

RG flows and cascades of $Lif_4^{(2)} \times S^1 \times S^5$ vacua**Harvendra Singh***Theory Division, Saha Institute of Nuclear Physics
1/AF, Bidhannagar, Kolkata 700064, India**and**Homi Bhabha National Institute, Anushakti Nagar, Mumbai 400094, India***Abstract**

The (F1,D2,D8) brane configuration produces $Lif_4^{(2)} \times S^1 \times S^5$ Lifshitz vacua supported by ‘massive’ B -field. We present exact deformations of this system under which new massless B -modes of strings also get excited. Due to these massless modes the deformed solutions flow to conformally $Lif_4^{(3)} \times S^1 \times S^5$ vacua in the IR. The latter types are supersymmetric solutions of ordinary type IIA theory. The massive and massless $B_{\mu\nu}$ modes segregate out in the IR. We confirm that the massive B mode and cosmological constant indeed decouple from the theory rendering the IR field dynamics controlled by massless fields only. A similar effect is observed on the UV side of the flow where a relativistic regime reappears. We also present ‘cascading’ Lifshitz vacua in which dynamical exponent has integral jumps along the flow; $Lif_4^{(3)} \rightarrow Lif_4^{(2)} \rightarrow Lif_4^{(1)}$. The critical $Lif_4^{(2)}$ theory separates ‘deconfining’ $Lif_4^{(3)}$ IR theory from the confining Yang-Mills phase in UV.

1 Introduction

The AdS-CFT holography [1, 2, 3] has produced a nonrelativistic version of itself where strongly coupled quantum theories at critical points have been the focus of several studies [4]-[25]. These systems may involve strongly coupled fermions at finite density or some gas of ultra-cold atoms [4, 5]. In some studies involving nonrelativistic Schrödinger spacetimes the 4-dimensional bulk spacetime geometry generically requires supporting Higgs like fields such as massive vector field [6, 9, 4]. We particularly discuss here separate examples of 10-dimensional Lifshitz spacetimes where Higgs like matter field is 2-rank antisymmetric tensor field which couples to excited massive modes of the strings. The two phenomena indeed are parallel from higher dimensional perspective, because a Kaluza-Klein compactification of 2-rank tensor field on S^1 gives rise to vector field in one lower dimensions.

In this work our aim is to study RG flows of the known $Lif_4^{(2)} \times S^1 \times S^5$ solution, a Lifshitz vacua with dynamical exponent of time as $a = 2$, in the ‘massive’ type IIA theory [26]. The massive type IIA string theory (mIIA) is unique 10-dimensional maximal supergravity where the string B field is explicitly massive and the theory also includes a positive cosmological constant. Because of this mIIA theory provides an unique setup to study Lifshitz and Schrödinger like nonrelativistic solutions, as these require massive gauge fields. The Lifshitz bulk spacetimes provide a holographic dual description of nonrelativistic quantum theories living on the spacetime boundary [16]. The recently found $Lif_4^{(2)} \times S^1 \times S^5$ solution is a background generated by a bound state of $(F1, D2, D8)$ branes [36]. The massive T-dual Lifshitz solutions have been shown to exist in type IIB theory with axionic flux switched on [19]. It is very important to study RG flows of these vacua because a compactification on $S^1 \times S^5$ immediately provides us the prototype $Lif_4^{(2)}$ background in four dimensions which is holographically dual to 3-dimensional Lifshitz theory. The RG flow will tell us how the physical changes occur in the theory at fundamental level at different energy scales. We shall study deformations of $Lif_4^{(2)} \times S^1 \times S^5$ background, which form exact solutions of mIIA and explicitly involve massive B -field excitations. The excitations induce running of string coupling as well. It is observed that the resulting RG flow to deep IR (and towards UV) can be described by ordinary type IIA string theory (oIIA) alone. The main reason that this to happen is that the contributions of the mass and the cosmological constant terms disappear from the field dynamics far away from the critical point.

The paper is organised as following. In section-2 we first review the fixed point Lifshitz solution $Lif_4^{(2)} \times S^1 \times S^5$ in massive type IIA theory. In section-3 we write down its deformed version where strings become excited. We discuss the RG flow where the string massive mode decouples in the IR and conformally $Lif_4^{(3)} \times S^1 \times S^5$ vacua of ordinary type IIA appear as spacetime. The section-4 contains UV

deformations of the $Lif_4^{(2)} \times S^1 \times S^5$ solutions. In the UV regime too the massive string field and cosmological term decouple. In section-5, we present a cascading type solution. Here we find that the difference $(a - \theta)$ remains constant along the RG flow through out the cascade. The results are summarised in section-6.

