Tsujikawa, Phys. Rev. D **97**, 124043 (2018) [arXiv:1804.00535 [gr-qc]].
- [41] J. Beltran Jimenez and L. Heisenberg, Phys. Lett. B **770**, 16 (2017) [arXiv:1610.08960 [hep-th]].
- [42] E. Allys, P. Peter and Y. Rodriguez, Phys. Rev. D **94**, 084041 (2016) [arXiv:1609.05870 [hep-th]]; R. Emami, S. Mukohyama, R. Namba and Y. I. Zhang, JCAP **1703**, 058 (2017) [arXiv:1612.09581 [hep-th]]; Y. Rodriguez and A. A. Navarro, Phys. Dark Univ. **19**, 129 (2018) [arXiv:1711.01935 [gr-qc]].
- [43] B. P. Abbott *et al.* [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. **119**, 161101 (2017) [arXiv:1710.05832 [gr-qc]].
- [44] A. Goldstein *et al.*, Astrophys. J. **848**, L14 (2017) [arXiv:1710.05446 [astro-ph.HE]].
- [45] P. Creminelli and F. Vernizzi, Phys. Rev. Lett. **119**, 251302 (2017) [arXiv:1710.05877 [astro-ph.CO]]; J. M. Ezquiaga and M. Zumalacarregui, Phys. Rev. Lett. **119**, 251304 (2017) [arXiv:1710.05901 [astro-ph.CO]]; J. Sakstein and B. Jain, Phys. Rev. Lett. **119**, 251303 (2017) [arXiv:1710.05893 [astro-ph.CO]]; T. Baker, E. Bellini, P. G. Ferreira, M. Lagos, J. Noller and I. Sawicki, Phys. Rev. Lett. **119**, 251301 (2017) [arXiv:1710.06394 [astro-ph.CO]]; L. Amendola, M. Kunz, I. D. Saltas and I. Sawicki, Phys. Rev. Lett. **120**, 131101 (2018) [arXiv:1711.04825 [astro-ph.CO]]; M. Crisostomi and K. Koyama, Phys. Rev. D **97**, 084004 (2018) [arXiv:1712.06556 [astro-ph.CO]]; R. Kase and S. Tsujikawa, Phys. Rev. D **97**, 103501 (2018) [arXiv:1802.02728 [gr-qc]].
- [46] E. D. Stewart and D. H. Lyth, Phys. Lett. B **302**, 171 (1993) [gr-qc/9302019].
- [47] X. Chen, M. x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007) [hep-th/0605045].
- [48] A. De Felice and S. Tsujikawa, Phys. Rev. D **91**, 103506 (2015) [arXiv:1411.0736 [hep-th]]; S. Tsujikawa, JCAP **1504**, 043 (2015) [arXiv:1412.6210 [hep-th]].
- [49] S. Kuroyanagi, C. Lin, M. Sasaki and S. Tsujikawa, Phys. Rev. D **97**, 023516 (2018) [arXiv:1710.06789 [gr-qc]].
- [50] B. A. Bassett, S. Tsujikawa and D. Wands, Rev. Mod. Phys. **78**, 537 (2006) [astro-ph/0507632].
- [51] C. Gordon, D. Wands, B. A. Bassett and R. Maartens, Phys. Rev. D **63**, 023506 (2000) [astro-ph/0009131]; N. Bartolo, S. Matarrese and A. Riotto, Phys. Rev. D **64**, 123504 (2001) [astro-ph/0107502]; D. Wands, N. Bartolo, S. Matarrese and A. Riotto, Phys. Rev. D **66**, 043520 (2002) [astro-ph/0205253].
- [52] S. Tsujikawa, D. Parkinson and B. A. Bassett, Phys. Rev. D **67**, 083516 (2003) [astro-ph/0210322].

- [53] R. Kallosh, A. Linde and D. Roest, JHEP **1311**, 198 (2013) [arXiv:1311.0472 [hep-th]].
- [54] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980).
- [55] A. De Felice and S. Tsujikawa, Living Rev. Rel. **13**, 3 (2010) [arXiv:1002.4928 [gr-qc]].