# BOLIB: Bilevel Optimization LIBrary of test problems

Shenglong Zhou, Alain B. Zemkoho, and Andrey Tin

**Abstract** This chapter presents the Bilevel Optimization LIBrary of the test problems (BOLIB–for short), which contains a collection of test problems, with continuous variables, to help support the development of numerical solvers for bilevel optimization. The library contains 173 examples with 138 nonlinear, 24 linear, and 11 simple bilevel optimization problems. This BOLIB collection is probably the largest bilevel optimization library of test problems. Moreover, as the library is computation-enabled with the MATLAB m-files of all the examples, it provides a uniform basis for testing and comparing algorithms. The library, together with all the related codes, is freely available at biopt.github.io/bolib.

**Keywords:** Bilevel optimization . Test problems . Numerical methods . Library of examples . MATLAB codes

## 1 Introduction

The bilevel optimization problem can take the form

$$\begin{aligned} \min_{x,y}\ & F(x,y) \\ \text{s.t.}\ & G(x,y) \le 0, \\ & y \in S(x) := \arg\min_{y} \left\{ f(x,y) \mid g(x,y) \le 0 \right\}, \end{aligned} \tag{1}$$

where the functions $G : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \longrightarrow \mathbb{R}^{n_G}$ and $g : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \longrightarrow \mathbb{R}^{n_g}$ define the upper-level and lower-level constraints, respectively. As for $F : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \longrightarrow \mathbb{R}$ and $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \longrightarrow \mathbb{R}$, they denote the upper-level and lower-level objective functions, respectively. The set-valued map $S : \mathbb{R}^{n_x} \rightrightarrows \mathbb{R}^{n_y}$ represents the opti-

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mal solution/argminimum mapping of the lower-level problem. Further recall that problem (1) as a whole is often called upper-level problem.

Our aim is to propose a computation-enabled library of test problems to help accelerate the development of numerical methods for bilevel programs in the form (1). For the sake of clarity, note that the bilevel optimization problems in this library, that we classify into the following three categories, only involve continuous variables:

- *Nonlinear bilevel programs*, which are problems in the form (1) with at least one of the functions involved being nonlinear.
- *Linear bilevel programs* are problems in the form (1) with functions $F$, $f$, and all the components of $G$ and $g$ being linear.
- *Simple bilevel programs* (term coined in [13]) are optimization problems where the feasible set is partly defined by the optimal solution set of a second optimization problem. But unlike in (1), the lower-level problem is not a parametric optimization problem. More precisely, a simple bilevel optimization has the form:

$$\begin{aligned} \min_{y}\ & F(y) \\ \text{s.t. } & G(y) \leq 0, \\ & y \in S := \arg\min_{y} \ \{f(y) \mid g(y) \leq 0\}, \end{aligned} \tag{2}$$

  where, similarly to (1), $G : \mathbb{R}^{n_y} \to \mathbb{R}^{n_G}$ and $g : \mathbb{R}^{n_y} \to \mathbb{R}^{n_g}$ describe the upper-level and lower-level constraints, respectively, while the real-valued function $F$ (resp. $f$), defined $\mathbb{R}^{n_y}$, represents the upper-level (resp. lower-level) objective function. The expression "simple bilevel program" is used in [32] to refer to bilevel optimization problems of the form (1), where $y$ (resp. $x$) is not involved in the upper-level (resp. lower-level) constraints.

The main contributions of the library are three-fold. First, BOLIB provides MATLAB codes for 173 examples, including 138 nonlinear, 24 linear, and 11 simple bilevel programs, ready to be used to test numerical algorithms. Secondly, it puts together the true or best known solutions and the corresponding references for all the examples included. Hence, can serve as a benchmark platform for numerical accuracy evaluation for methods designed to solve problem (1). Thirdly, all examples as well as their gradients and Hessians are programmed and stored in the MATLAB m-files. Thus, facilitating the use of the examples and corresponding derivatives in the implementation of numerical methods, where such information is necessary.

To the best of our knowledge, this is the largest library of test examples for bilevel optimization, especially for the nonlinear class of the problem. It includes bilevel optimization problems from Colson's BIPA [11], Leyffer's MacMPEC [35], as well as from Mitsos and Barton's technical report [39]. We would like to emphasize that the fundamental objective that we hope to achieve with BOLIB is the acceleration of numerical software development for bilevel optimization, as it is our opinion that the level of expansion of applications of the problem has outpaced the development rate for numerical solvers, especially for the nonlinear class of the problem.

In the next section, we describe the library with details on the inputs and outputs of the codes, as well as some useful insights on the examples. In the subsequent section, a guideline is given on how to access the library.

## 2 Description of the library

This section describes the structure of the library, while focusing on the inputs and outputs of each example, as well as the list of all examples together with their true or best known solutions and corresponding references. Before we proceed, note that each `m-file` contains information about the corresponding example, which include the first and second order derivatives of the input functions. For the upper-level objective function $F : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \to \mathbb{R}$, these derivatives are defined as follows

$$\begin{aligned}
\nabla_x F(x,y) &= \begin{bmatrix} \nabla_{x_1} F \\ \vdots \\ \nabla_{x_{n_x}} F \end{bmatrix} \in \mathbb{R}^{n_x}, \\
\nabla^2_{xx} F(x,y) &= \begin{bmatrix} \nabla^2_{x_1 x_1} F & \cdots & \nabla^2_{x_{n_x} x_1} F \\ \vdots & \ddots & \vdots \\ \nabla^2_{x_1 x_{n_x}} F & \cdots & \nabla^2_{x_{n_x} x_{n_x}} F \end{bmatrix} \in \mathbb{R}^{n_x \times n_x}, \\
\nabla^2_{xy} F(x,y) &= \begin{bmatrix} \nabla^2_{x_1 y_1} F & \cdots & \nabla^2_{x_{n_x} y_1} F \\ \vdots & \ddots & \vdots \\ \nabla^2_{x_1 y_{n_x}} F & \cdots & \nabla^2_{x_{n_x} y_{n_x}} F \end{bmatrix} \in \mathbb{R}^{n_y \times n_x}.
\end{aligned} \tag{1}$$

