
DISCRETE ADAPTIVE CONTROL ALLOCATION

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ABSTRACT

The main purpose of a control allocator is to distribute a total control effort among redundant actuators. This paper proposes a discrete adaptive control allocator for over-actuated sampled-data systems in the presence of actuator uncertainty. The proposed method does not require uncertainty estimation or persistency of excitation. Furthermore, the presented algorithm employs a closed loop reference model, which provides fast convergence without introducing excessive oscillations. To generate the total control signal, an LQR controller with reference tracking is used to guarantee the outer loop asymptotic stability. The discretized version of the Aerodata Model in Research Environment (ADMIRE) is used as an over-actuated system, to demonstrate the efficacy of the proposed method.

Keywords First keyword · Second keyword · More

1 Introduction

Increasing the number of actuators in dynamical systems is one of the ways to improve maneuverability and fault tolerance [1, 2]. Thanks to the advances in microprocessors and progress in actuator miniaturization, which leads to actuator cost reduction, over-actuated systems are becoming ubiquitous in engineering applications. Aerial vehicles [3–9], marine vehicles [10, 11], and automobiles [12–15] can be counted as examples of systems where redundant actuators are employed.

Allocating control signals among redundant actuators can be achieved via several different control allocation methods which can be categorized into the following categories: Pseudo-inverse-based, optimization-based and dynamic control allocation. Pseudo-inverse-based methods [2, 16], which have the lowest computational complexity among the others, are implemented by manipulating the null space of the control input matrix. Optimization-based methods [17–21] are performed by minimizing a cost function that penalizes the difference between the desired and achieved total control inputs. Dynamic control allocation methods [1, 22, 23], on the other hand, are based on solving differential equations that model the control allocation goals. These methods can be extended to consider actuator limits [24, 25]. A survey of control allocation methods can be found in [26].

Actuator effectiveness uncertainty is an inevitable problem in several dynamical systems and it can be handled by control allocation methods in over-actuated systems. However, most of the control allocation methods that are proposed to solve this problem require uncertainty estimation and persistently exciting input signals [2, 27]. Adaptive control allocation [1, 28, 29], on the other hand, is able to manage redundancy and uncertainty of actuators without these requirements.

The majority of control algorithms are implemented using digital technology. Therefore, before their application, continuous-time controllers need an intermediate discretization step, which may lead to loss of stability margins [30, 31].

In this paper, a discrete adaptive control allocation method is introduced for sampled-data systems. The approach is inspired by the continuous control allocation algorithm that is recently proposed in [1]. It does not require uncertainty estimation or persistency of excitation assumption. Furthermore, the method is implemented using a closed loop

reference model [32], which proved to speed up the system response without causing excessive oscillations [33]. To the best of our knowledge, a discrete adaptive control allocator with these properties is not available in the prior literature.

This paper is organized as follows: Section 2 introduces the notations and definitions employed during the paper. Section 3 presents control allocation problem statement. The discrete adaptive control allocation is presented in Section 4. Controller design is presented in section 5. Section 6 illustrates the effectiveness of the proposed method in the simulation environment. Finally, Section 7 summarizes the paper.

2 NOTATIONS

In this section, we collect several definitions and basic results which are used in the following sections. Throughout this paper, $\lambda_{\min}(\cdot)$, $\lambda_{\max}(\cdot)$ and $\lambda_i(\cdot)$ refer to the minimum, maximum and the i^{th} eigenvalue of a matrix, respectively. I_r is the identity matrix of dimension $r \times r$, $0_{r \times m}$ is the zero matrix of dimension $r \times m$. $\text{tr}(\cdot)$ refers to the trace operation and $\text{diag}([\cdot])$ symbolizes a diagonal matrix with the elements of a vector $[\cdot]$. \mathbb{R} , \mathbb{R}^n and $\mathbb{R}^{n \times m}$ denote the set of real numbers, real column vectors with n elements, and $n \times m$ real matrices, respectively. \mathbb{Z}^+ denotes the set of non-negative integers. In the discrete time, the \mathcal{L}_2 and \mathcal{L}_∞ signal norms are defined as

$$\|x(k)\| = \|x(k)\|_2 = \sqrt{\sum_{k=0}^{\infty} (x_1^2(k) + \dots + x_n^2(k))}, \quad (1)$$

$$\|x(k)\|_\infty = \sup_{k \in \mathbb{Z}^+} \max_{1 \leq i \leq n} |x_i(k)|, \quad (2)$$

where x_1, \dots, x_n are the elements of x . $x(k) \in L_2$ if $\|x(k)\|_2 < \infty$. Also, $x(k) \in L_\infty$ if $\|x(k)\|_\infty < \infty$.

