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On axial current in rotating and accelerating medium

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Abstract

Statistical average of the axial current is evaluated on the basis of the covariant Wigner function. In the resulting formula, chemical potential μ, angular velocity Ω and acceleration a enter in combination $\mu \pm (\Omega \pm ia)/2$. The limiting cases of zero mass and zero temperature are investigated in detail. In the zero-mass limit, the axial current is described by a smooth function only at temperatures higher than the Unruh temperature. At zero temperature, the axial current, as a function of the angular velocity and chemical potential, vanishes in a two-dimensional plane region.

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I. INTRODUCTION

Recently many remarkable effects related to the properties of relativistic fluids have been discovered at theoretical level. The nature of these effects, on one hand, is associated with fundamental properties of matter, and, on the other hand, they are expected to be observable experimentally. Two best known examples of this kind are the Chiral Magnetic (CME) and the Chiral Vortical Effect (CVE) manifested in electromagnetic and axial currents, respectively. For detailed discussion of the effects we refer the reader to the rich existing literature, see in particular [1–14].

In [11, 15] the mean value of the axial current was calculated on the basis of the ansatz for the covariant Wigner function proposed in [16]. The resulting formula reduces to the standard formula for the CVE in the approximation linear in vorticity.

Moreover, as is emphasized in [15], the entire series of expansion in the thermal vorticity can be summed up. The result contains information on corrections to the standard CVE. Some of these higher order terms have been derived earlier within other approaches [1, 2]. Here we demonstrate that the formula obtained can be greatly simplified and reduced to a form in which the angular velocity and acceleration enter as a real and imaginary chemical potentials, respectively. Moreover, in the zero-mass limit at temperatures below the Unruh temperature, additional terms appear in the axial current, resulting in a jumplike behavior of the current, as a function of the temperature. According to [17] for linearly accelerated systems, the Unruh temperature is the lowest possible temperature. Our observation on existence of discontinuities in the behavior of the axial current at temperatures below the Unruh temperature supports this conclusion. Note that existence of a boundary temperature proportional to the Unruh temperature was also derived in [18] starting from the condition of positivity of energy density.

Evaluation of the axial current might have important phenomenological implications. Indeed, the appearance of a significant baryon polarization in heavy ion collisions can be one of most important experimental signatures of the CVE. In particular, papers in Refs. [19–22], relate the polarization of baryons to an anomalous axial charge of quarks. On the other hand, the polarization effects can be investigated within the framework of the relativistic hydrodynamics of baryons [23–25], based on the Wigner function introduced in [16], from which the CVE can also be derived. Note that the carriers of the axial charge differ in the two approaches. This situation served as a motivation for us to study the effects in the axial current [15, 16], connected with a finite mass of particles. An interesting phenomenon, which we find in this case, is the existence of a planar two-dimensional domain in the coordinates Ω, μ, where the axial current vanishes. Qualitatively, such a picture is associated with the above-mentioned observation that the angular velocity plays the role of a new chemical potential.

The system of units $\hbar = c = k = 1$ is used.

II. ANALYSIS OF THE EFFECTS OF MOTION OF THE MEDIUM ON THE BASIS OF THE WIGNER FUNCTION

As is known, kinetic properties of a medium can be derived from the quantum field theory using the Wigner function, see, e.g., [26]. In the Ref. [16] an ansatz for the Wigner function was proposed to describe media with fermionic constituents in the state of a local thermodynamic equilibrium. Moreover, it was checked that the ansatz reproduces correctly some known limiting cases. Based on this ansatz, the effects associated with thermal vorticity were investigated in various physical quantities [11, 15, 16, 18]. In particular, in [11, 15], the axial current was first calculated, while an exact formula within the framework of this formalism was obtained in [15].

The Wigner function in [16] is expressed in terms of the distribution function $X(x,p)$, which has the form of a modified Fermi-Dirac distribution

$$X(x,p) = \Big(\exp[\beta_\mu p^\mu - \zeta] \exp\Big[-\frac{1}{2}\varpi_{\mu\nu}\Sigma^{\mu\nu}\Big] + I \Big)^{-1}, \tag{2.1}$$

where $\zeta = \frac{\mu}{T}$, $\varpi_{\mu\nu}$ is the thermal vorticity tensor, and $\Sigma_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]$. Mean values of various physical quantities can be found by integrating the trace of the operator of the quantity considered with the function $X(x,p)$ over the momentum space. Thus, for the axial current we have the following formula [16]