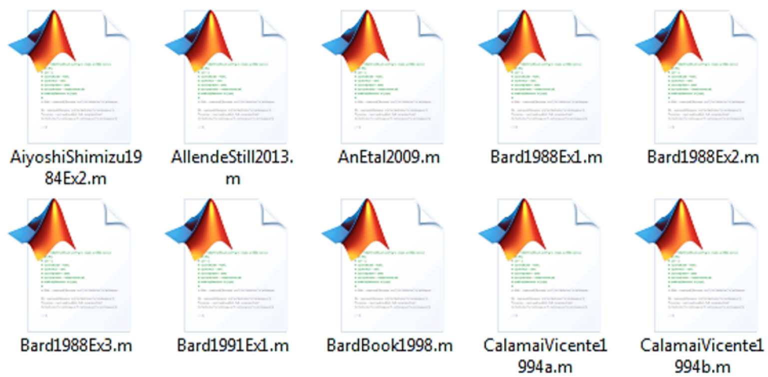
Similar expressions are valid for $\nabla_y F(x,y) \in \mathbb{R}^{n_y}$, $\nabla^2_{yy} F(x,y) \in \mathbb{R}^{n_y \times n_y}$, and the lower-level objective function $f$. As the constraint functions are vector-valued, we use the following notations to refer to derivative information in the context of the upper-level constraint function $G : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \to \mathbb{R}^{n_G}$, for instance:

$$
\begin{aligned}
\nabla_x G(x,y) &= \begin{bmatrix} \nabla_x G_1 \\ \vdots \\ \nabla_x G_{n_G} \end{bmatrix} = \begin{bmatrix} \nabla_{x_1} G_1 & \cdots & \nabla_{x_{n_x}} G_1 \\ \vdots & \ddots & \vdots \\ \nabla_{x_1} G_{n_G} & \cdots & \nabla_{x_{n_x}} G_{n_G} \end{bmatrix} \in \mathbb{R}^{n_G \times n_x}, \\
\nabla^2_{xx} G(x,y) &= \begin{bmatrix} \nabla^2_{xx} G_1 \\ \vdots \\ \nabla^2_{xx} G_{n_G} \end{bmatrix} = \begin{bmatrix} \nabla^2_{x_1 x_1} G_1 & \cdots & \nabla^2_{x_{n_x} x_1} G_1 \\ \vdots & \ddots & \vdots \\ \nabla^2_{x_1 x_{n_x}} G_1 & \cdots & \nabla^2_{x_{n_x} x_{n_x}} G_1 \\ \vdots & \ddots & \vdots \\ \nabla^2_{x_1 x_1} G_{n_G} & \cdots & \nabla^2_{x_{n_x} x_1} G_{n_G} \\ \vdots & \ddots & \vdots \\ \nabla^2_{x_1 x_{n_x}} G_{n_G} & \cdots & \nabla^2_{x_{n_x} x_{n_x}} G_{n_G} \end{bmatrix} \in \mathbb{R}^{(n_G n_x) \times n_x}, \\
\nabla^2_{xy} G(x,y) &= \begin{bmatrix} \nabla^2_{xy} G_1 \\ \vdots \\ \nabla^2_{xy} G_{n_G} \end{bmatrix} = \begin{bmatrix} \nabla^2_{x_1 y_1} G_1 & \cdots & \nabla^2_{x_{n_x} y_1} G_1 \\ \vdots & \ddots & \vdots \\ \nabla^2_{x_1 y_{n_y}} G_1 & \cdots & \nabla^2_{x_{n_x} y_{n_y}} G_1 \\ \vdots & \ddots & \vdots \\ \nabla^2_{x_1 y_1} G_{n_G} & \cdots & \nabla^2_{x_{n_x} y_1} G_{n_G} \\ \vdots & \ddots & \vdots \\ \nabla^2_{x_1 y_{n_y}} G_{n_G} & \cdots & \nabla^2_{x_{n_x} y_{n_y}} G_{n_G} \end{bmatrix} \in \mathbb{R}^{(n_G n_y) \times n_x}.
\end{aligned} \tag{2}
$$

Similar formulas are also valid for $\nabla_y G(x,y) \in \mathbb{R}^{n_G \times n_y}$, $\nabla^2_{yy} G(x,y) \in \mathbb{R}^{n_G n_y \times n_y}$, and the lower-level constraint $g$. It is important to emphasize that in the context of the constraints, $\nabla_x G(x,y) \in \mathbb{R}^{1 \times n_x}$, for example, is a row vector when $n_G = 1$. However, $\nabla_x F(x,y) \in \mathbb{R}^{n_x}$ and $\nabla_x f(x,y) \in \mathbb{R}^{n_x}$ are column vectors.

## 2.1 Inputs and outputs

The `BOLIBver2` folder (see Section 3 on how to access the library), which contains all the library material, includes 3 sub-folders named `Nonlinear`, `Linear`, and `Simple`. In the `Nonlinear` subfolder, there are 138 MATLAB m-files. Each one specifies a nonliner bilevel optimization test example, named by a combination of authors' surnames, year of publication, and when necessary, the order of the example in the corresponding reference. For example, as in following figure (showing a partial list of the examples), `AiyoshiShimizu1984Ex2.m` stands for Example 2 in the paper by Aiyoshi and Shimizu published in 1984 [1]. However, for a few examples (`DesignCentringP1`, `NetworkDesignP1`, etc.), the problem naming is based on previous use in the literature and therefore could help to easily recognize them.




In folder `Linear`, there are 24 MATLAB m-files defining 24 liner bilevel optimization test examples. The rule of naming each example is same as in the nonlinear case. Similarly, folder `Simple` contains 11 simple bilevel optimization test examples.

Now we describe the inputs and outputs of the m-file of a given example. Each file has the function handle named in the following way:

$$\texttt{w = example\_name(x, y, keyf, keyxy)}. \tag{3}$$

For the inputs, we have

$$\begin{aligned} x \in \mathbb{R}^{n_x}, \quad y \in \mathbb{R}^{n_y}&, \\ \texttt{keyf} \in \{\texttt{'F', 'G', 'f', 'g'}\}&, \\ \texttt{keyxy} \in \{[\,], \texttt{'x', 'y', 'xx', 'xy', 'yy'}\}&, \end{aligned}$$

where `'F'`, `'G'`, `'f'`, and `'g'`, respectively stand for the four functions involved in (1). `'x'` and `'y'` represent the first order derivative with respect to $x$ and $y$, respectively. Finally, `'xx'`, `'xy'`, and `'yy'` correspond to the second order derivative of the function $F$, $G$, $f$, and $g$, with respect to $xx$, $xy$, and $yy$, respectively.