2 $Lif_4^{(2)} \times S^1 \times S^5$ vacua

The Romans type IIA theory is the only known maximal supergravity in ten dimensions which allows massive string $B_{\mu\nu}$ field. The theory is described by the following bosonic action

$$S = \frac{1}{G_N} \int \left[e^{-2\phi} \left(*R + 4(d\phi)^2 - \frac{1}{2}(H_{(3)})^2 \right) - \frac{1}{2}(G_{(2)})^2 - \frac{1}{2}(G_{(4)})^2 - * \frac{m^2}{2} \right] \quad (1)$$

where topological terms have been dropped because these would be vanishing for the Lifshitz backgrounds we are studying in this paper, see for details in [26, 34].¹ The field strengths are defined as

$$H_{(3)} = dB_{(2)}, \quad G_{(2)} = dC_{(1)} + mB_{(2)}, \quad G_{(4)} = dC_{(3)} + B_{(2)} \wedge dC_{(1)} + \frac{m}{2} B_{(2)} \wedge B_{(2)} \quad (2)$$

where m is the mass parameter and m^2 a positive cosmological constant of the 10-dimensional theory. The cosmological constant generates a nontrivial potential term for the dilaton field. Other than the well known Freund-Rubin $AdS_4 \times S^6$ vacua [26], some supersymmetric solutions of the theory include D8-branes [27, 28, 29, 30, 31, 32], the $K3$ compactifications [33], the $(D6, D8)$ and $(D4, D6, D8)$ bound states [34, 35]. Under the ‘massive’ T-duality [28] the D8-brane (domain walls) can be mapped to D7-brane of type IIB theory. The string B -field is explicitly massive and it plays important role in obtaining nonrelativistic $a = 3$ Schrödinger solutions [17]. The massive type IIA theory however never admits a Minkowski solution.

An observed feature in four-dimensional AdS gravity theories has been that in order to obtain Schrödinger or Lifshitz type non-relativistic solutions one needs to include a massive (Proca like) gauge fields in the theory [4, 5]. (Although massless gauge fields can give rise to nonrelativistic vacua however, in simple cases of Dp -branes compactified along a worldvolume lightcone coordinate [20], these solutions straight forwardly give rise to conformal (or hyperscaling) Lifshitz and Schrödinger vacua [22].)

2.1 F1-D2-D8 system: Lifshitz vacua and massive strings

The $AdS_4 \times S^6$ maximally symmetric Freund-Rubin vacua in Romans’ theory [26] is constituted by D2 and D8-branes. The D8-brane couples to a 10-form field strength

¹ We are adopting a convention: $\int (H_{(p)})^2 = \int H_{(p)} \wedge *H_{(p)} = \frac{1}{p!} \int d^{10}x \sqrt{-g} H_{\mu_1 \dots \mu_p} H^{\mu_1 \dots \mu_p}$ and for scalar quantities like curvature scalar: $\int *R = \int d^{10}x \sqrt{-g} R$.

$F_{(10)} = \star m$, where m is the cosmological constant in massive type IIA supergravity. It has recently been shown that one can even construct Lifshitz vacua which are constituted by ‘massive’ strings, D2 and D8 branes [36]. The string field in the Lifshitz solutions becomes massive after gobbling up the D0-branes. Thus massive B -field carries additional degrees of freedom as compacted to the massless one. The $Lif_4^{(2)} \times S^1 \times S^5$ Lifshitz solutions are given by (in string metric and $\alpha' = 1$)

$$\begin{aligned} ds^2 &= L^2 \left(-\frac{dt^2}{z^4} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} + \frac{dy^2}{q^2} + d\Omega_5^2 \right), \\ e^\phi &= g_a, \quad C_{(3)} = -\frac{L^3}{g_a} \frac{1}{z^4} dt \wedge dx_1 \wedge dx_2, \\ B_{(2)} &= \frac{L^2}{qz^2} dt \wedge dy, \end{aligned} \tag{3}$$

with L being related to m as $L = \frac{2}{g_a m l_s}$, and m being the mass parameter in the mIIA action and equations of motion. The constant q is a free (length) parameter and g_a we take to be weak string coupling ($g_a < 1$) in this massive type IIA vacuum. Note L , which is dimensionless, determines overall radius of curvature of the 10-dimensional spacetime. While m a parameter of the lagrangian theory is related to L . Therefore Romans’ theory with $m \ll \frac{2}{g_a l_s}$ would be preferred so that we can have $L \gg 1$ in the solutions (3), else these classical vacua cannot be trusted.²

The Lifshitz configuration (3) describes parallel stack of D2-branes with brane directions stretched along (x_1, x_2) and ‘massive’ fundamental strings that are aligned along y direction. The D8-branes would wrap around S^5 and remaining part of the worldvolume is stretched along the patch (x_1, x_2, y) .

3 $Lif_4^{(2)} \times S^1 \times S^5$ vacua with IR deformations

The $a = 2$ Lifshitz vacua allows following type of deformations

$$\begin{aligned} ds^2 &= L^2 \left(-\frac{dt^2}{z^4 h} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} + \frac{dy^2}{q^2 h} + d\Omega_5^2 \right), \\ e^\phi &= g_a h^{-1/2}, \quad C_{(3)} = -\frac{L^3}{g_a} \frac{1}{z^4} dt \wedge dx_1 \wedge dx_2, \\ B_{(2)} &= \frac{L^2}{qz^2} h^{-1} dt \wedge dy, \end{aligned} \tag{4}$$

where the function

$$h = 1 + \frac{z^2}{z_0^2}.$$

²Note, from the D8 brane/domain wall idea discussed in [28], one typically expects $m \approx \frac{g_a N_{D8}}{l_s}$, a value which is definitely well within $\frac{2}{g_a l_s}$ for finite number, N_{D8} , of D8 branes in the background.

The excitations involve (t, y) metric components, leaving the x_1, x_2 plane of D2-branes unaffected. These excitations also induce a running dilaton field. The string B -field along y direction also gets coupled to the excitations. Since $h \sim 1$ near $z = 0$, these excitations are normalizable modes (usually z_0 corresponds to switching on relevant operators in the Lifshitz theory). It is clear that the solution (4) flows to a weakly coupled fixed point solution (3) in the UV. The string coupling always stays weak so long as $g_a < 1$ even with the deformations.

