Renormalizing gravity: a new insight into an old problem

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Abstract

It is well-known that perturbative quantum gravity is non-renormalizable. The metric or vierbein has generally been used as the variable to quantize in perturbative quantum gravity. In this essay, we show that one can use the spin connection instead, in which case it is possible to obtain a ghost-free renormalizable theory of quantum gravity. Furthermore in this approach, gravitational analogs of particle physics phenomena can be studied. In particular, we study the gravitational Higgs mechanism using spin connection as a gauge field, and show that this provides a mechanism for the effective reduction in the dimensionality of spacetime.

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arXiv:1805.05665v3 [gr-qc] 19 Jul 2018

General relativity (GR) and quantum field theory (QFT) are theoretically well-founded and experimentally verified theories. Even though QFTs have divergences, they can be dealt with by renormalization, for theories where the interactions are bounded by the free action [1]-[2]. In four dimensions, interactions are bounded by the free action in theories with quartic interactions terms because the first Sobolev norm bounds the volume integral of such interactions (as is the case of Yang-Mills theories) [1]-[2]. Since all interactions in the Standard Model (SM) of particle physics are described by such theories, SM is renormalizable. On the other hand, for perturbative quantum gravity (QG) in four dimensions, governed by the action $S = (1/16\pi G_N) \int d^4x \sqrt{-g} \, R$ (where R = curvature scalar, G_N = Newton's constant), the interactions are not bounded by the free action and thus perturbative QG is non-renormalizable [3]-[6]. Furthermore, in the above action, perturbation theory breaks down as the perturbations can exceed the values of the original metric or vierbein [1]-[2]. The situation changes, however, if one adds higher curvature terms to the action, namely

$$S = \int d^4x \sqrt{-g} \left[-\frac{R}{16\pi G_N} - \alpha R_{\mu\nu} R^{\mu\nu} + \beta R^2 \right] . \tag{1}$$

In this case, although one has interactions greater than quartic, the theory is still renormalizable [7], as now the second Sobolev norm bounds the pointwise value of perturbations, and hence the free action bounds the interactions [1]-[2]. However, it is well-known that this theory contains Ostrogradsky ghosts, giving rise to negative norm states and negative probabilities [8]-[9]. The origin of these ghosts is the presence of higher derivative terms, which occur in this theory when the curvature scalar R is expressed in terms of the metric ($g_{\mu\nu}$) or the vierbein $(e_\mu^a)^1$.

In this essay, we show that the above problem does not occur if one uses the spin connection (ω_μ^{ab}) as the variable to quantize. It may be noted that the spin connection has also been used in Loop QG [10]. However they have not been studied in detail in perturbative QG. Note that in classical gravity the use of the spin connection gives rise to identical predictions as with metric variables, including experimentally measurable (gauge invariant) quantities. However their behavior in quantum theory can be quite different.

In this picture, gravity can be considered as a gauge theory with the spin connection as a $SO(3,1)$ gauge field [11]. The metric is given by $g_{\mu\nu} = e_\mu^a e_{a\nu}$ with $e = \sqrt{|g|}$. Therefore, the

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[1] Greek letters denote spacetime indices and run from 0 to d, while Latin letters are used as tangent space indices and run from 1 to d, in a $(d+1)$-dimensional spacetime.

curvature tensor can be written as the field strength of the spin connection $R_{\mu\nu}^{ab} \equiv F_{\mu\nu}^{ab} = \partial_{[\mu}\omega_{\nu]}^{ab} - \omega_{[\mu}^{ca}\omega_{\nu]c}^{b}$. Then the curvature scalar is $R = R(\omega)$, while the action is $S = S[e, \omega]$ (the e comes from the measure). The expression for the spin connection in terms of vierbeins $\omega_{\mu}^{ab} = -e^{\nu a}\nabla_{\mu}e_{\nu}^{b}$ and the Einstein equations are obtained by varying S with respect to ω_{μ}^{ab} and e_{μ}^{a} respectively [11]. Next, one can write a higher curvature action for gravity as a topological field theory coupled to Yang-Mills theory in four dimensions as follows

$$S = \int d^4x \; e \left[-\frac{B_{\mu\nu}^{ab}F_{ab}^{\mu\nu}}{16\pi G_N} - \frac{1}{4g^2} F_{ab}^{\mu\nu}F_{\mu\nu}^{ab} \right] \tag{2}$$