For the outputs, `w = example_name(x,y,keyf)` or `w = example_name(x,y,keyf,[])` returns the function value of `keyf` while `w=example_name(x,y,keyf,keyxy)` can additionally return the first or second order derivative of `keyf` w.r.t. the choice of `keyxy` as described above. We summarize all the scenarios in Table 1:

**Table 1** Input–output scenarios from the m-files containing the examples

| keyf/keyxy | [ ] | 'x' | 'y' | 'xx' | 'xy' | 'yy' |
|---|---|---|---|---|---|---|
| 'F' | $F(x,y)$ | $\nabla_x F(x,y)$ | $\nabla_y F(x,y)$ | $\nabla^2_{xx} F(x,y)$ | $\nabla^2_{xy} F(x,y)$ | $\nabla^2_{yy} F(x,y)$ |
| 'G' | $G(x,y)$ | $\nabla_x G(x,y)$ | $\nabla_y G(x,y)$ | $\nabla^2_{xx} G(x,y)$ | $\nabla^2_{xy} G(x,y)$ | $\nabla^2_{yy} G(x,y)$ |
| 'f' | $f(x,y)$ | $\nabla_x f(x,y)$ | $\nabla_y f(x,y)$ | $\nabla^2_{xx} f(x,y)$ | $\nabla^2_{xy} f(x,y)$ | $\nabla^2_{yy} f(x,y)$ |
| 'g' | $g(x,y)$ | $\nabla_x g(x,y)$ | $\nabla_y g(x,y)$ | $\nabla^2_{xx} g(x,y)$ | $\nabla^2_{xy} g(x,y)$ | $\nabla^2_{yy} g(x,y)$ |

For the dimension of $w$ in each scenario, see (1)–(2). If $n_G = 0$ (or $n_g = 0$), all outputs related to $G$ (or $g$) should be empty, namely, $w = [\ ]$. To further clarify the outputs, let us look at some specific usage:

- $w$ = `example_name`$(x, y,$ `'F'`$)$ or $w$ = `example_name`$(x, y,$ `'F'`$, [\ ])$ returns the function value of $F$, i.e., $w = F(x, y)$; this is similar for $G$, $f$, and $g$;
- $w$ = `example_name`$(x, y,$ `'F'`, `'x'`$)$ returns the partial derivative of $F$ with respect to $x$, i.e., $w = \nabla_x F(x, y)$;
- $w$ = `example_name`$(x, y,$ `'G'`, `'y'`$)$ returns the Jacobian matrix of $G$ with respect to $y$, i.e., $w = \nabla_y G(x, y)$;
- $w$ = `example_name`$(x, y,$ `'f'`, `'xy'`$)$ returns the Hessian matrix of $f$ with respect to $xy$, i.e., $w = \nabla^2_{xy} f(x, y)$;
- $w$ = `example_name`$(x, y,$ `'g'`, `'yy'`$)$ returns the second order derivative of $g$ with respect to $yy$, i.e., $w = \nabla^2_{yy} g(x, y)$.

We now use two examples to illustrate the definitions above. The first one is nonlinear while the second one is a simple bilevel program.

**Example**

Shimizu et al. (1997), see [50], considered the bilevel program (1) with

$$\begin{aligned} F(x, y) &:= (x-5)^2 + (2y+1)^2, \\ f(x, y) &:= (y-1)^2 - 1.5xy, \\ g(x, y) &:= \begin{bmatrix} -3x + y + 3 \\ x - 0.5y - 4 \\ x + y - 7 \end{bmatrix}. \end{aligned}$$

Here, we have dimensions $n_x = 1$, $n_y = 1$, $n_G = 0$, and $n_g = 3$. The `m-file` is named by `ShimizuEtal1997a` (i.e., `exmaple_name` = `ShimizuEtal1997a` ) and was coded in MATLAB as it can be seen in Table 2. If we are given some inputs (as in left column of the table below), then `ShimizuEtal1997a` will return us corresponding results as in the right column:

| **Inputs** | **Outputs** |
|---|---|
| `x   = 4` | `x   = 4` |
| `y   = 0` | `y   = 0` |
| `F   = ShimizuEtal1997a(x,y,'F')` | `F   = 2` |
| `Fx  = ShimizuEtal1997a(x,y,'F','x')` | `Fx  = -2` |
| `Gy  = ShimizuEtal1997a(x,y,'G','y')` | `Gy  = []` |
| `fxy = ShimizuEtal1997a(x,y,'f','xy')` | `fxy = -1.5` |
| `gyy = ShimizuEtal1997a(x,y,'g','yy')` | `gyy = [0;0;0]` |

**Table 2** Matlab code for `ShimizuEtal1997a`.

```
function w=ShimizuEtal1997a(x,y,keyf,keyxy)
if nargin<4 || isempty(keyxy)
    switch keyf
    case ‘F’; w = (x-5)^2+(2*y+1)^2;
    case ‘G’; w = [];
    case ‘f’; w = (y-1)^2-1.5*x*y;
    case ‘g’; w = [-3*x+y+3; x-0.5*y-4; x+y-7];
    end
else
    switch keyf
    case ‘F’
        switch keyxy
        case ‘x’ ; w = 2*(x-5);
        case ‘y’ ; w = 4*(2*y+1);
        case ‘xx’; w = 2;
        case ‘xy’; w = 0;
        case ‘yy’; w = 8;
        end
    case ‘G’
       switch keyxy
        case ‘x’ ; w = [];
        case ‘y’ ; w = [];
        case ‘xx’; w = [];
        case ‘xy’; w = [];
        case ‘yy’; w = [];
        end
    case ‘f’
        switch keyxy
        case ‘x’ ; w = -1.5*y;
        case ‘y’ ; w = 2*(y-1)-1.5*x;
        case ‘xx’; w = 0;
        case ‘xy’; w = -1.5;
        case ‘yy’; w = 2;
        end
    case ‘g’
        switch keyxy
        case ‘x’ ; w = [-3;  1;  1];
        case ‘y’ ; w = [ 1;-0.5; 1];
        case ‘xx’; w = [ 0;  0;  0];
        case ‘xy’; w = [ 0;  0;  0];
        case ‘yy’; w = [ 0;  0;  0];
        end
     end
end
end
```

**Example**

Franke et al. (2018), see [73], considered the bilevel program (1) with

$$\begin{aligned} F(y) &:= -y_2, \\ f(y) &:= y_3, \\ g(y) &:= \begin{bmatrix} y_1^2 - y_3 \\ y_1^2 + y_2^2 - 1 \\ -y_3 \end{bmatrix}. \end{aligned}$$

Here, we have dimensions $n_x = 0, n_y = 3, n_G = 0$, and $n_g = 3$. The `m-file` is named by `FrankeEtal2018Ex513` (i.e., `exmaple_name = FrankeEtal2018Ex513`) and is equally coded in MATLAB as described in Table 3.