3 PROBLEM STATEMENT

Consider the following discretized plant dynamics

$$\begin{aligned} x(k+1) &= Ax(k) + B_u u(k) \\ &= Ax(k) + B_v B u(k), \end{aligned} \quad (3)$$

where $k \in \mathbb{Z}^+$ is the sampling instant, $x \in \mathbb{R}^n$ is the system states vector, $u \in \mathbb{R}^m$ is the control input vector, $A \in \mathbb{R}^{n \times n}$ is the known state matrix and $B_u \in \mathbb{R}^{n \times m}$ is the known control input matrix.

Remark 1 *It is noted that in the development of the control allocator, the matrix A being known or unknown is irrelevant. This matrix will play a role in the controller (not control allocator) design and since the contribution of the paper is a novel discrete time adaptive control allocator, the matrix A is taken to be known to facilitate the controller (not control allocator) development.*

Redundancy of actuators leads B_u in (3) to be rank deficient, that is, $\text{rank}(B_u) = r < m$. Therefore, B_u can be decomposed into the known matrices $B_v \in \mathbb{R}^{n \times r}$ and $B \in \mathbb{R}^{r \times m}$ with $\text{rank}(B_v) = \text{rank}(B) = r$. To model the actuator degradation, a diagonal matrix $\Lambda \in \mathbb{R}^{m \times m}$ with uncertain positive elements, belong to $(0, 1]$, is introduced to the system dynamics as

$$\begin{aligned} x(k+1) &= Ax(k) + B_v B \Lambda u(k) \\ &= Ax(k) + B_v v(k), \end{aligned} \quad (4)$$

where $v \in \mathbb{R}^r$ denotes the bounded control input produced by the controller. The boundedness of the control input can be guaranteed by using a soft saturation on the control signal v , before feeding it to the control allocator. An example of this can also be seen in [1].

The control allocation problem is to achieve

$$B \Lambda u(k) = v(k), \quad (5)$$

without using any matrix identification methods. Since Λ is unknown, conventional control allocation methods do not apply.

4 DISCRETE ADAPTIVE CONTROL ALLOCATION

Consider the following dynamics

$$\xi(k+1) = A_m \xi(k) + B \Lambda u(k) - v(k), \quad (6)$$

where $A_m \in \mathbb{R}^{r \times r}$ is stable matrix, that is, eigenvalues of A_m are inside the unit circle. A reference model dynamics is chosen as

$$\xi_m(k+1) = A_m \xi_m(k). \quad (7)$$

Defining the control input as a mapping from v to u ,

$$u(k) = \theta_v^T(k)v(k), \quad (8)$$

where $\theta_v \in \mathbb{R}^{r \times m}$ represents the adaptive parameter matrix to be determined, and substituting (8) into (6), it is obtained that

$$\xi(k+1) = A_m \xi(k) + (B\Lambda\theta_v^T(k) - I_r)v(k). \quad (9)$$

Defining $\theta_v(k) = \theta_v^* + \tilde{\theta}_v(k)$, where $\theta_v^* = ((B\Lambda)^T(B\Lambda\Lambda B^T)^{-1})^T$ is the ideal value of θ_v , which corresponds to the pseudo inverse of $B\Lambda$, and $\tilde{\theta}_v$ is the deviation of θ_v from its ideal value, equation (9) can be rewritten as

$$\xi(k+1) = A_m \xi(k) + B\Lambda\tilde{\theta}_v^T(k)v(k). \quad (10)$$

Defining the error $e(k) = \xi(k) - \xi_m(k)$, and using (7) and (10), the error dynamics is obtained as

$$e(k+1) = A_m e(k) + B\Lambda\tilde{\theta}_v^T(k)v(k). \quad (11)$$

Assumption 1 The design matrix A_m is chosen such that [34]:

- (i) $|\lambda_i(A_m)| \leq 1, i = 1, \dots, r$,
- (ii) All controllable modes of $(A_m, B\Lambda)$ are inside the unit circle,
- (iii) The eigenvalues of A_m on the unit circle have a Jordan block of size one.