The vacua (4) form exact solutions of the m-IIA theory. This enables us to also study the IR region of $a = 2$ Lifshitz theory. In the deep IR region, $z \gg z_0$, where $h \approx \frac{z^2}{z_0^2}$, the Lifshitz vacua is driven to a weakly coupled regime. For $z \gg z_0$, the IR geometry becomes a conformally $Lif_4^{(3)} \times S^1 \times S^5$ solution:

$$\begin{aligned}
ds_{IR}^2 &\sim L^2 \left(-\frac{z_0^2 dt^2}{z^6} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} + \frac{z_0^2 dy^2}{z^2 q^2} + d\Omega_5^2 \right), \\
e^\phi &\sim g_a \frac{z_0}{z}, \quad C_{(3)} = -\frac{L^3}{g_a} \frac{1}{z^4} dt \wedge dx_1 \wedge dx_2, \\
B_{(2)} &\sim \frac{L^2 z_0^2}{q z^4} dt \wedge dy. \tag{5}
\end{aligned}$$

In the IR regime the y circle essentially becomes smaller. (Once it becomes sub-stringy, in that situation we should opt to go to type IIB dual theory that will provide a better holographic description.) Otherwise also, if y coordinate is a flat noncompact direction, then the IR geometry would resemble $Lif_4^{(3)} \times R^1 \times S^5$ spacetime describing a 4-dimensional $a = 3$ Lifshitz theory on its boundary at $z = 0$. Along this IR flow the massive modes of bulk fields completely decouple in deep IR region leaving behind a massless field description, at very weak string coupling. Due to weak string coupling fluctuations are suppressed near $z \sim \infty$. (This may indicate the presence of a horizon.)

The constant z_0 in (5) has no particular importance as it can be removed by redefining g_a, t, y suitably. For example, with the definitions: $g'_a = g_a z_0$, $t' = z_0 t$, $x_3 = \frac{z_0}{q} y$, we can get

$$\begin{aligned}
ds^2 &= L^2 \left(-\frac{(dt')^2}{z^6} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} + \frac{dx_3^2}{z^2} + d\Omega_5^2 \right), \\
e^\phi &= \frac{g'_a}{z}, \quad C_{(3)} = -\frac{L^3}{g'_a} \frac{1}{z^4} dt' \wedge dx_1 \wedge dx_2, \\
B_{(2)} &= \frac{L^2}{z^4} dt' \wedge dx_3, \tag{6}
\end{aligned}$$

It is required to note that conformally $Lif_4^{(3)} \times S^1 \times S^5$ background (6) are described by the ordinary type IIA string theory, of which this is an exact solution. (These same solutions appear in the eq.(20) of the work [20]. The $SO(3)$ symmetry of the

(x_1, x_2, x_3) patch is explicitly broken by B_{tx_3} .) These massless vacua are constituted by D2-branes and F-strings only. This essentially indicates that a little or no effect of D8-branes (cosmological constant) is seen in the $z \gg z_0$ IR region! How did it actually happen? It will be important to understand how do the deformed solutions (4) interpolate between a massive type vacua (in UV) and the massless one (in IR). Let us understand the basic reasons behind this mechanism.

Actually the $B_{\mu\nu}$ field in the background eq.(4) plays a nontrivial double role. A massive field carries additional degrees of freedom (modes) as compared to the ordinary field. We will show that for the solutions (4) these degrees of freedom get separated out from each other in different regions of the spacetime. From eq.(4) we find that the string coupling and corresponding B field in $z \sim \infty$ IR region become (leading behaviour)

$$e^\phi \sim g_a \frac{z_0}{z} + \dots, \quad B_{ty}^{massless} \sim \frac{L^2 z_0^2}{q z^4} + \dots \quad (7)$$

and the corresponding expressions in $z \sim 0$ UV region are

$$e^\phi \sim g_a + \dots, \quad B_{ty}^{Higgsed} \sim \frac{L^2}{q z^2} + \dots \quad (8)$$

where dots indicate subleading terms. These expressions tell us that the massive (Higgsed) B mode $\sim \frac{1}{z^2}$ is dominant in the UV Lifshitz vacuum, while the massless mode $\sim \frac{1}{z^4}$ is relevant in the IR region. Their different scaling behaviors also make them distinct and well segregated modes in two separate energy regimes. Therefore the B field in the background (4) should only be thought of as composite of these two types of modes,

$$B_{ty} = B_{ty}^{Higgsed} + B_{ty}^{massless}. \quad (9)$$

It is meaningful because a massive tensor field carries more degrees of freedom than the usual massless tensor. The Higgsed B -mode decouples from the $z \gg z_0$ IR region whereas the massless B -mode takes over there, and exactly opposite phenomenon occurs towards $z \ll z_0$ UV region. Since it is the massless B mode which explicitly depends on z_0 , it will carry the tag of an *excitation*. It is remarkable that it happens to be like this.

Further, as an explicit check, we shall demonstrate in the following that all m dependent terms indeed decouple from the field equations, rendering the field dynamics being described by the massless fields only, being dominant in the IR region.