It is worth mentioning that despite the lack of variable $x$ in the latter example, we still treat it as an input, for the sake of unifying the inputs of the function handle as in (3). Hence, for all the simple bilevel optimization examples, we input `x` as a scalar. In this way, `x` has no impact on the example itself.

## 2.2 Useful details on the examples

The details related to each example presented in the BOLIB library are in a column of Table 4 below. As we mentioned before, those examples are classified into 3 categories: nonliner, linear and simple bilevel optimisation test examples. The first column of the table provides the list of problems, as they appear in the `Examples` subfolder of the `BOLIBver2` folder. The second column gives the reference in the literature where the example might have first appeared. The third column combines the labels corresponding to the nature of the functions involved in (1). Precisely, "N" and "L" will be used to indicate whether the functions $F$, $G$, $f$, and $g$ are nonlinear (N) or linear (L), while "$O$" is used to symbolize that there is either no function $G$ or $g$ present in problem (1). Then follows the column with $n_x$ and $n_y$ for the upper and lower-level variable dimensions, as well as $n_G$ (resp. $n_g$) to denote the the number of components of the upper (resp. lower)-level constraint function. On the other hand, $F^*$ and $f^*$ denote the best known optimal upper and lower-level objective function values, respectively, according to the reference that is listed in the last column.

Note that examples `Zlobec2001b` and `MitsosBartonEx32` have no optimal solutions. There are 4 examples involving parameters; i.e., `CalamaiVicente1994a` with $\rho \geq 1$ (its $F^*$ and $f^*$ listed in the table are under $\rho = 1$, other cases can be found in [7]), `HenrionSurowiec2011` with $c \in \mathbb{R}$, `IshizukaAiyoshi1992a` with $M > 1$ and `RobustPortfolioP1` with $\delta \in [1, +\infty]$ (its $F^*$ and $f^*$ listed in the

**Table 3** Matlab code for `FrankeEtal2018Ex513`.

```
function w=FrankeEtal2018Ex513(x,y,keyf,keyxy)
if nargin<4 || isempty(keyxy)
    switch keyf
    case 'F'; w = -y(2);
    case 'G'; w = [];
    case 'f'; w = y(3);
    case 'g'; w = [y(1)^2-y(3); y(1)^2+y(2)^2-1; -y(3)];
    end
else
    switch keyf
    case 'F'
        switch keyxy
        case 'x' ; w = 0;
        case 'y' ; w = [0; -1; 0];
        case 'xx'; w = 0;
        case 'xy'; w = zeros(3,1);
        case 'yy'; w = zeros(3,3);
        end
    case 'G'
       switch keyxy
        case 'x' ; w = [];
        case 'y' ; w = [];
        case 'xx'; w = [];
        case 'xy'; w = [];
        case 'yy'; w = [];
        end
    case 'f'
        switch keyxy
        case 'x' ; w = 0;
        case 'y' ; w = [0; 0; 1];
        case 'xx'; w = 0;
        case 'xy'; w = zeros(3,1);
        case 'yy'; w = zeros(3,3);
        end
    case 'g'
        switch keyxy
        case 'x' ; w = zeros(3,1);
        case 'y' ; w = [2*y(1) 0 -1; 2*y(1) 2*y(2) 0; 0 0 -1];
        case 'xx'; w = zeros(3,1);
        case 'xy'; w = zeros(9,1);
        case 'yy'; w = [2 0 0;0 0 0;0 0 0;2 0 0; 0 2 0;zeros(4,3)];
        end
     end
end
end
```

table are under $\delta = 2$). Dimensions $n_x$, $n_y$, $n_G$ or $n_g$ of examples `OptimalControl`, `RobustPortfolioP1`, `RobustPortfolioP2`, and `ShehuEtal2019Ex42` can be altered to get problems of different sizes, as necessary.

Table 4: List of bilevel programs with related labels and known solutions.