Theorem 1 Consider the system $x(k+1) = \hat{A}x(k) + \hat{B}u(k)$, which satisfies Assumption 1. There exist positive constants m_1 and m_2 , independent of k and N , such that

$$\|x(k)\| \leq m_1 + m_2 \max_{0 \leq \tau \leq N} \|u(\tau)\|, \quad (12)$$

for all k , such that $0 \leq k \leq N$.

Proof 1 The proof can be found in [34]. ■

Theorem 2 Consider the error dynamics (11), which satisfies Assumption 1. If the update law

$$\theta_v(k+1) = \theta_v(k) + \Gamma v(k)\epsilon^T(k)B \quad (13)$$

is used, where $0 < \Gamma = \Gamma^T \in \mathbb{R}^{r \times r}$ is the adaptation rate matrix, and $\epsilon(k) \in \mathbb{R}^r$ is defined as

$$\epsilon(k) = \frac{v(k) - B\Lambda u(k)}{\sigma^2(k)}, \quad (14)$$

with $\sigma(k) \equiv \sqrt{1 + \lambda_{\max}(B\Lambda B^T)v^T(k)\Gamma v(k)}$, then the adaptive parameter $\theta_v(k)$, the error signal $e(k)$ and all signals remain bounded. Furthermore, $\lim_{k \rightarrow \infty} e(k) = 0$.

Proof 2 Consider the scalar positive definite function

$$V(k) = \text{tr} \left\{ \tilde{\theta}_v^T(k)\Gamma^{-1}\tilde{\theta}_v(k)\Lambda \right\}, \quad (15)$$

where $\Gamma = \Gamma^T > 0$. The time increment of (15) can be calculated as

$$V(k+1) - V(k) = \text{tr} \left\{ \tilde{\theta}_v^T(k+1)\Gamma^{-1}\tilde{\theta}_v(k+1)\Lambda \right\} - \text{tr} \left\{ \tilde{\theta}_v^T(k)\Gamma^{-1}\tilde{\theta}_v(k)\Lambda \right\}. \quad (16)$$

Using (13) and the fact that $\theta_v(k) = \theta_v^* + \tilde{\theta}_v(k)$, it is obtained that

$$\begin{aligned} \tilde{\theta}_v(k+1) &= \theta_v(k+1) - \theta_v^* \\ &= \theta_v(k) + \Gamma v(k)\epsilon^T(k)B - \theta_v^* \\ &= \tilde{\theta}_v(k) + \Gamma v(k)\epsilon^T(k)B. \end{aligned} \quad (17)$$

Substituting (17) in (16), and using the trace property, $\text{tr}\{A + B\} = \text{tr}\{A\} + \text{tr}\{B\}$ for two square matrices A and B , we have

$$\begin{aligned} V(k+1) - V(k) &= \text{tr} \left\{ \tilde{\theta}_v^T(k) \Gamma^{-1} \tilde{\theta}_v(k) \Lambda + \tilde{\theta}_v^T(k) v(k) \epsilon^T(k) B \Lambda + B^T \epsilon(k) v^T(k) \tilde{\theta}_v(k) \Lambda \right. \\ &\quad \left. + B^T \epsilon(k) v^T(k) \Gamma v(k) \epsilon^T(k) B \Lambda \right\} - \text{tr} \left\{ \tilde{\theta}_v^T(k) \Gamma^{-1} \tilde{\theta}_v(k) \Lambda \right\} \\ &= \text{tr} \left\{ 2B^T \epsilon(k) v^T(k) \tilde{\theta}_v(k) \Lambda + B^T \epsilon(k) v^T(k) \Gamma v(k) \epsilon^T(k) B \Lambda \right\}. \end{aligned} \quad (18)$$

Since $u(k) = \theta_v^T v(k)$ and $v(k) = B \Lambda \theta_v^{*T} v(k)$, (14) can be rewritten as

$$\epsilon(k) = \frac{-B \Lambda \tilde{\theta}_v^T(k) v(k)}{\sigma^2(k)}. \quad (19)$$