3.1 Decoupling of mass and cosmological terms in the IR regime

We demonstrate here that for the excited vacua (4) the mass terms as well as the cosmological constant term in mIIA theory indeed decouple from the IR dynamics.

To determine this the terms in massive supergravity action are evaluated. Using the IR expressions of fields in (5) we get the following estimates

$$\begin{aligned}
(d\phi)^2 &\sim O\left(\frac{1}{\sqrt{z}}\right) \\
e^{-\phi}(dB)^2 &\sim O\left(\frac{1}{\sqrt{z}}\right) \\
e^{\phi/2}(dC)^2 &\sim O\left(\frac{1}{\sqrt{z}}\right) \\
m^2 e^{3\phi/2}(B_{\mu\nu})^2 &\sim O\left(\frac{1}{z^{5/2}}\right) \\
m^2 e^{5\phi/2} &\sim O\left(\frac{1}{z^{5/2}}\right)
\end{aligned} \tag{10}$$

We clearly see that in these expressions valid for $z \rightarrow \infty$ region the last two m -dependent terms indeed are vanishing as compared to first three quantities. It is mainly due to weak string coupling, $e^\phi \sim \frac{1}{z} \sim 0$, that the mass and the cosmological terms cannot play a significant role in the low energy dynamics of these fields. Since these m -dependent terms in the action (also in equations of motion) become subleading and irrelevant, the bulk dynamics will be effectively described by ordinary type IIA theory in deep IR. This phenomenon simply demonstrates that the decoupling of Higgsed mode of B_{ty} which although has got a strong pivot in the $z \sim 0$ region, but the weak string coupling renders it ineffective in the IR region. Furthermore, *it is rather usual for QFT that massive modes get decoupled from a low energy regime, but what is unusual here is that even the cosmological constant term (which couples to dilaton only) gets decoupled from the supergravity in the deformed $Lif_4^{(2)} \times S^1 \times S^5$ vacua!* This demonstrates that massive type IIA theory is only an effective theory valid near UV scale in the deformed $Lif_4^{(2)} \times S^1 \times S^5$ solutions (4). At small scales, where the energy of the system is lower, the mass/cosmological terms get screened out by a weak string coupling, and dynamically these terms become subleading.

Let us now study the UV regime close to $z \sim 0$. The excited solution (4) flows to scale invariant Lifshitz background described by the fixed point solution (3). We again evaluate the following expressions in UV region

$$\begin{aligned}
(\partial_\mu \phi)^2 &\sim 0 \\
e^{-\phi}(dB)^2 &\sim O(1) \\
e^{\phi/2}(dC)^2 &\sim O(1) \\
m^2 e^{3\phi/2} B^2 &\sim O(1) \\
m^2 e^{5\phi/2} &\sim O(1).
\end{aligned} \tag{11}$$

Here we find that the mass terms are of the same order as the kinetic terms of the fields. These covariants are also independent of holographic coordinate z , meaning

that the theory is at the fixed point. Thus the mass terms in Romans theory indeed play an important role in the field dynamics near UV fixed point, where the vacuum is described by $Lif_4^{(2)} \times S^1 \times S^5$ geometry. The typical scale where this crossover of the mIIA-oIIA theories (and the respective vacuas) happens is governed by z_0 scale.

4 $Lif_4^{(2)} \times S^1 \times S^5$ with a flow towards UV

Having discussed IR flows, we next present completely different solutions of massive type IIA theory which drive the flow from towards a relativistic UV vacua.³ The $a = 2$ Lifshitz vacua allows following exact deformation

$$\begin{aligned} ds^2 &= L^2 \left(-\frac{dt^2}{z^4 f} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} + \frac{dy^2}{q^2 f} + d\Omega_5^2 \right), \\ e^\phi &= g_a f^{-1/2}, \quad C_{(3)} = -\frac{L^3}{g_a} \frac{1}{z^4} dt \wedge dx_1 \wedge dx_2, \\ B_{(2)} &= \frac{L^2}{q z^2} f^{-1} dt \wedge dy, \end{aligned} \tag{12}$$

where new function

$$f = 1 + \frac{z_c^2}{z^2},$$

the parameter z_c corresponds to switching on irrelevant operators. These involve growing type of modes towards UV. The vacua (12) are exact solutions of mIIA theory. The deformations induce running of the string coupling which start flowing to weakly coupled ($g_a < 1$) regime at the UV end. Especially in $z \sim 0$ region, where $f \approx \frac{z_c^2}{z^2}$, the solution (12) becomes:

$$\begin{aligned} ds^2 &\simeq L^2 \left(-\frac{z_c^2 dt^2}{z^2} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} + \frac{z^2 dy^2}{q^2 z_c^2} + d\Omega_5^2 \right), \\ e^\phi &= g_a \frac{z}{z_c}, \quad C_{(3)} = -\frac{L^3}{g_a} \frac{1}{z^4} dt \wedge dx_1 \wedge dx_2, \\ B_{(2)} &= \frac{L^2 z_c^2}{q} dt \wedge dy, \end{aligned} \tag{13}$$

In these asymptotic (UV) solutions the z_c is arbitrary and can be absorbed by redefining g_a, t, y . We remark that the UV vacua (13) are well described by ordinary type IIA theory. However, the y direction in (13), as being compact, will develop a pinching type singularity in the near UV region. But it remains stable because the interactions are weakened due to weak coupling. It would be appropriate to study them by going over to type IIB T-dual background. It is rather straightforward to

³ There however already exist known examples where the RG flow directly takes $Lif^{(3)}$ vacua (in the IR) to AdS_5 vacua in the UV, see [21].

convince oneself that on the type IIB side the solution (13) has got a nicer description as deformed $AdS_5 \times S^5$.