| **Example name** | **RefI** | $F$-$G$-$f$-$g$ | $n_x$ | $n_y$ | $n_G$ | $n_g$ | $F^*$ | $f^*$ | **RefII** |
|---|---|---|---|---|---|---|---|---|---|
| | | | | **Nonlinear bilevel programs** | | | | | |
| AiyoshiShimizu1984Ex2 | [1] | L-L-N-L | 2 | 2 | 5 | 6 | 5 | 0 | [1] |
| AllendeStill2013 | [2] | N-L-N-N | 2 | 2 | 5 | 2 | 1 | -0.5 | [2] |
| AnEtal2009 | [3] | N-L-N-L | 2 | 2 | 6 | 4 | 2251.6 | 565.8 | [3] |
| Bard1988Ex1 | [4] | N-L-N-L | 1 | 1 | 1 | 4 | 17 | 1 | [4] |
| Bard1988Ex2 | [4] | N-L-N-L | 4 | 4 | 9 | 12 | -6600 | 54 | [11] |
| Bard1988Ex3 | [4] | N-N-N-N | 2 | 2 | 3 | 4 | -12.68 | -1.02 | [8] |
| Bard1991Ex1 | [5] | L-L-N-L | 1 | 2 | 2 | 3 | 2 | 12 | [5] |
| BardBook1998Ex832 | [6] | N-L-L-L | 2 | 2 | 4 | 7 | 0 | 5 | |
| CalamaiVicente1994a | [7] | N-O-N-L | 1 | 1 | 0 | 3 | 0 | 0 | [7] |
| CalamaiVicente1994b | [7] | N-O-N-L | 4 | 2 | 0 | 6 | 0.3125 | -0.4063 | [7] |
| CalamaiVicente1994c | [7] | N-O-N-L | 4 | 2 | 0 | 6 | 0.3125 | -0.4063 | [7] |
| CalveteGale1999P1 | [9] | L-L-L-N | 2 | 3 | 2 | 6 | -29.2 | 0.31 | [9, 23] |
| ClarkWesterberg1990a | [10] | N-L-N-L | 1 | 1 | 2 | 3 | 5 | 4 | [47] |
| Colson2002BIPA1 | [11] | N-L-N-L | 1 | 1 | 3 | 3 | 250 | 0 | |
| Colson2002BIPA2 | [11] | N-L-N-L | 1 | 1 | 1 | 4 | 17 | 2 | [8] |
| Colson2002BIPA3 | [11] | N-L-N-L | 1 | 1 | 2 | 2 | 2 | 24.02 | [8] |
| Colson2002BIPA4 | [11] | N-L-N-L | 1 | 1 | 2 | 2 | 88.79 | -0.77 | [8] |
| Colson2002BIPA5 | [11] | N-L-N-N | 1 | 2 | 1 | 6 | 2.75 | 0.57 | [8] |
| Dempe1992a | [12] | L-N-N-N | 2 | 2 | 1 | 2 | $\times$ | $\times$ | |
| Dempe1992b | [12] | N-O-N-N | 1 | 1 | 0 | 1 | 31.25 | 4 | [8] |
| DempeDutta2012Ex24 | [14] | N-O-N-N | 1 | 1 | 0 | 1 | 0 | 0 | [14] |
| DempeDutta2012Ex31 | [14] | L-N-N-N | 2 | 2 | 4 | 2 | -1 | 4 | [14] |
| DempeEtal2012 | [15] | L-L-N-L | 1 | 1 | 2 | 2 | -1 | -1 | [15] |
| DempeFranke2011Ex41 | [16] | N-L-N-L | 2 | 2 | 4 | 4 | 5 | -2 | [16] |
| DempeFranke2011Ex42 | [16] | N-L-N-L | 2 | 2 | 4 | 3 | 2.13 | -3.5 | [16] |
| DempeFranke2014Ex38 | [17] | L-L-N-L | 2 | 2 | 4 | 4 | -1 | -4 | [17] |
| DempeLohse2011Ex31a | [18] | N-O-N-L | 2 | 2 | 0 | 4 | -5.5 | 0 | [18] |
| DempeLohse2011Ex31b | [18] | N-O-N-L | 3 | 3 | 0 | 5 | -12 | 0 | |
| DeSilva1978 | [19] | N-O-N-L | 2 | 2 | 0 | 4 | -1 | 0 | [8] |
| FalkLiu1995 | [20] | N-O-N-L | 2 | 2 | 0 | 4 | -2.1962 | 0 | [8] |
| FloudasEtal2013 | [21] | L-L-N-L | 2 | 2 | 4 | 7 | 0 | 200 | [52] |
| FloudasZlobec1998 | [22] | N-L-L-N | 1 | 2 | 2 | 6 | 1 | -1 | [23, 39] |
| GumusFloudas2001Ex1 | [23] | N-L-N-L | 1 | 1 | 3 | 3 | 2250 | 197.75 | [39] |
| GumusFloudas2001Ex3 | [23] | L-L-N-L | 2 | 3 | 4 | 9 | -29.2 | 0.31 | [39] |
| GumusFloudas2001Ex4 | [23] | N-L-N-L | 1 | 1 | 5 | 2 | 9 | 0 | [39] |
| GumusFloudas2001Ex5 | [23] | L-L-N-N | 1 | 2 | 2 | 6 | 0.19 | -7.23 | [39] |
| HatzEtal2013 | [24] | L-O-N-L | 1 | 2 | 0 | 2 | 0 | 0 | [24] |
| HendersonQuandt1958 | [25] | N-L-N-L | 1 | 1 | 2 | 1 | -3266.7 | -711.11 | [25] |
| HenrionSurowiec2011 | [26] | N-O-N-O | 1 | 1 | 0 | 0 | $-c^2/4$ | $-c^2/8$ | [27] |
| IshizukaAiyoshi1992a | [28] | N-L-L-L | 1 | 2 | 1 | 5 | 0 | -M | [28] |
| KleniatiAdjiman2014Ex3 | [29] | L-L-N-L | 1 | 1 | 2 | 2 | -1 | 0 | [29] |
| KleniatiAdjiman2014Ex4 | [29] | N-N-N-N | 5 | 5 | 13 | 11 | -10 | -3.1 | [29] |
| LamparSagrat2017Ex23 | [30] | L-L-N-L | 1 | 2 | 2 | 2 | -1 | 1 | [30] |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| LamparSagrat2017Ex31 | [31] | N-L-L-L | 1 | 1 | 1 | 1 | 1 | 0 | [31] |
| LamparSagrat2017Ex32 | [31] | N-O-N-O | 1 | 1 | 0 | 0 | 0.5 | 0 | [31] |
| LamparSagrat2017Ex33 | [31] | N-L-L-L | 1 | 2 | 1 | 3 | 0.5 | 0 | [31] |
| LamparSagrat2017Ex35 | [31] | N-L-L-L | 1 | 1 | 2 | 3 | 0.8 | -0.4 | [31] |
| LucchettiEtal1987 | [33] | N-L-N-L | 1 | 1 | 2 | 2 | 0 | 0 | [33] |
| LuDebSinha2016a | [34] | N-L-N-O | 1 | 1 | 4 | 0 | 1.14 | 1.69 | [34] |
| LuDebSinha2016b | [34] | N-L-N-O | 1 | 1 | 4 | 0 | 0 | 1.66 | [34] |
| LuDebSinha2016c | [34] | N-L-N-O | 1 | 1 | 4 | 0 | 1.12 | 0.06 | [34] |
| LuDebSinha2016d | [34] | L-N-L-N | 2 | 2 | 11 | 3 | × | × | |
| LuDebSinha2016e | [34] | N-L-L-N | 1 | 2 | 6 | 3 | × | × | |
| LuDebSinha2016f | [34] | L-N-N-O | 2 | 1 | 9 | 0 | × | × | |
| MacalHurter1997 | [36] | N-O-N-O | 1 | 1 | 0 | 0 | 81.33 | -0.33 | [36] |
| Mirrlees1999 | [38] | N-O-N-O | 1 | 1 | 0 | 0 | 1 | 0.06 | [38] |
| MitsosBarton2006Ex38 | [39] | N-L-N-L | 1 | 1 | 4 | 2 | 0 | 0 | [39] |