By substituting (19) in (18) and using the trace properties, $\text{tr}\{cA\} = c \times \text{tr}\{A\}$, for a square matrix A and a scalar c , and $a^T b = \text{tr}\{ba^T\}$, for two column vectors a and b , (18) can be rewritten as

$$\begin{aligned} V(k+1) - V(k) &= \text{tr} \left\{ \frac{-2B^T B \Lambda \tilde{\theta}_v^T(k) v(k) v^T(k) \tilde{\theta}_v(k) \Lambda}{\sigma^2(k)} + \frac{B^T B \Lambda \tilde{\theta}_v^T(k) v(k) v^T(k) \Gamma v(k) v^T(k) \tilde{\theta}_v(k) \Lambda B^T B \Lambda}{\sigma^4(k)} \right\} \\ &= \frac{1}{\sigma^2} \text{tr} \left\{ -2B^T B \Lambda \tilde{\theta}_v^T(k) v(k) v^T(k) \tilde{\theta}_v(k) \Lambda + \frac{B^T B \Lambda \tilde{\theta}_v^T(k) v(k) v^T(k) \Gamma v(k) v^T(k) \tilde{\theta}_v(k) \Lambda B^T B \Lambda}{\sigma^2(k)} \right\} \\ &= \frac{1}{\sigma^2(k)} \text{tr} \left\{ -2v^T(k) \tilde{\theta}_v(k) \Lambda B^T B \Lambda \tilde{\theta}_v^T(k) v(k) \right. \\ &\quad \left. + \frac{v^T(k) \Gamma v(k) v^T(k) \tilde{\theta}_v(k) \Lambda B^T B \Lambda B^T B \Lambda \tilde{\theta}_v^T(k) v(k)}{\sigma^2(k)} \right\}. \end{aligned} \quad (20)$$

Using the inequality $a^T A a \leq \lambda_{\max}(A) a^T a$ for a symmetric matrix A and a column vector a , an upper bound for (20) can be obtained as

$$\begin{aligned} V(k+1) - V(k) &\leq \frac{1}{\sigma^2(k)} \text{tr} \left\{ -2v^T(k) \tilde{\theta}_v(k) \Lambda B^T B \Lambda \tilde{\theta}_v^T(k) v(k) \right. \\ &\quad \left. + \lambda_{\max}(B \Lambda B^T) \frac{v^T(k) \Gamma v(k)}{\sigma^2(k)} v^T(k) \tilde{\theta}_v(k) \Lambda B^T B \Lambda \tilde{\theta}_v^T(k) v(k) \right\} \\ &= \frac{1}{\sigma^2(k)} \text{tr} \left\{ v^T(k) \tilde{\theta}_v(k) \Lambda B^T B \Lambda \tilde{\theta}_v^T(k) v(k) \times \left(-2 + \lambda_{\max}(B \Lambda B^T) \frac{v^T(k) \Gamma v(k)}{\sigma^2(k)} \right) \right\}. \end{aligned} \quad (21)$$

Considering the definition of $\sigma(k)$, which is given after (14), it can be obtained that $-2 \leq \left(-2 + \lambda_{\max}(B \Lambda B^T) \frac{v^T(k) \Gamma v(k)}{\sigma^2(k)} \right) < -1$. Therefore, an upper bound for (21) can be written as

$$V(k+1) - V(k) < \frac{-1}{\sigma^2(k)} v^T(k) \tilde{\theta}_v(k) \Lambda B^T B \Lambda \tilde{\theta}_v^T(k) v(k) \leq 0. \quad (22)$$

This shows that $V(k) \in \mathcal{L}_\infty$ and therefore $\tilde{\theta}_v(k) \in \mathcal{L}_\infty$, which implies that $\theta_v(k) \in \mathcal{L}_\infty$. In addition, since $V(k)$ is decreasing and positive definite, it has a limit as $k \rightarrow \infty$, that is, $\lim_{k \rightarrow \infty} V(k) = V_\infty$. Furthermore, using Theorem 1, the error dynamics (11), and the boundedness of $\tilde{\theta}_v$ and v , it can be shown that $e(t) \in \mathcal{L}_\infty$. Finally, since ξ_m in (7) is bounded, ξ is also bounded.