with $B_{\mu\nu}^{ab} = e_{\mu}^{a}e_{\nu}^{b}$ and g is a coupling constant. It may be noted that in this formalism, Einstein gravity (the first term) is a topological theory similar to a BF theory [12]. Furthermore, the second term is of the form of the higher curvature terms in Eq.(1), using the fact that $\int d^4x \; e \left[R_{\mu\nu}^{ab}R_{ab}^{\mu\nu} - 4R_{\mu}^{a}R_{a}^{\mu} + R^2 \right]$ is the (Gauss-Bonnet) topological invariant in four-dimensions, and vanishes for a topologically trivial background. It may be noted that even though consistency of Newtonian limit of this theory with observational data needs further investigation, this theory is clearly a ghost-free renormalizable theory in these variables. This is because, as seen from Eq.(2), the interactions therein are quartic, just as in the case of SM. Here instead of perturbing the metric or vierbein, one now studies the theory by perturbing the spin connection

$$\omega_{\mu}^{ab} = \tilde{\omega}_{\mu}^{ab} + \bar{\omega}_{\mu}^{ab} \; . \tag{3}$$

where $\bar{\omega}_{\mu}^{ab}$ is the background, and one quantizes the fluctuations $\tilde{\omega}_{\mu}^{ab}$ around it. This can then be used to compute scattering processes involving gravity, now described in terms of these fluctuations.

Having proposed a ghost-free renormalizable theory of gravity as a gauge theory, one can explore gravity analogs of SM phenomena. In particular, in what follows, we study one such example, namely spontaneous symmetry breaking (SSB) or the Higgs mechanism in gravity. Gravitational Higgs mechanism has been studied in the past in various contexts, including in a few instances using the spin connection [13]-[27], but none with the aim of spontaneous dimensional reduction, to the best of our knowledge. For generality, we will analyze SSB in $(d+1)$-spacetime dimensions. We start with a Higgs field Φ which transforms under the *vector* representation of the $(d+1)$-dimensional Lorentz group (note that the vector representation was also used in the Georgi-Glashow model for weak interactions [28]). Then,

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Φ has $(d+1)$ real components. We choose a non-negative Higgs potential of the form

$$V(\Phi^\dagger\Phi) = \frac{m^2}{\phi_0^2}\left[\Phi^\dagger\Phi - \phi_0^2\right]^2 \geq 0 , \tag{4}$$

where m and ϕ_0 are constants. The vacuum minimizing the above potential is $\Phi = (0,\ldots,\phi_0)^T$. Fluctuations around this vacuum are denoted by h, i.e. $\Phi = (0,\ldots,\phi_0+h)^T$. Next, using the covariant derivative $D_\mu\Phi = (\partial_\mu + \frac{i}{2}\omega_\mu)\Phi$,[2] the Lagrangian for the Higgs field coupled to gravity via spin connection can be written as follows, with subsequent expansions around the vacuum

$$\begin{aligned}
\mathcal{L} &= (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi^\dagger\Phi) \\
&= \frac{\phi_0^2}{4}\left[\omega_\mu^{0d}\,\omega^{0d\mu} + \omega_\mu^{1d}\,\omega^{1d\mu} + \cdots + \omega_\mu^{d-1,d}\,\omega^{d-1,d\mu}\right] \\
&\quad + 2m^2h^2 + \frac{1}{2}\,\partial_\mu h\partial^\mu h + \left[\frac{h^2}{8} + \frac{h\phi_0}{2\sqrt{2}}\right]\times \\
&\quad \left[\omega_\mu^{0d}\,\omega^{0d\mu} + \omega_\mu^{1d}\,\omega^{1d\mu} + \cdots + \omega_\mu^{d-1,d}\,\omega^{d-1,d\mu}\right] \\
&\quad + \frac{m^2}{\phi_0^2}\left[\sqrt{2}\,\phi_0\,h^3 + \frac{1}{4}h^4\right] .
\end{aligned} \tag{5}$$