| MitsosBarton2006Ex39 | [39] | L-L-N-L | 1 | 1 | 3 | 2 | -1 | -1 | [39] |
| MitsosBarton2006Ex310 | [39] | L-L-N-L | 1 | 1 | 2 | 2 | 0.5 | -0.1 | [39] |
| MitsosBarton2006Ex311 | [39] | L-L-N-L | 1 | 1 | 2 | 2 | -0.8 | 0 | [39] |
| MitsosBarton2006Ex312 | [39] | N-L-N-L | 1 | 1 | 2 | 2 | 0 | 0 | [39] |
| MitsosBarton2006Ex313 | [39] | L-L-N-L | 1 | 1 | 2 | 2 | -1 | 0 | [39] |
| MitsosBarton2006Ex314 | [39] | N-L-N-L | 1 | 1 | 2 | 2 | 0.25 | -0.08 | [39] |
| MitsosBarton2006Ex315 | [39] | L-L-N-L | 1 | 1 | 2 | 2 | 0 | -0.83 | [39] |
| MitsosBarton2006Ex316 | [39] | L-L-N-L | 1 | 1 | 2 | 2 | -2 | 0 | [39] |
| MitsosBarton2006Ex317 | [39] | N-L-N-L | 1 | 1 | 2 | 2 | 0.19 | -0.02 | [39] |
| MitsosBarton2006Ex318 | [39] | N-L-N-L | 1 | 1 | 2 | 2 | -0.25 | 0 | [39] |
| MitsosBarton2006Ex319 | [39] | N-L-N-L | 1 | 1 | 2 | 2 | -0.26 | 0 | [39] |
| MitsosBarton2006Ex320 | [39] | N-L-N-L | 1 | 1 | 2 | 2 | 0.31 | -0.08 | [39] |
| MitsosBarton2006Ex321 | [39] | N-L-N-L | 1 | 1 | 2 | 2 | 0.21 | -0.07 | [39] |
| MitsosBarton2006Ex322 | [39] | N-L-N-N | 1 | 1 | 2 | 3 | 0.21 | -0.07 | [39] |
| MitsosBarton2006Ex323 | [39] | N-N-L-N | 1 | 1 | 3 | 3 | 0.18 | -1 | [39] |
| MitsosBarton2006Ex324 | [39] | N-L-N-L | 1 | 1 | 2 | 2 | -1.75 | 0 | [39] |
| MitsosBarton2006Ex325 | [39] | N-N-N-N | 2 | 3 | 6 | 9 | -1 | -2 | [39] |
| MitsosBarton2006Ex326 | [39] | N-N-N-L | 2 | 3 | 7 | 6 | -2.35 | -2 | [39] |
| MitsosBarton2006Ex327 | [39] | N-N-N-N | 5 | 5 | 13 | 13 | 2 | -1.1 | [39] |
| MitsosBarton2006Ex328 | [39] | N-N-N-N | 5 | 5 | 13 | 13 | -10 | -3.1 | [39] |
| MorganPatrone2006a | [40] | L-L-N-L | 1 | 1 | 2 | 2 | -1 | 0 | [40] |
| MorganPatrone2006b | [40] | L-O-N-L | 1 | 1 | 0 | 4 | -1.25 | 0 | [40] |
| MorganPatrone2006c | [40] | L-O-N-L | 1 | 1 | 0 | 4 | -1 | -0.25 | [40] |
| MuuQuy2003Ex1 | [41] | N-L-N-L | 1 | 2 | 2 | 3 | -2.08 | -0.59 | [41] |
| MuuQuy2003Ex2 | [41] | N-L-N-L | 2 | 3 | 3 | 4 | 0.64 | 1.67 | [41] |
| NieEtal2017Ex34 | [42] | L-L-N-N | 1 | 2 | 2 | 2 | 2 | 0 | [42] |
| NieEtal2017Ex52 | [42] | N-N-N-N | 2 | 3 | 5 | 2 | -1.71 | -2.23 | [42] |
| NieEtal2017Ex54 | [42] | N-N-N-N | 4 | 4 | 3 | 2 | -0.44 | -1.19 | [42] |
| NieEtal2017Ex57 | [42] | N-N-N-N | 2 | 3 | 5 | 2 | -2 | -1 | [42] |
| NieEtal2017Ex58 | [42] | N-N-N-N | 4 | 4 | 3 | 2 | -3.49 | -0.86 | [42] |
| NieEtal2017Ex61 | [42] | N-N-N-N | 2 | 2 | 5 | 1 | -1.02 | -1.08 | [42] |
| Outrata1990Ex1a | [43] | N-O-N-L | 2 | 2 | 0 | 4 | -8.92 | -6.05 | [43] |
| Outrata1990Ex1b | [43] | N-O-N-L | 2 | 2 | 0 | 4 | -7.56 | -0.58 | [43] |
| Outrata1990Ex1c | [43] | N-O-N-L | 2 | 2 | 0 | 4 | -12 | -112.71 | [43] |
| Outrata1990Ex1d | [43] | N-O-N-L | 2 | 2 | 0 | 4 | -3.6 | -2 | [43] |
| Outrata1990Ex1e | [43] | N-O-N-L | 2 | 2 | 0 | 4 | -3.15 | -16.29 | [43] |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Outrata1990Ex2a | [43] | N-L-N-L | 1 | 2 | 1 | 4 | 0.5 | -14.53 | [43] |
| Outrata1990Ex2b | [43] | N-L-N-L | 1 | 2 | 1 | 4 | 0.5 | -4.5 | [43] |
| Outrata1990Ex2c | [43] | N-L-N-L | 1 | 2 | 1 | 4 | 1.86 | -10.93 | [43] |
| Outrata1990Ex2d | [43] | N-L-N-N | 1 | 2 | 1 | 4 | 0.92 | -19.47 | [43] |
| Outrata1990Ex2e | [43] | N-L-N-N | 1 | 2 | 1 | 4 | 0.90 | -14.94 | [43] |
| Outrata1993Ex31 | [44] | N-L-N-N | 1 | 2 | 1 | 4 | 1.56 | -11.67 | [44] |
| Outrata1993Ex32 | [44] | N-L-N-N | 1 | 2 | 1 | 4 | 3.21 | -20.53 | [44] |
| Outrata1994Ex31 | [45] | N-L-N-N | 1 | 2 | 2 | 4 | 3.21 | -20.53 | [45] |
| OutrataCervinka2009 | [46] | L-L-N-L | 2 | 2 | 1 | 3 | 0 | 0 | [46] |
| PauloviciusEtal2017a | [47] | N-L-N-L | 1 | 1 | 4 | 2 | 0.25 | 0 | [47] |
| PauloviciusEtal2017b | [47] | L-L-N-L | 1 | 1 | 4 | 2 | -2 | -1.5 | [47] |
| SahinCiric1998Ex2 | [48] | N-L-N-L | 1 | 1 | 2 | 3 | 5 | 4 | [48] |
| ShimizuAiyoshi1981Ex1 | [49] | N-L-N-L | 1 | 1 | 3 | 3 | 100 | 0 | [49] |
| ShimizuAiyoshi1981Ex2 | [49] | N-L-N-L | 2 | 2 | 3 | 4 | 225 | 100 | [49] |
| ShimizuEtal1997a | [50] | N-O-N-L | 1 | 1 | 0 | 3 | × | × | |
| ShimizuEtal1997b | [50] | N-L-N-L | 1 | 1 | 2 | 2 | 2250 | 197.75 | [50] |
| SinhaMaloDeb2014TP3 | [51] | N-N-N-N | 2 | 2 | 3 | 4 | -18.68 | -1.02 | [51] |
| SinhaMaloDeb2014TP6 | [51] | N-L-N-L | 1 | 2 | 1 | 6 | -1.21 | 7.62 | [51] |
| SinhaMaloDeb2014TP7 | [51] | N-N-N-L | 2 | 2 | 4 | 4 | -1.96 | 1.96 | [51] |
| SinhaMaloDeb2014TP8 | [51] | N-L-N-L | 2 | 2 | 5 | 6 | 0 | 100 | [51] |
| SinhaMaloDeb2014TP9 | [51] | N-O-N-L | 10 | 10 | 0 | 20 | 0 | 1 | [51] |