Summing both sides of (22) from $k = 0$ to ∞ , it is obtained that

$$\begin{aligned} & \sum_{k=0}^{\infty} \frac{1}{\sigma^2(k)} \left(v^T(k) \tilde{\theta}_v(k) \Lambda B^T B \Lambda \tilde{\theta}_v^T(k) v(k) \right) \leq V(0) - V_{\infty} \leq \infty, \\ & \Rightarrow \lambda_{\min}(\Lambda B^T B \Lambda) \sum_{k=0}^{\infty} \frac{1}{\sigma^2(k)} \left(v^T(k) \tilde{\theta}_v(k) \tilde{\theta}_v^T(k) v(k) \right) \leq V(0) - V_{\infty} \leq \infty, \\ & \Rightarrow \sum_{k=0}^{\infty} \frac{v^T(k) \tilde{\theta}_v(k) \tilde{\theta}_v^T(k) v(k)}{\sigma^2(k)} \leq \frac{V(0) - V_{\infty}}{\lambda_{\min}(\Lambda B^T B \Lambda)} \leq \infty. \end{aligned} \quad (23)$$

This leads to the conclusion that $\frac{\tilde{\theta}_v^T(k) v(k)}{\sigma(k)} \in \mathcal{L}_2$. Therefore [35],

$$\lim_{k \rightarrow \infty} \frac{\tilde{\theta}_v^T(k) v(k)}{\sigma(k)} = 0. \quad (24)$$

Since $v(k)$ is bounded, $\sigma(k)$ is also bounded. Furthermore, $\sigma(k) \geq 1$ by definition. Therefore, using (24) it is obtained that

$$\lim_{k \rightarrow \infty} \tilde{\theta}_v^T(k) v(k) = 0_{m \times 1}. \quad (25)$$

Using (25) and (19), it can be concluded that $\epsilon(k)$ converges to zero. Considering (13) and the convergence of $\epsilon(k)$ to zero, we deduce that $\theta_v(k)$ converges to a constant value as $k \rightarrow \infty$. Finally, (25) and the error dynamics (11) lead to the conclusion that $\lim_{k \rightarrow \infty} e(k) = 0$. ■

Remark 2 To realize (6), the signal $B \Lambda u(k)$ is required. In motion control applications, this signal corresponds to the net external forces and moments [36], which can be obtained via an inertial measurement unit (IMU). Examples of measuring/estimating this signal, without introducing delay or noise amplification, and employing it in real applications can be found in [37] and [38].

Remark 3 To calculate σ , whose definition is given after (14), $\lambda_{\max}(B \Lambda B^T)$ needs to be computed. Although Λ is an unknown matrix, the range of its elements is known, which is $(0, 1]$. Therefore, the maximum eigenvalue of the matrix multiplication $B \Lambda B^T$ can be calculated.

To obtain fast convergence without introducing excessive oscillations, the open loop reference model (7) is modified as a closed loop reference model [32] as follows

$$\xi_{m1}(k+1) = A_m \xi_{m1}(k) - (\xi(k) - \xi_{m1}(k)) \quad (26)$$

$$= A_m^c \xi_{m1}(k) - \xi(k), \quad (27)$$

where $A_m^c \equiv A_m + I_r$ and l is a scalar design parameter. Defining the error $e_1(k) = \xi(k) - \xi_{m1}(k)$, and using (26) and (10), the error dynamics is obtained as

$$\begin{aligned} e_1(k+1) &= A_m e_1(k) + B \Lambda \tilde{\theta}_v^T(k) v(k) - e_1(k) \\ &= \bar{A}_m e_1(k) + B \Lambda \tilde{\theta}_v^T(k) v(k), \end{aligned} \quad (28)$$

where $\bar{A}_m \equiv A_m - I_r$. It is noted that the design parameter needs to be chosen such that both A_m^c in (27) and \bar{A}_m in (28) satisfy Assumption 1.

Theorem 3 Consider the error dynamics (28), with the update law (13). The adaptive parameter $\theta_v(k)$, error signal $e_1(k)$ and all other signals remain bounded, and $\lim_{k \rightarrow \infty} e_1(k) = 0$.

Proof 3 The proof is similar to that of Theorem 2's and therefore omitted here for brevity. ■

5 CONTROLLER DESIGN

In this section, a discrete Linear Quadratic Regulator (LQR) is designed to generate the total control input for reference tracking.