In the remaining section, we wish to make sure that the mass terms in the theory completely decouple leaving behind a perfectly massless vacua (governed by ordinary type IIA theory) at UV side. For this, using the asymptotic backgrounds (13), in $z \sim 0$ region, we evaluate the following quantities

$$\begin{aligned}
(\partial_\mu \phi)^2 &\sim O(\sqrt{z}) \\
e^{-\phi}(dB)^2 &\sim 0 \\
e^{\phi/2}(dC)^2 &\sim O(\sqrt{z}) \\
m^2 e^{5\phi/2} &\sim O(z^{5/2}) \\
m^2 e^{3\phi/2} B^2 &\sim O(z^{5/2})
\end{aligned} \tag{14}$$

We clearly observe that the last two m -dependent terms indeed become subleading as compared to the kinetic terms for $z \sim 0$. Once again due to weakened dilatonic coupling, i.e. $e^\phi \sim z \simeq 0$, the mass and cosmological constant do not play any significant role in the UV dynamics of the fields. Since the mass terms in the action (and in equations of motion) become subleading and irrelevant, the bulk dynamics in the UV regime is better described by oIIA theory. From eq.(12) one can find that the string coupling and corresponding B field in the $z \sim 0$ region become

$$e^\phi \sim g_a \frac{z}{z_c} + \dots, \quad B_{ty}^{massless} \sim \frac{L^2 z_c^2}{q} + \dots \tag{15}$$

while corresponding expressions in $z \sim \infty$ region are

$$e^\phi \sim g_a + \dots, \quad B_{ty}^{Higgsed} \sim \frac{L^2}{qz^2} + \dots \tag{16}$$

where dots indicate subleading terms. These expressions tell us that the Higgsed B -mode $\sim \frac{1}{z^2}$ is dominant in the IR Lifshitz vacuum, while a constant (but massless) B -mode is important in the boundary region. Their power behaviors also make them distinct and well segregated modes in two separate energy regimes. Therefore the net B field in the background (12) ought to be thought of as composite mode: $B_{ty} = B_{ty}^{Higgsed} + B_{ty}^{massless}$. This implies that there is explicit decoupling of Higgsed B -mode from UV dynamics, which although has a strong support in the IR.

5 Cascade of dynamical exponents along RG flow:

$$Lif_4^{(3)} \rightarrow Lif_4^{(2)} \rightarrow Lif_4^{(1)}$$

The cascading Lifshitz theories are obtained in which dynamical exponent jumps as a consequence of RG flow. These solutions are the combinations of previous two

type of solutions. In the following we write down these vacua as solutions of mIIA theory.

$$\begin{aligned}
ds^2 &= L^2 \left(-\frac{dt^2}{z^4 f} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} + \frac{dy^2}{q^2 f} + d\Omega_5^2 \right), \\
e^\phi &= g_a f^{-1/2}, \quad C_{(3)} = -\frac{L^3}{g_a} \frac{1}{z^4} dt \wedge dx_1 \wedge dx_2, \\
B_{(2)} &= \frac{L^2}{qz^2} f^{-1} dt \wedge dy, \tag{17}
\end{aligned}$$

with function $f = \frac{z_c^2}{z^2} + 1 + \frac{z_0^2}{z^2}$, where z_c, z_0 are widely separated scales, $z_c \ll z_0$, but otherwise remain free parameters. An explicit compactification along S^1 (y-circle) and S^5 produces the following 4-dimensional metric (Einstein) and a dilatonic scalar

$$\begin{aligned}
ds_{Einstein}^2 &\sim L^2 f^{\frac{1}{2}} \left(-\frac{dt^2}{z^4 f} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} \right), \\
e^{2\phi_4} &= g_4^2 f^{-1/2}, \tag{18}
\end{aligned}$$

and associated gauge field $A_{(1)} \sim \frac{L^2}{qz^2 f} dt$. The four-dimensional coupling is given by $g_4^2 = g_a^2 q/L$. Since the function $f \geq 1$ always, the curvature of the spacetime still remains small due to $L \gg 1$. One can easily see that, in the UV region $z \ll z_c$, where $f \approx \frac{z_c^2}{z^2}$, the solution (18) reduces to the $a = 1$, $\theta = -1$ Lifshitz vacua

$$\begin{aligned}
ds_{Einstein}^2 &\sim L^2 \frac{z_c}{z} \left(-\frac{dt^2}{z^2} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} \right), \\
e^{2\phi_4} &\sim g_4^2 \frac{z}{z_c} \ll g_4^2, \quad A_0 \sim \frac{L^2}{qz_c^2} \tag{19}
\end{aligned}$$

discussed earlier.⁴ Since the coupling gets weaker at higher energies, this phase of the theory describe asymptotically free 3D Yang-Mills type (relativistic) theory on the boundary.⁵ While the ordinary D2-brane theory in the IR flows to strongly coupled fixed point where M2-branes arise [37]. The difference in the present case arises in the IR regime. Due to nontrivial A_0 , in the IR regime the CFT_3 flows towards a Lifshitz theory, as we will see next. At some intermediate scale $z = z_i$, such that $z_c \ll z_i \ll z_0$, where $f \approx 1$, the solution (18) would resemble with following $a = 2$, $\theta = 0$ (scale invariant) Lifshitz vacua,

$$\begin{aligned}
ds_{Einstein}^2 &\sim L^2 \left(-\frac{dt^2}{z^4} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} \right), \\
e^{2\phi_4} &\sim g_4^2, \quad A_0 \sim \frac{L^2}{qz^2} \tag{20}
\end{aligned}$$