$$\langle j^5_\mu \rangle = -\frac{1}{16\pi^3}\epsilon_{\mu\alpha\nu\beta} \int \frac{d^3p}{\varepsilon} p^\alpha \Big\{ \mathrm{tr}\big(X\Sigma^{\nu\beta}\big) - \mathrm{tr}\big(\bar{X}\Sigma^{\nu\beta}\big) \Big\}, \tag{2.2}$$

where $\langle\cdot\rangle$ denotes statistical averaging with normal ordering, $\bar{X}$ describes the contribution of antiparticles and differs from (2.1) in sign of ζ and ϖ. The matrix traces in (2.2) were exactly found in [15] in formula (4.3)

$$\mathrm{tr}\big(X\Sigma^{\nu\beta}\big) = \Big\{ \big(\exp\big[(\beta p) - \zeta - \frac{g_\omega}{2T} + i\frac{g_a}{2T}\big] + 1\big)^{-1} - \big(\exp\big[(\beta p) - \zeta + \frac{g_\omega}{2T} - i\frac{g_a}{2T}\big] + 1\big)^{-1} \Big\} \\ \frac{T}{2(g_\omega - i g_a)} [\varpi^{\nu\beta} - i\,\mathrm{sgn}(\varpi_{\mu\alpha}\widetilde{\varpi}^{\mu\alpha})\widetilde{\varpi}^{\nu\beta}] + c.c., \tag{2.3}$$

where $\widetilde{\varpi}^{\nu\beta}$ is the tensor dual to $\varpi^{\nu\beta}$, while g_ω and g_a are scalar quantities that depend on acceleration $a^\mu = u^\nu \partial_\nu u^\mu$ and vorticity $\omega_\mu = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}u^\nu \partial^\alpha u^\beta$

$$g_\omega = \frac{1}{\sqrt{2}} \big(\sqrt{(a^2-\omega^2)^2 + 4(\omega a)^2} + a^2 - \omega^2 \big)^{1/2}, \\ g_a = \frac{1}{\sqrt{2}} \big(\sqrt{(a^2-\omega^2)^2 + 4(\omega a)^2} - a^2 + \omega^2 \big)^{1/2}. \tag{2.4}$$

Substituting (2.3) into (2.2), we obtain

$$\langle j^5_\mu \rangle = \frac{\omega_\mu + i\,\mathrm{sgn}(\omega a) a_\mu}{2(g_\omega - i g_a)} \int \frac{d^3p}{(2\pi)^3} \Big\{ n_F(E_p - \mu - g_\omega/2 + i g_a/2) - \\ n_F(E_p - \mu + g_\omega/2 - i g_a/2) + n_F(E_p + \mu - g_\omega/2 + i g_a/2) - \\ n_F(E_p + \mu + g_\omega/2 - i g_a/2) \Big\} + c.c., \tag{2.5}$$

which is another form of Eq. (4.6) from [15]. Here $n_F(E) = (e^{E/T} + 1)^{-1}$ is the Fermi-Dirac distribution, $a^\mu = u^\nu\partial_\nu u^\mu$ and $\omega_\mu = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}u^\nu\partial^\alpha u^\beta$ are the four-acceleration and vorticity, respectively.

It is useful to consider a particular case by going into the comoving reference system and assuming that $\mathbf{\Omega} \,||\, \mathbf{a}$, that is, the acceleration directed along the rotation axis. Then $g_\omega = \Omega$,

$g_a = a$, where Ω and a are the moduli of three dimensional angular velocity and acceleration in the comoving frame, and (2.5) leads to

$$\langle \mathbf{j}^5 \rangle = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\Big\{ n_F(E_p-\mu-\frac{\Omega}{2}+i\frac{a}{2}) - n_F(E_p-\mu+\frac{\Omega}{2}+i\frac{a}{2}) + n_F(E_p+\mu-\frac{\Omega}{2}+i\frac{a}{2}) - n_F(E_p+\mu+\frac{\Omega}{2}+i\frac{a}{2}) + c.c.\Big\}\,\mathbf{e}_\Omega\,, \tag{2.6}$$

where $\mathbf{e}_\Omega = \frac{\mathbf{\Omega}}{\Omega}$ is the unit vector in the direction of the angular velocity.

Eq. (2.6) demonstrates that Ω and a come in a certain combination with the chemical potential. Thus, the effect of rotation and acceleration reduces to a modification of the chemical potential and introduction of a kind of an imaginary chemical potential. This conclusion is worthy of further discussion.