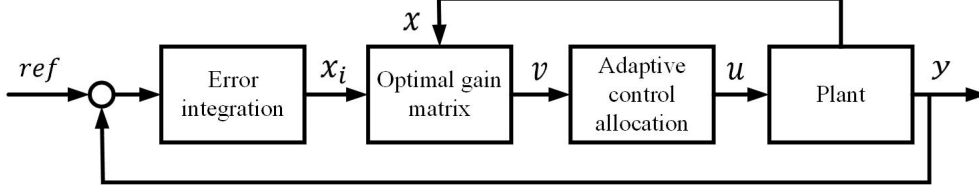


Figure 1: Block diagram of the closed loop system.

Consider the discrete dynamical system given in (4), together with the output vector $y \in \mathbb{R}^r$ as

$$\begin{aligned} x(k+1) &= Ax(k) + B_v v(k), \\ y(k) &= Cx(k), \end{aligned} \quad (29)$$

where $C \in \mathbb{R}^{r \times n}$ is the output matrix. In order to design a controller for reference tracking, we define a new state as

$$x_{new}(k+1) = x_{new}(k) + \Delta t(ref(k) - y(k)), \quad (30)$$

where $x_{new} \in \mathbb{R}^r$ is created by integrating the tracking error, $ref \in \mathbb{R}^r$ is the reference input and $\Delta t \in \mathbb{R}^+$ is the sampling interval. Augmenting (30) with (29), we have

$$\begin{bmatrix} x(k+1) \\ x_{new}(k+1) \end{bmatrix} = \begin{bmatrix} A & 0_{n \times r} \\ -\Delta t C & I_r \end{bmatrix} \begin{bmatrix} x(k) \\ x_{new}(k) \end{bmatrix} + \begin{bmatrix} B_v \\ 0_{r \times r} \end{bmatrix} v(k) + \begin{bmatrix} 0_{n \times r} \\ \Delta t I_r \end{bmatrix} ref(k). \quad (31)$$

Defining $z(k) \equiv [x^T(k) \ x_{new}^T(k)]^T \in \mathbb{R}^{n+r}$ as the state vector of the aggregate system, the cost function required to solve the LQR problem becomes

$$J = \sum_{k=0}^{\infty} (z^T(k) Q z(k) + v^T(k) R v(k)), \quad (32)$$

where $Q \in \mathbb{R}^{(n+r) \times (n+r)}$ is a positive semi-definite matrix and $R \in \mathbb{R}^{r \times r}$ is a positive definite matrix.

Using (32) and solving the standard LQR problem for discrete systems lead to the optimal gain matrix $K \in \mathbb{R}^{r \times (n+r)}$ and the control signal is obtained as $v(k) = -Kz(k)$. The structure of the controller is shown in Figure 1.

6 SIMULATION RESULTS

6.1 ADMIRE model

The ADMIRE model is an over-actuated aircraft model introduced in [39] and [36]. We use a version of this model that is linearized at Mach 0.22 and altitude 3km. Considering the actuator loss of effectiveness matrix Λ , and discretizing the continuous model using a 0.1s sampling interval, the discrete time dynamics is obtained as

$$\begin{aligned} x(k+1) &= Ax(k) + B_u u(k) \\ &= Ax(k) + B_v B u(k) \\ &= Ax(k) + B_v v(k), \end{aligned} \quad (33)$$

where $v = Bu$ is the control input, $x = [\alpha \ \beta \ p \ q \ r]^T$ is the state matrix with α, β, p, q and r denoting the angle of attack, sideslip angle, roll rate, pitch rate and yaw rate, respectively. The vector $u = [u_c \ u_{re} \ u_{le} \ u_r]^T$ represents the control surface deflections of canard wings, right and left elevons and the rudder. The state and control matrices are given as

$$A = \begin{bmatrix} 1.0214 & 0.0054 & 0.0003 & 0.4176 & -0.0013 \\ 0 & 0.6307 & 0.0821 & 0 & -0.3792 \\ 0 & -3.4485 & 0.3979 & 0 & 1.1569 \\ 1.1199 & 0.0024 & 0.0001 & 1.0374 & -0.0003 \\ 0 & 0.3802 & -0.0156 & 0 & 0.8062 \end{bmatrix}, \quad (34)$$