⁴ Here θ parameter stands for hyper-scaling (or effective conformal dimension) of the 4D Einstein metric as per the convention: $ds_{Einstein}^2 \sim z^\theta \left(-\frac{dt^2}{z^{2a}} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} \right)$

⁵ This CFT behaviour is similar to and is consistent with the UV behaviour of the known nonconformal CFT_3 of the D2-branes.

It describes a critical point behaviour in a nonrelativistic Lifshitz like theory. If we further lower the energies, in the deep IR region, $z \gg z_0$, where $f \approx \frac{z^2}{z_0^2}$, the solution (18) becomes more like $a = 3$, $\theta = 1$ Lifshitz vacua:

$$\begin{aligned} ds_{Einstein}^2 &\sim L^2 \frac{z}{z_0} \left(-\frac{z_0^2 dt^2}{z^6} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} \right), \\ e^{2\phi_4} &\sim g_4^2 \frac{z_0}{z} \ll g_4^2, \quad A_0 \sim \frac{L^2 z_0^2}{q z^4}. \end{aligned} \quad (21)$$

Thus in the $z \gg z_0$ IR region, the coupling gets weaker at farther distances. Hence this Lifshitz $a = 3$ phase exhibits ‘deconfinement’ behaviour at low energies, like usual electrodynamics. See the complete plot of the flow of the coupling in figure (2). Due to identical background fields all these three Lifshitz behaviours can be characterized through:

$$\begin{aligned} ds_{Einstein}^2 &\sim \left(\frac{z}{\lambda}\right)^\theta \left(-\frac{dt^2}{z^{2a}} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} \right), \\ e^{2\phi_4} &\sim \left(\frac{\lambda}{z}\right)^\theta, \quad A_t \sim \frac{\lambda^\theta}{z^{a+\theta}}. \end{aligned} \quad (22)$$

where the gauge field is Maxwellian (for $a = 3, 1$) and Proca type (for $a = 2$). The scale λ defines the range of validity. For $z \ll \lambda$ the solutions describe $a = 1$, $\theta = -1$ UV relativistic region well, whereas for $z \gg \lambda$ the solutions describe $a = 3$, $\theta = 1$ IR Lifshitz region. Hence these composite kind of solutions can also be called the Lifshitz ‘wormholes’ in which $Lif_4^{(3)}$ world tunnels through a $Lif_4^{(2)}$ throat region and finally emerging into a relativistic Yang-Mills phase as we start from IR region and tune towards $z \sim 0$ region. An interesting aspect is that the difference

$$a - \theta = 2$$

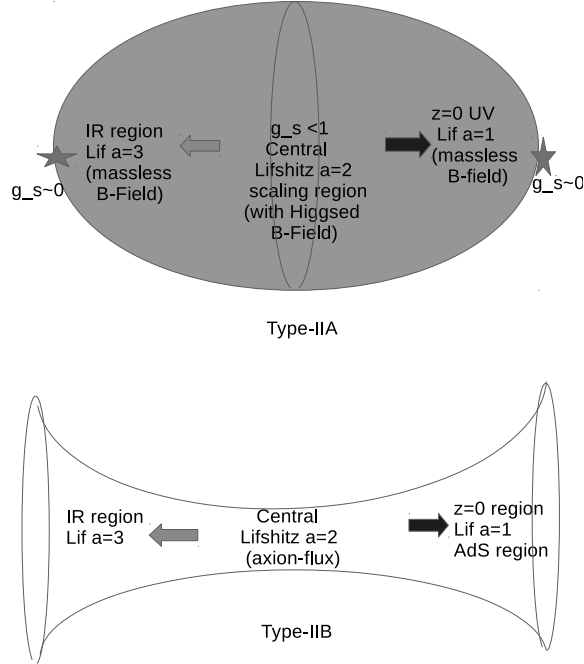
remains fixed along all three Lifshitz regions. It is sort of a quantity which remains conserved all along the z -flow.

To gain further insight let us evaluate the running of effective string coupling, $g_4 \equiv e^{\phi_4}$, with respect to scale $\lambda \equiv z$. This would indicate to us about the nature of the flow of the coupling. We obtain it in the following way

$$\begin{aligned} \beta_{UV} &= \frac{dg_4}{d \ln \lambda} \sim g_4, \quad \text{for } z \sim 0 \\ \beta_{IR} &= \frac{dg_4}{d \ln \lambda} \sim -g_4, \quad \text{for } z \sim \infty \end{aligned} \quad (23)$$

We find that on the either side of the intermediate scale, z_i , the β 's change the sign. It implies that z_i would be a fixed point, where $\beta = 0$. The fixed point corresponds to $Lif_4^{(2)}$ spacetime of constant string coupling g_4 . There is a resulting

Figure 1: The figures are drawn for the cascading $Lif_4^{(a)} \times S^1 \times S^5$. In the top figure the vertical size is indicative of physical radius of y -circle in various z regions. The end stars indicate possible S^1 pinching. In the lower figure the same situation is viewed on type-IIB theory side. The size of T-dual \tilde{y} -circle is minimum at the throat, supported by constant χ flux. The dynamical effect of flux get diluted in the far IR and UV region. The geometry appears like a traversable wormhole.