It is seen from the above, that the d spin-connections, namely ω_μ^{ad}, with $a = 0,\ldots,d-1$ have each acquired a mass $M_\omega = \phi_0/\sqrt{2}$. The corresponding interactions are therefore short-ranged. The Higgs field also acquires a mass m. The remaining $d(d-1)/2$ spin connections remain massless, accounting from the long-ranged nature of gravity. The symmetry of the theory spontaneously reduces from: $SO(d,1) \to SO(d-1,1)$. One can easily show that the total number of degrees of freedom (d.o.f.) before and after SSB is the same. Before SSB, one has $(d-1)$ d.o.f. for each of the $d(d+1)/2$ massless spin connection and one for each of the $(d+1)$ scalar components, i.e. a total of $d(d+1)(d-1)/2+(d+1) = (d^3+d)/2+1$ d.o.f. After SSB, one adds up the d.o.f. for the d massive spin connections (each with d d.o.f.), $d(d-1)/2$ massless spin connections (each with $(d-1)$ d.o.f.) and one residual massless scalar field. This results in $d \times d + d(d-1)/2 \times (d-1) + 1 = (d^3+d)/2+1$ exactly same as before.

It is natural to equate the SSB scale to the Planck scale in d-spacetime dimensions, such that the mass acquired by massive spin connections is of the order of Planck mass $M_{pl}^{(d)}$. This implies that they cannot be accessed by low-energy phenomena, and the dynamics in

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[2] Here, $\omega_\mu = \omega_\mu^{ab}\Sigma_{ab}$ with $\Sigma_{ab} = -i[\gamma_a,\gamma_b]/4$ and $[D_\mu,D_\nu] = \frac{i}{2}R_{\mu\nu}^{ab}\Sigma_{ab}$.

spacetime is effectively described by a lower, d-dimensional theory. In other words, SSB has caused an effective dimensional reduction from $(d + 1) \to d$ dimensions. This mechanism provides an alternative to Kaluza-Klein compactification as a means of dimensional reduction, and may have potential applications in string/M-theory for dimensional reduction from ten/eleven-dimensions to the observed four-dimensions.

We conclude from the above that gravitational Higgs mechanism presents a viable method for the emergence of the observed 4-dimensional spacetime from a 5-dimensional one, near the Planck energy scale $M_{Pl}c^2 \approx 10^{16} \ TeV$. Our earlier comments about the renormalizability of gauge and gravity theories continue to hold in the final four-dimensional theory.

It is worth noting that in this case of $5 \to 4$ dimensional reduction via SSB, for matter coupled to the residual *massive* spin connections, the dimensional Newton's constant G_N appears naturally from a dimensionless coupling λ. The easiest way to see this is to start with a matter current j_{ab}^{μ} coupled to the massive spin connections via an interaction Lagrangian, $\mathcal{L}_{int} = -\lambda \sum j_{ab}^{\mu} \, \omega_{\mu}^{ab}$, where the sum is over the massive spin-connections. Then, at energies below $M_{\omega} = M_{Pl}$, the mass terms in Eq.(5) dominate and the effective Lagrangian is given by $\mathcal{L}_{eff} = \lambda \sum \left[\frac{1}{2} M_{\omega}^2 \, \omega_{ab\mu} \omega^{ab\mu} - \lambda j_{ab\mu} \omega_{ab}^{\mu} \right]$. Varying this with respect to ω_{μ}^{ab}, and substituting the stationary solution $\omega_{ab\mu} = \frac{\lambda}{M_{\omega}^2} j_{ab\mu}$ back in the effective action yields

$$\mathcal{L}_{eff} = -\sum G_N \, j_{\mu}^{ab} j_{ab}^{\mu} \quad \text{with} \quad G_N \equiv \left(\frac{\lambda}{\sqrt{2} M_{\omega}} \right)^2 . \tag{6}$$

This phenomenon is similar to the emergence of the effective dimensional Fermi constant G_F from the dimensionless $SU(2)$ coupling constant g_2, namely $G_F = g_2^2 / 4\sqrt{2} M_W^2$ with M_W as the W boson mass [29]-[30]. Note that this only occurs for massive spin connections, and not for the massless long-ranged spin connections.

In summary, we have shown here that one can study perturbative QG using spin connections as the dynamical variable, and that a higher curvature theory of gravity written in terms of these connections gives a ghost-free and renormalizable theory of QG. Furthermore, it is possible to study gravitational analogs of phenomena in particle physics in this picture. In particular, we have studied SSB due to gravitational Higgs mechanism. Further work in this direction may include computing corrections to Newton's law and other scattering processes involving particles interacting via spin connections to show that they are finite.

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