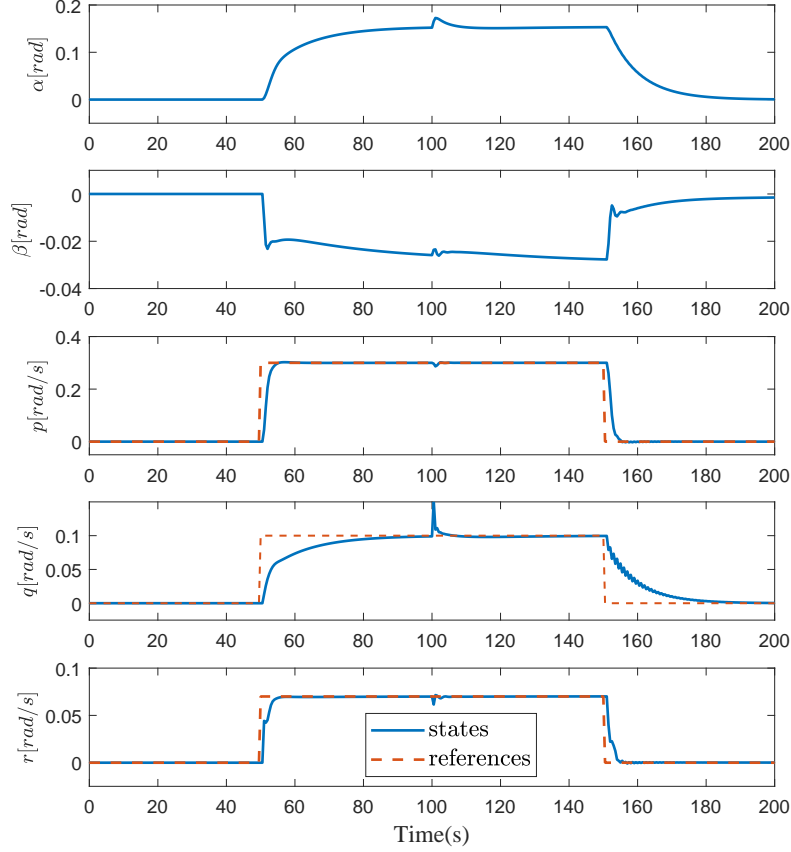


Figure 2: System states and reference tracking.

$$B_u = \begin{bmatrix} 0.1823 & -0.1798 & -0.1795 & 0.0008 \\ 0 & -0.0639 & 0.0639 & 0.1396 \\ 0 & -1.584 & 1.584 & 0.2937 \\ 0.8075 & -0.6456 & -0.6456 & 0.0013 \\ 0 & -0.1005 & 0.1005 & -0.4113 \end{bmatrix}. \quad (35)$$

The uncertainty which is considered as actuator loss of effectiveness occurs at $t = 100$ s and reduces the actuator effectivenesses by 30%. It is noted that for the design of the control allocator, the first two rows of the matrix B_u is taken to be zero, which makes the control surfaces pure moment generators [18]. However, the original B_u matrix given in (35) is used for the plant dynamics in the simulations.

6.2 Design parameters

The design parameters of the controller are the matrices R and Q , which are selected as $R = \text{diag}([1, 1, 0.1])$ and $Q = I_8$. Control allocation design parameters are Γ and A_m . These two matrices are chosen as $\Gamma = \text{diag}([1, 1, 0.1])$ and $A_m = \text{diag}([0.5, 0.5, 0.5])$. To improve the transient response, the design parameter for the closed loop reference model approach is chosen as $\tau = 0.1$.

6.3 Simulation results

Figure 2 illustrates the aircraft states together with the three references. It is seen that the first two states (α and β) remain bounded while the other three states (p , q and r) track their references. The effect of the actuator effectiveness uncertainty, which is introduced at $t = 100$ s, can also be observed in this figure. Figure 3 shows the time evolution