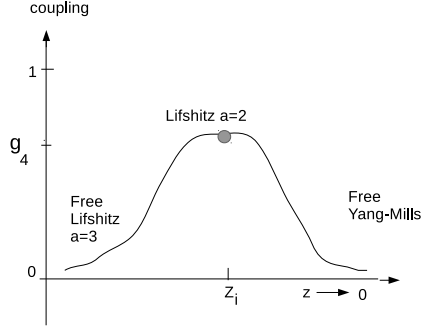


scale invariance in the boundary Lifshitz theory. The running string coupling is also sketched in the figure (2).

We comment that the physical size of y -circle becomes minimum near the two extremities; viz $z = \infty$ (horizon) and $z = 0$ (boundary), where the associated string couplings become vanishing. It means that the fluctuations are stable and highly suppressed and controlled. The configurations also preserve some supersymmetry. It can be better appreciated by going to a dual type-IIB set up near the two asymptotic regions. It is an straight forward analysis and we are avoiding full discussion here (one can see the solution given in the Appendix). Thus we indeed observe a cascade of Lifshitz theories with integral jumps in the dynamical exponents. These basic ideas are depicted in the figure (1). A few observations can be drawn from these cascading Lifshitz solutions involving particular combination of background fields:

- The dynamical exponent a decreases along the flow towards UV region. This behaviour is quite reasonable because covariance is expected to get restored

Figure 2: *There is a fixed point at $z = z_i$ where coupling is maximum g_4 . But on the either side of it the coupling decreases. The two phases both of weak couplings, the ‘confining’ Yang-Mills type (for $z_c \neq 0$) and ‘deconfined’ Lifshitz $a = 3$ type (if $z_0 \neq 0$) are well separated.*



at higher energies.

- The associated hyperscaling parameter θ also has decreasing integral jumps towards higher energy scales. But the quantity $(a - \theta)$ is found to be conserved as we pass through various energy scales.
- At least 8 supersymmetries are intact in the IR asymptotic region. In the asymptotic UV region full supersymmetry of the relativistic theory would be recovered.

6 Summary

We have presented new class of solutions for the massive type IIA theory under which massive string modes become excited. The geometry is such that the Lifshitz $Lif_4^{(2)} \times S^1 \times S^5$ vacua emerges near the UV fixed point of the RG flow. In the deep IR region these solutions flow towards conformal $Lif_4^{(3)} \times S^1 \times S^5$ background which belongs to ordinary type IIA theory. Thus the deep IR region is governed by ordinary type-IIA theory where B field is massless, while the UV region requires a massive B field in massive type-IIA theory. We have explicitly verified that in the deep IR region all mass terms including the cosmological term indeed decouple

from the field equations, rendering the whole IR field dynamics in $Lif_4^{(3)} \times S^1 \times S^5$ vacua controlled by the massless fields only. The latter vacua form 1/4-BPS states of ordinary type IIA theory. There is a crossover scale where the swap in the theories happens which is governed by the deformation scale z_0 . The crossover does happen for any z_0 value though. We have also presented another kind of solutions which describe a flow starting from $Lif_4^{(2)} \times S^1 \times S^5$ IR vacua to $Lif_4^{(1)} \times S^1 \times S^5$ relativistic vacua in $z \sim 0$ region. These relativistic vacua at the UV end point would be maximally supersymmetric. In both of these RG flows the massive mode of the B -field, particularly $B_{ty}^{Higgsed} \sim \frac{1}{z^2}$, gets decoupled along with associated cosmological constant term (of the same order) in the Roman's theory. We believe primarily it happens due to vanishing string interactions with the massive mode, primarily the interactions being diminished by vanishing string couplings in two asymptotic regions.

We have also presented a full solution where a cascading flow of dynamical exponents $Lif_4^{(3)} \rightarrow Lif_4^{(2)} \rightarrow Lif_4^{(1)}$ is observed along the RG flow from IR to UV and vice versa. The $a = 2, \theta = 0$ Lifshitz vacua in the center flows to $a = 3, \theta = 1$ and $a = 1, \theta = -1$ vacuas in the IR and UV regions respectively. Though along the entire cascading flow the quantity $(a - \theta) = 2$ remains fixed. The string coupling remains maximum for the $a = 2$ solution, but it flows to weaker couplings on the either side of it. The two phases of weak couplings, the 'confining' Yang-Mills type in the UV (provided $z_c \neq 0$) and a 'deconfining' Lifshitz $a = 3$ type in the IR (once $\frac{1}{z_0} \neq 0$) are well separated by a fixed point (critical) phase. Generically all physical (not ideal systems) situations would involve $z_c \neq 0, \frac{1}{z_0} \neq 0$ type fluctuations, no matter however small values these quantities take, the smooth flow described by our cascading solution appears to be realistic. It would be interesting to explore 2d planar systems at ultra low energies where this picture might be realized.