| SinhaMaloDeb2014TP10 | [51] | N-O-N-L | 10 | 10 | 0 | 20 | 0 | 1 | [51] |
| TuyEtal2007 | [52] | N-L-L-L | 1 | 1 | 2 | 3 | 22.5 | -1.52 | [52] |
| Vogel2002 | [53] | N-L-N-L | 1 | 1 | 2 | 1 | 1 | -2 | [53] |
| WanWangLv2011 | [54] | N-O-L-L | 2 | 3 | 0 | 8 | 10.63 | -0.5 | [54] |
| YeZhu2010Ex42 | [55] | N-L-N-L | 1 | 1 | 2 | 1 | 1 | -2 | [55] |
| YeZhu2010Ex43 | [55] | N-L-N-L | 1 | 1 | 2 | 1 | 1.25 | -2 | [55] |
| Yezza1996Ex31 | [56] | N-L-N-L | 1 | 1 | 2 | 2 | 1.5 | -2.5 | [56] |
| Yezza1996Ex41 | [56] | N-O-N-L | 1 | 2 | 0 | 2 | 0.5 | 2.5 | [56] |
| Zlobec2001a | [57] | N-O-L-L | 1 | 2 | 0 | 3 | -1 | -1 | [57] |
| Zlobec2001b | [57] | L-L-L-N | 1 | 1 | 2 | 4 | no | solution | [57] |
| DesignCentringP1 | [71] | N-N-N-N | 3 | 6 | 3 | 3 | × | × | |
| DesignCentringP2 | [71] | N-N-N-N | 4 | 6 | 5 | 3 | × | × | |
| DesignCentringP3 | [71] | N-N-N-N | 6 | 6 | 3 | 3 | × | × | |
| DesignCentringP4 | [71] | N-N-N-N | 4 | 6 | 3 | 12 | × | × | |
| NetworkDesignP1 | [11] | N-L-N-L | 5 | 5 | 5 | 6 | 300.5 | 419.8 | [8] |
| NetworkDesignP2 | [11] | N-L-N-L | 5 | 5 | 5 | 6 | 142.9 | 81.95 | [8] |
| OptimalControl | [37] | N-N-N-L | 2 | $n_y$ | 3 | $2n_y$ | × | × | |
| RobustPortfolioP1 | [71] | L-N-N-N | N+1 | N | N+3 | N+1 | 1.15 | 0 | [71] |
| RobustPortfolioP2 | [71] | L-N-N-N | N+1 | N | N+3 | N+1 | 1.15 | 0 | [71] |
| TollSettingP1 | [11] | N-L-N-L | 3 | 8 | 3 | 18 | -7 | 12 | [8] |
| TollSettingP2 | [11] | N-L-N-L | 3 | 18 | 3 | 38 | -4.5 | 32 | [8] |
| TollSettingP3 | [11] | N-L-N-L | 3 | 18 | 3 | 38 | - 3.5 | 32 | [8] |
| TollSettingP4 | [11] | N-O-N-L | 2 | 4 | 0 | 8 | -4 | 14 | [8] |
| TollSettingP5 | [11] | N-O-N-L | 1 | 4 | 0 | 8 | -2.5 | 14 | [8] |
| **Linear bilevel programs** | | | | | | | | | |
| AnandalinghamWhite1990 | [58] | L-L-L-L | 1 | 1 | 1 | 6 | -49 | 15 | [58] |
| Bard1984a | [75] | L-L-L-L | 1 | 1 | 1 | 5 | 28/9 | -60/9 | [75] |
| Bard1984b | [75] | L-L-L-L | 1 | 1 | 1 | 5 | -37.6 | 1.6 | [75] |
| Bard1991Ex2 | [5] | L-L-L-L | 1 | 2 | 1 | 5 | -1 | -1 | [5] |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| BardFalk1982Ex2 | [59] | L-L-L-L | 2 | 2 | 2 | 5 | -3.25 | -4 | [59] |
| Ben-AyedBlair1990a | [76] | L-L-L-L | 1 | 2 | 2 | 4 | -2.5 | -5 | [76] |
| Ben-AyedBlair1990b | [76] | L-L-L-L | 1 | 1 | 1 | 4 | -6 | 5 | [76] |
| BialasKarwan1984a | [77] | L-L-L-L | 1 | 2 | 1 | 7 | -2 | -0.5 | [77] |
| BialasKarwan1984b | [77] | L-L-L-L | 1 | 1 | 1 | 6 | -11 | 11 | [77] |
| CandlerTownsley1982 | [60] | L-L-L-L | 2 | 3 | 2 | 6 | -29.2 | 3.2 | [60] |
| ClarkWesterberg1988 | [61] | L-O-L-L | 1 | 1 | 0 | 3 | -37 | 14 | [61] |
| ClarkWesterberg1990b | [10] | L-L-L-L | 1 | 2 | 2 | 5 | -13 | -4 | [10] |
| GlackinEtal2009 | [62] | L-L-L-L | 2 | 1 | 3 | 3 | 6 | 0 | [62] |
| HaurieSavardWhite1990 | [78] | L-O-L-L | 1 | 1 | 0 | 4 | 27 | -3 | [78] |
| HuHuangZhang2009 | [63] | L-L-L-L | 1 | 2 | 1 | 5 | -76/9 | -41/9 | [63] |
| LanWenShihLee2007 | [64] | L-L-L-L | 1 | 1 | 1 | 7 | -85.09 | 50.17 | [64] |
| LiuHart1994 | [67] | L-L-L-L | 1 | 1 | 1 | 4 | -16 | 4 | [67] |
| MershaDempe2006Ex1 | [65] | L-L-L-L | 1 | 1 | 1 | 5 | $\times$ | $\times$ | [65] |
| MershaDempe2006Ex2 | [65] | L-L-L-L | 1 | 1 | 2 | 2 | -20 | -6 | [65] |
| TuyEtal1993 | [79] | L-L-L-L | 2 | 2 | 3 | 4 | -3.25 | -6 | [79] |
| TuyEtal1994 | [80] | L-L-L-L | 2 | 2 | 3 | 3 | 6 | 0 | [80] |
| TuyEtal2007Ex3 | [52] | L-L-L-L | 10 | 6 | 12 | 13 | -467.46 | -11.62 | [52] |
| VisweswaranEtal1996 | [68] | L-L-L-L | 1 | 1 | 1 | 5 | 28/9 | -60/9 | [68] |
| WangJiaoLi2005 | [66] | L-L-L-L | 1 | 2 | 2 | 2 | -1000 | -1 | [66] |
| **Simple bilevel programs** | | | | | | | | | |
| FrankeEtal2018Ex53 | [73] | N-L-N-L | 0 | 2 | 4 | 4 | 1 | 1 | [73] |
| FrankeEtal2018Ex511 | [73] | N-O-L-L | 0 | 3 | 0 | 4 | 3 | 0 | [73] |
| FrankeEtal2018Ex513 | [73] | L-O-L-N | 0 | 3 | 0 | 3 | -1 | 0 | [73] |
| FrankeEtal2018Ex521 | [73] | L-O-L-N | 0 | 2 | 0 | 3 | -1 | 0 | [73] |
| MitsosBarton2006Ex31 | [39] | L-L-L-L | 0 | 1 | 2 | 2 | 1 | -1 | [39] |
| MitsosBarton2006Ex32 | [39] | L-L-L-L | 0 | 1 | 3 | 2 | no | solution | [39] |
| MitsosBarton2006Ex33 | [39] | L-L-N-N | 0 | 1 | 2 | 3 | -1 | 1 | [39] |
| MitsosBarton2006Ex34 | [39] | L-L-N-L | 0 | 1 | 2 | 2 | 1 | -1 | [39] |
| MitsosBarton2006Ex35 | [39] | L-L-N-L | 0 | 1 | 2 | 2 | 0.5 | -1 | [39] |
| MitsosBarton2006Ex36 | [39] | L-L-N-L | 0 | 1 | 2 | 2 | -1 | -1 | [39] |
| ShehuEtal2019Ex42 | [72] | N-O-N-O | 0 | $n_y$ | 0 | 0 | $\times$ | $\times$ | |