In the limiting case of massless fermions, $m = 0$, the integrals in (2.5) can be found analytically and expressed in terms of polylogarithms in the same way as was done in [15]. Using the following property of the polylogarithms [27]

$$\mathrm{Li}_3(-e^{a+ib}) - \mathrm{Li}_3(-e^{-a-ib}) = -\frac{1}{6}\Big\{a + 2\pi i\Big[\frac{b}{2\pi} - \lfloor \frac{b}{2\pi} + \frac{\mathrm{sgn}(b)}{2}\rfloor\Big]\Big\}^3 - \frac{\pi^2}{6}\Big\{a + 2\pi i\Big[\frac{b}{2\pi} - \lfloor \frac{b}{2\pi} + \frac{\mathrm{sgn}(b)}{2}\rfloor\Big]\Big\}\,, \tag{2.7}$$

we obtain

$$\langle j^5_\mu \rangle = \Big(\frac{1}{6}\big[T^2 + \frac{a^2-\omega^2}{4\pi^2}\big] + \frac{\mu^2}{2\pi^2}\Big)\omega_\mu + \frac{1}{12\pi^2}(\omega a)\,a_\mu + \omega_\mu\Big[-\frac{4\pi T g_a}{g_a^2+g_\omega^2}(\frac{T^2}{6}+\frac{\mu^2}{2\pi^2}-\frac{g_a^2}{8\pi^2}-\frac{g_\omega^2}{8\pi^2})\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\rfloor - 2T^2\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\rfloor^2 + \frac{8\pi T^3 g_a}{3(g_a^2+g_\omega^2)}\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\rfloor^3\Big] + a_\mu \mathrm{sgn}(\omega a)\Big[-\frac{4\pi T g_\omega}{g_a^2+g_\omega^2}(\frac{T^2}{6}+\frac{\mu^2}{2\pi^2}+\frac{g_a^2}{8\pi^2}+\frac{g_\omega^2}{8\pi^2})\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\rfloor + \frac{8\pi T^3 g_\omega}{3(g_a^2+g_\omega^2)}\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\rfloor^3\Big]\,, \tag{2.8}$$

where $\lfloor\cdot\rfloor$ is the integer part. Note that in [15] particular case $|\frac{b}{2\pi}+\frac{\mathrm{sgn}(b)}{2}| < 1$ was considered under which formula (2.7) leads to the Eq. (4.9) from [15], which means that resulting formula Eq. (4.11) from [15] corresponds to the case $T > \frac{g_a}{2\pi}$. Due to contributions from $\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\rfloor$ for $T < \widetilde{T}_U$, where $\widetilde{T}_U$ is

$$\widetilde{T}_U = \frac{g_a}{2\pi}\,, \tag{2.9}$$

the formula (2.8) has discontinuities. For $T > \widetilde{T}_U$ the formula (2.8) takes the form of Eq. (4.11) from [15], derived in approximation $T > \widetilde{T}_U$

$$\langle j^5_\mu \rangle = \Big(\frac{1}{6}\big[T^2 + \frac{a^2-\omega^2}{4\pi^2}\big] + \frac{\mu^2}{2\pi^2}\Big)\omega_\mu + \frac{1}{12\pi^2}(\omega a)\,a_\mu\,. \tag{2.10}$$

It is interesting to note that in the case of $\mathbf{\Omega}\,||\,\mathbf{a}$ or $\mathbf{\Omega} = 0$ the condition $T > \widetilde{T}_U$ results in $T > \frac{a}{2\pi}$, that is, the temperature is to be greater than the Unruh temperature $T_U = \frac{a}{2\pi}$.

The appearance of the Unruh temperature in Eq. (2.8) is a direct consequence of the fact that in (2.5) and (2.6) the acceleration enters as an imaginary chemical potential. If both acceleration and angular velocity are nonzero and directed arbitrarily, the boundary temperature is generalized to $T_U \to \widetilde{T}_U(\Omega, a, \theta)$, where θ is the angle between $\mathbf{a}$ and $\boldsymbol{\Omega}$ in the comoving reference system.

According to [17], the Unruh temperature is the minimum temperature that an accelerated medium can have. Apparently, this fact is the reason why the behavior of the axial current in (2.8) changes qualitatively at $T < \widetilde{T}_U$. A similar result on the existence of a boundary temperature proportional to the Unruh temperature on the basis of the same Wigner function [16] was recently obtained in [18] by considering the energy-momentum tensor and the condition of positivity of the energy density. We note, however, that in [18] the boundary temperature is twice that of the Unruh temperature, which may be due to the fact that in [18] the Boltzmann limit was investigated [1].