A Cascading Lifshitz vacua in type-IIB theory

For the completeness we note down the type-IIB string vacua with constant axion 'flux' which are 'massive' T-dual of the Lifshitz solutions (17) in massive type IIA theory. The background is

$$\begin{aligned}
 ds^2 &= L^2 \left(-\frac{2dt dy}{z^2} + \frac{g}{z^2} dy^2 + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} + d\Omega_5^2 \right), \\
 e^\phi &= g_b, \quad \chi = \frac{2\tilde{q}}{g_b} y,
 \end{aligned} \tag{24}$$

supported by self-dual 5-form $F_{(5)}$ field strength. The function g is given by $g(z) = \tilde{z}_c^2 + \tilde{q}^2 z^2 + \frac{z^4}{\tilde{z}_0^2}$. The new constants \tilde{z}_c, \tilde{z}_0 are two far separated scales, $\tilde{z}_c \ll \tilde{z}_0$, but they remain free parameters in type IIB also. But the parameter \tilde{q} is tightly related to

the axion field. The type IIB/A string couplings are related as $g_b = \frac{g_a g}{L}$, $\tilde{q} = mg_b/2$ etc. A Kaluza-Klein reduction along $S^1 \times S^5$ will produce four-dimensional solutions same as given in (18). It is well known that [28], because the axionic field strength $F_{(1)} = d\chi = \text{constant}$, these constant fluxes produce massive supergravity theories under generalized Scherk-Schwarz compactification on S^1 [38, 28, 39].

A.1 Supersymmetries in the cascade

The supersymmetry of these type IIB background can be found by evaluating the fermionic variations in type IIB theory. These variations can be obtained from the works [28, 40]. To simplify the effort, we will set $L = 1, g_b = 1$. From eq.(24) the vielbeins are defined as

$$e^+ = \frac{1}{z}(dt - gdy), \quad e^- = \frac{1}{z}dy, \quad e^1 = \frac{1}{z}dx^1, \quad e^2 = \frac{1}{z}dx^2, \quad e^3 = \frac{1}{z}dz, \dots \quad (25)$$

so that ten-dimensional line element becomes

$$ds^2 = -2e^+e^- + e^1e^1 + e^2e^2 + e^3e^3 + \dots$$

and the self-dual 5-form as: $F_{(5)} \sim (1 + \star_{10})e^+ \wedge e^- \wedge e^1 \wedge e^2 \wedge e^3$. The dots imply similar expression for S^5 . Using the differential geometric identity $de^a + \omega_b^a \wedge e^b = 0$, the spin connection 1-forms can be evaluated. These mostly will be of the same type as in the case of exact $AdS_5 \times S^5$ geometry, except the following connection component

$$\omega_3^+ = -\frac{1}{z}dt - z\partial_z\left(\frac{g}{z}\right)dy \quad (26)$$

which has a new contribution from g dependent term. Now when we evaluate the dilatino variation for the above background, it reduces to

$$0 = \delta\lambda = \partial_y\chi\Gamma^y\epsilon = 2\tilde{q}e_-^y\gamma^-\epsilon \quad (27)$$

The vanishing fermionic variations put a rigid condition on the Killing spinors that

$$\gamma_+\epsilon = 0 \quad (28)$$

At this stage we have chosen the Killing spinors to be precisely that of anti de Sitter spacetime $\epsilon \equiv \epsilon_{AdS \times S}$. (Note $(\gamma_+)^2 = 0$, and γ_a 's are undressed gamma matrices.) Such a restriction would break all sixteen supernumerary Killing spinors of AdS spacetime. The condition however will allow eight Poincare (ordinary) type Killing spinors remaining intact.

We next have to evaluate the gravitino variations: $\delta\Psi_\mu = 0$, to obtain further conditions, if any. We keep in mind that $\omega_{,y}^{+3}$ is nontrivial and given the condition

(28). We do not present full details of these calculations here, but all gravitino equations will get satisfied except the following one

$$0 = \delta\Psi_y = (\partial_y + \frac{1}{4}\omega_{,y}^{ab}\gamma_{ab})\epsilon + \frac{i}{4}\epsilon\partial_y\chi - \frac{i}{192}\Gamma^{(5)}\Gamma_y F_{(5)}\epsilon \quad (29)$$

which needs to be evaluated separately as it contains g dependent contribution. Substituting the background fields from (24), this simplifies to

$$\partial_y\epsilon + \frac{i\tilde{q}}{2}\epsilon = 0. \quad (30)$$

That has an immediate solution of the type: $\epsilon = e^{\frac{-i\tilde{q}y}{2}}\epsilon_{AdS\times S}$, along with the condition (28). Thus the Killing spinors will also have explicit y dependence, but no new condition is required. When $\tilde{q} = 0$ this dependence on y will drop out. However, the condition $\gamma_+\epsilon = 0$ will still be there so long as z'_0 is nontrivial. Thus the supersymmetry count for the background (24) reduces to 8 Killing spinors being intact. We expect at least these many supersymmetries will survive if we T-dualise (24) back to get mIIA cascading solution (17). The explicit y dependence in the Killing spinors is induced only due to the axion flux. Generically a massive T-duality preserves supersymmetry. It is not clear whether these Killing spinors would survive after compactification. However, if the axion flux is switched off, then there are certainly 8 Killing spinors. Thus we comment that, in the two asymptotic regions where the axion flux is sufficiently weakened the supersymmetry will be regained. In conclusion, we have got a supergravity vacua where supersymmetry can be gained or lost dynamically, both in the IR region as well as in the UV regime.

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