It is worth pointing out that some examples involve equalities constraints in the upper or lower-level problems. As only 8% of the BOLIB problems have such constraints, we preserve the uniformity in the structure of the codes by converting equalities constraints into inequalities. For the sake of clarity, we list all the examples with equality constraints below.

Table 5: List of bilevel programs with equalities constraints.

| **Example name** | **Ref.** | $F$-$G$-$H$-$f$-$g$-$h$ | $n_x$ | $n_y$ | $n_G$ | $n_H$ | $n_g$ | $n_h$ |
|---|---|---|---|---|---|---|---|---|
| DempeDutta2012Ex31 | [14] | L-L-N-N-N-O | 2 | 2 | 2 | 1 | 2 | 0 |
| DempeFranke2011Ex41 | [16] | N-L-L-N-L-O | 2 | 2 | 2 | 1 | 4 | 0 |
| DempeFranke2011Ex42 | [16] | N-L-L-N-L-O | 2 | 2 | 2 | 1 | 3 | 0 |
| Zlobec2001b | [57] | L-L-O-L-L-N | 1 | 1 | 2 | 0 | 2 | 1 |
| NetworkDesignP1 | [11] | N-L-O-N-O-L | 5 | 5 | 5 | 0 | 0 | 3 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| NetworkDesignP2 | [11] | N-L-O-N-O-L | 5 | 5 | 5 | 0 | 0 | 3 |
| OptimalControl | [37] | N-N-O-N-L-L | 2 | $n_y$ | 3 | 0 | $n_y$ | $\frac{1}{2}n_y$ |
| RobustPortfolioP1 | [71] | L-N-L-N-N-O | N+1 | N | N+1 | 1 | N+1 | 0 |
| RobustPortfolioP2 | [71] | L-N-L-N-N-O | N+1 | N | N+1 | 1 | N+1 | 0 |
| TollSettingP1 | [11] | N-L-O-N-L-L | 3 | 8 | 3 | 0 | 8 | 5 |
| TollSettingP2 | [11] | N-L-O-N-L-L | 3 | 18 | 3 | 0 | 18 | 10 |
| TollSettingP3 | [11] | N-L-O-N-L-L | 3 | 18 | 3 | 0 | 18 | 10 |
| TollSettingP4 | [11] | N-O-O-N-L-L | 2 | 4 | 0 | 0 | 2 | 4 |
| TollSettingP5 | [11] | N-O-O-N-L-L | 1 | 4 | 0 | 0 | 2 | 4 |

## 3 How to access the library?

The library can be accessed through the dedicated website biopt.github.io/bolib. Under this link, you will find the zipped folder named BOLIBver2 containing all the relevant files for the version of the library presented in this paper. The folder contains the subfolder named `Examples`, which contains all the m-files with the codes of the examples, as described in the previous section. The pdf file named `Formulas` collects all the mathematical formulas of all the examples in this library. To start with the library, it is advised to consult the `readme` file for some further useful instructions on how to use it.

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