Note that (2.10) in the first order in ω^μ leads to the standard formula for CVE [11, 15], while $(-\frac{\omega^2}{24\pi^2})\omega_\mu$ is consistent with the results of [1, 2] (see also [28] for recent progress in the geometric approach, developed in [1]).

III. EFFECTS OF FINITE MASS

There exist various approaches to calculating the polarization of baryons in heavy ion collisions. In particular, in the [19–22] the axial charge of quarks, acquired by them due to the CVE, is considered, and this charge is associated with the polarization of baryons. On the other hand, in [23–25], the polarization is calculated on the basis of the Wigner function for a medium consisting of baryons, assuming equilibrium of the spin degrees of freedom.

Note that the CVE, which is essential for calculating the polarization in [19–22], arises in the approach of Refs. [23–25] as well. However, in [19–22], quarks are considered as carriers of the axial charge, while in [23–25] they are baryons, that is, particles with different masses. In view of this, it is useful to consider the effects of a finite mass in an axial current.

The most characteristic features in the behavior of the axial current arise at $T = 0$. For simplicity, we also assume that $a_\mu = 0$. Going into the comoving reference frame, we obtain $g_a = 0$ and $g_W = \Omega$ in (2.4). The integrals in (2.6) can be evaluated analytically, and we get a simple formula

$$\begin{aligned}\langle \mathbf{j}^5 \rangle = \frac{1}{6\pi^2}\Big\{&\theta(\mu + \frac{\Omega}{2} - m)\big[(\mu + \frac{\Omega}{2})^2 - m^2\big]^{3/2} - \\ &\theta(\mu - \frac{\Omega}{2} - m)\big[(\mu - \frac{\Omega}{2})^2 - m^2\big]^{3/2} + \\ &\theta(-\mu + \frac{\Omega}{2} - m)\big[(\mu - \frac{\Omega}{2})^2 - m^2\big]^{3/2} - \\ &\theta(-\mu - \frac{\Omega}{2} - m)\big[(\mu + \frac{\Omega}{2})^2 - m^2\big]^{3/2}\Big\}\,\mathbf{e}_\Omega\,, \end{aligned} \tag{3.1}$$

where θ is the Heaviside function. From (3.1) it follows, in particular, that for $\Omega < 2(m - |\mu|)$ the axial current is zero. This is in accord with the absence of chemical-potential effect if μ is smaller than the corresponding physical masses. Moreover, we find out that in case

[1] We are grateful to W. Florkowski, E. Speranza for pointing out the possibility of such an explanation.

of a rotating medium, this is true for the "effective" chemical potential incorporating the angular velocity.

The behavior of $j_5 = |\mathbf{j_5}|$, see Eq. (3.1), as a function of Ω and μ is shown in Fig.1. For $\Omega \gg m$ and $\mu \gg m$ the axial current asymptotically tends to its value at zero mass (2.10), $j_5(m=0) = (\frac{\Omega^2}{24\pi^2} + \frac{\mu^2}{2\pi^2})\Omega$, as it should be. In general, due to the effects associated with the mass, j_5 in the massive case is always smaller than in the massless limit, as can be seen from Fig.1.

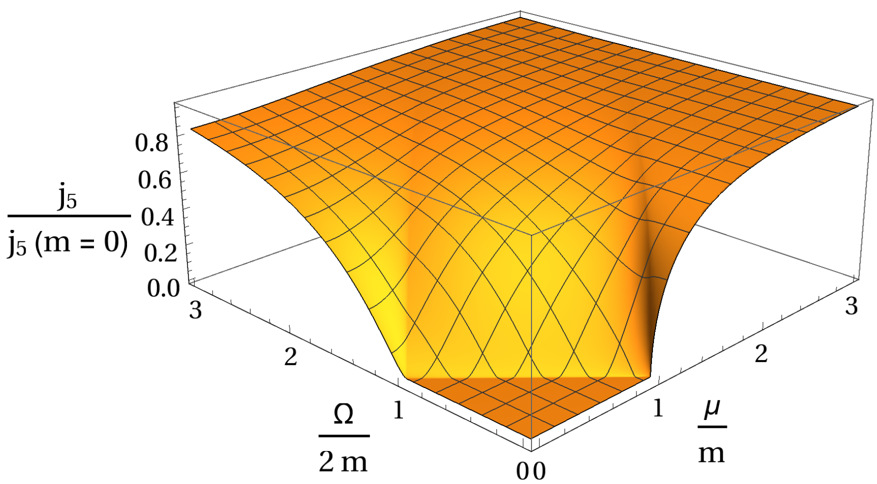

FIG. 1: Axial current (3.1), as a function of the chemical potential and angular velocity at zero temperature. The value of $|\mathbf{j_5}|$ is normalized to its value (2.10) at zero mass.