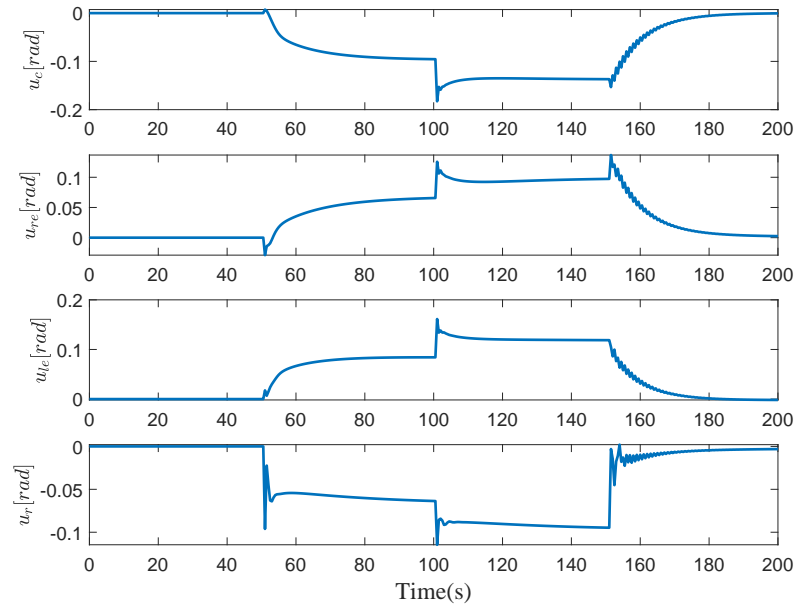


Figure 3: Control surface deflections.

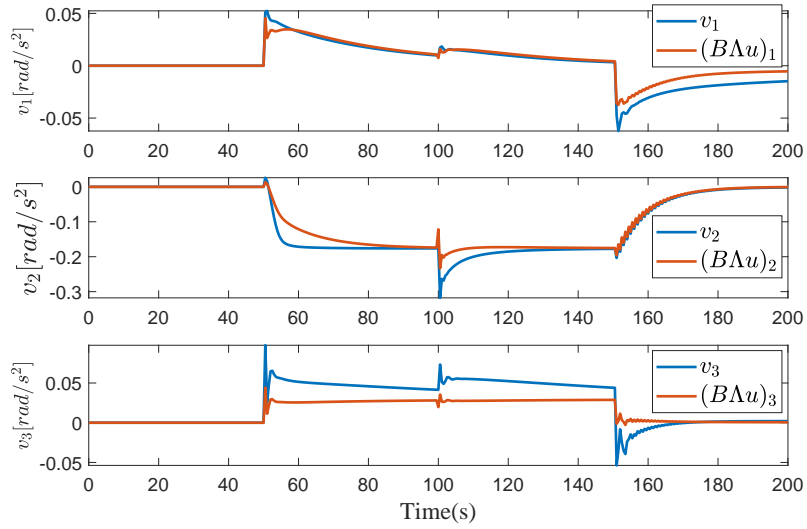


Figure 4: Control input signal, v , tracking.

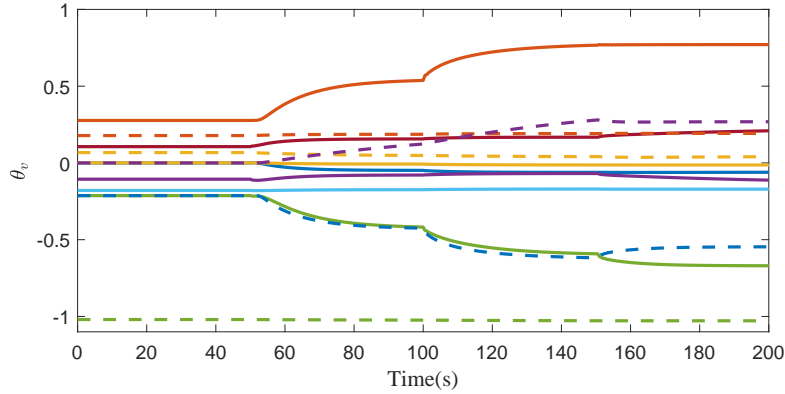


Figure 5: Adaptive parameters.

of control surfaces, where no excessive deflections are observed. It is seen in Figure 4 that the total control signals, $v_i, i = 1, 2, 3$, are realized by the control allocator. The figure shows that $B\Delta u$ is converging to v , which implies that the control allocation error is converging to zero. Adaptive parameters' time evolutions are demonstrated in Figure 5. The elements of θ_v remain bounded throughout the simulation and eventually converge to constant values.

7 Summary

A discrete adaptive control allocation is proposed in this paper. This method is able to distribute the total control signals of a sampled-data system among redundant actuators in the presence of actuator effectiveness uncertainty. The proposed control allocation method does not require uncertainty estimation or persistency of excitation. Simulation results demonstrate the effectiveness of the method.

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