IV. DISCUSSION

Eq. (2.6) exhibits some features which are challenging to explain theoretically. Let us start with the "imaginary acceleration", ia. Originally, see [17] and references therein, the acceleration a enters the density operator $\hat{\rho}$ as a real number. Namely, in absence of rotation:

$$\hat{\rho} \;=\; (1/Z)\exp\Big(-\hat{H}/T_0 + a\hat{K}_z/T_0\Big)\,, \tag{4.1}$$

where $\hat{H}$ is Hamiltonian and $\hat{K}_z$ is the generator of a Lorentz boost along the z axis. Note that $\ln\hat{\rho}$ is a Hermitian operator for real a. In this sense the Eq. (4.1) looks as a straightforward generalization of the standard equilibrium density operator.

However, when applied to spinors in irreducible representations the boost operators result in a complex number, see, e.g., [29]. Indeed, the angular momentum $\hat{J}$ and boost generator $\hat{K}$ are combining to

$$\hat{N} = \hat{J} + i\hat{K}, \quad \hat{N}^\dagger \;=\; \hat{J} - i\hat{K}\,, \tag{4.2}$$

where the eigenvalues $N \neq 0, N^\dagger = 0$ for left-handed spinors and $N^\dagger \neq 0, N = 0$ for the right-handed spinors.

This leads to the different signs of acceleration of left and right fermions and could be called "chiral gravity". Axial current is a natural probe of such a solution.

In this sense, the density matrix (4.1) does not correspond to a genuine equilibrium if we stick to its interpretation in terms of flat space. This is of no surprise, of course. Indeed, it is well known, for example, that in presence of an external gravitational field the ordinary conservation of a current, $\partial_\alpha j^\alpha = 0$, is becoming a covariant conservation, $\nabla_\alpha J^\alpha = 0$. Re-interpreted in terms of the flat space the covariant conservation becomes a non-conservation, $\partial_\alpha j^\alpha \sim O(a)$. Similarly, the expression for the divergence of the axial current obtained above gives $\partial_\alpha j^\alpha \neq 0$ even in the limit of exact chiral symmetry if $a \neq 0$.

Note also that appearance of two signs of ia may indicate the emergence of dissipating and unstable states which might also tunnel to each other.

Turn now to the "modified chemical potential", $\mu + \Omega/2$ emerging in Eq. (2.6). Note that the possibility of considering the angular velocity as a chemical potential has already been noticed in the literature, see Ref. [1]. What we would like to emphasize here is that the coefficient $1/2$ in front of Ω can be interpreted as a consequence of the equivalence principle according to which spin and angular momentum precess with the same angular velocity [30]. In other words, the spin precession (for Dirac fermions) is twice slower than in the case of magnetic field. This factor of $1/2$, in turn, destroys the balance producing a zero mode in the electromagnetic case. There is no zero mode in the gravitomagnetic field and, as a result, the axial anomaly in gravitational field is proportional to the curvature rather than connection.

All these remarks can be considered as independent checks of Eq. (2.6) and support its validity.

V. CONCLUSIONS

Basing on the ansatz for the Wigner function proposed in [16], we obtained simple formulas for the axial current in the general case of massive fermions, see Eqs. (2.5) and (2.6). In these formulas, the angular velocity and acceleration enter the Fermi-Dirac distribution in combination with the chemical potential. The zero-mass limit (2.8), (2.10), which is consistent in the linear approximation with the standard formula for the CVE, was studied. It is shown that in case that the acceleration and rotation are directed along the same axis, at a temperature lower than the Unruh temperature, the axial current has a series of discontinuities. In more general case of an arbitrary mutual orientation of the acceleration and angular velocity, the temperature (2.9) appears as a boundary, instead of the Unruh temperature.

Dependence of the axial current on the mass of constituents implied by Eq. (2.5) was investigated. In the limit $T = a_\mu = 0$, (2.5) reduces to (3.1), and the axial current, as a function of the angular velocity and chemical potential, vanishes in the two-dimensional region $\Omega < 2(m - |\mu|)$, as is shown in Fig.1.

One can see, that the Wigner-function approach in the zero-mass limit reproduces, after the integration over momenta, the anomaly induced contribution to the axial current, establishing the relation between different approaches to polarization. One can even say, that the thermodynamical approach contains the "hidden anomaly".

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