

Quantum Scheduling for Millimeter-Wave Observation Satellite Constellation

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Abstract—In beyond 5G and 6G network scenarios, the use of satellites has been actively discussed for extending target monitoring areas, even for extreme circumstances, where the monitoring functionalities can be realized due to the usage of millimeter-wave wireless links. This paper designs an efficient scheduling algorithm which minimizes overlapping monitoring areas among observation satellite constellation. In order to achieve this objective, a quantum optimization based algorithm is used because the overlapping can be mathematically modelled via a max-weight independent set (MWIS) problem which is one of well-known NP-hard problems.

Index Terms—Satellite Constellation, Quantum Optimization, Scheduling, Maximum Weight Independent Set (MWIS)

I. INTRODUCTION

The use of satellite constellation is widely and actively used in next generation wireless network system design and implementation [1]–[4]. Especially, low earth orbit (LEO) satellites are getting a lot of attentions for various 6G applications such as target area observations [5] and flexible/robust network coverage extensions [2]. Both of them require high-capacity satellite communications.

In order to realize the high-capacity satellite communications, (mmWave) frequencies are used in order to take care of the huge traffic demands and the service continuity requirements of next-generation 6G applications [1], [6]–[10]. Thus, large-scale surveillance and target area observation can be realized [11]. In the observation satellite systems, having server duplicated/overlapped monitoring areas among satellites is not efficient even though millimeter-wave high-capacity communications can be realized. Thus, scheduling algorithms in order to minimize the overlapping monitoring areas have been actively studied and proposed, e.g., [12].

The modeling of the overlapping area scheduling for observation satellite constellation can be realized with maximum weight independent set (MWIS) formulation [11], [13], [14], which is one of the well-known NP-hard problems [13]. In order to approximately solve the NP-hard problems, many algorithms have been investigated. Among them, the use of message-passing algorithms is one of the well-known solutions [13], [14], whereas this paper proposes a new novel algorithm that utilizes quantum optimization and approximation methodologies [15], [16].

Based on the advances in quantum optimization methodologies, many algorithms have been investigated for approximating combinatorial problems (e.g., MWIS [17], max-flow/min-

cut [18] and graph cut segmentation [19]) and deep learning training/inference problems (e.g., Quantum Convolutional Neural Network (QCNN) [20], Quantum Random Access Memory (QRAM) [21], and Quantum Graph Recurrent Neural Network (QGRNN) [22]). In this paper, we design a quantum-based approximation algorithm for MWIS-based overlapping monitoring area scheduling in observation satellite constellation.

The rest of this paper is organized as follows. Sec. II presents the formulation of MWIS scheduling in satellite observation modeling. Sec. III and Sec. IV describe the preliminaries of QAOA and QAOA-based MWIS scheduling for observation satellite constellation. Sec. V concludes this paper and presents future research directions.

II. MAXIMUM WEIGHT INDEPENDENT SET (MWIS) FORMULATION FOR SATELLITE OBSERVATION SCHEDULING

We consider a network which consists of a set of observation areas [14]. According to the high data transmission rate in millimeter-wave wireless links among satellites, the transmission queue backlog in satellites can be filled in an instant, with the observation image data via synthetic aperture radar (SAR) [5]. For the scheduling of observation satellite constellation, a conflict graph is constructed with the set of nodes (physically, observation areas) and edges where two nodes are connected by an edge if the corresponding observation areas are overlapped more than threshold among adjacent observation satellites. The edges between node s_i (observation area in satellite i) and node s_j (observation area in satellite j) of the conflict graph, i.e., $E_{(i,j)}$, can be modelled as,

$$E_{(i,j)} = \begin{cases} 1, & \text{if } s_i \text{ is overlapped with } s_j \text{ where} \\ & s_i \in S, s_j \in S, \text{ and } i \neq j, \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

where S stands for the set of nodes (i.e., observation areas of satellites).

For scheduling problems, the main objective is to find the set of nodes (i.e., observation areas in satellite constellation) where two adjacent nodes those are connected via an edge cannot be simultaneously selected because it is not allowed to have huge overlapping monitoring areas among observation satellites. This situation is obviously equivalent to the case

which maximizes the summation of weights of all independent sets in a given conflict graph. Note that the weight is defined as the degree of overlapping or the number of observation data in satellite constellation. Thus, it is obvious that this scheduling problem can be modelled with the form of MWIS as [14],

$$\max : \sum_{\forall s_k \in S} w_k \cdot \mathcal{I}_k, \quad (2)$$

$$\text{s.t. } \mathcal{I}_i + \mathcal{I}_j + E_{(i,j)} \leq 2, \forall s_i \in S, \forall s_j \in S, \quad (3)$$

$$\mathcal{I}_i \in \{0, 1\}, \forall s_i \in S, \quad (4)$$

where w_k stands for the weight of satellite k (a positive integer), and

$$\mathcal{I}_i = \begin{cases} 1, & \text{if } s_i \text{ is scheduled where } s_i \in S, \\ 0, & \text{otherwise,} \end{cases} \quad (5)$$

where this formulation aims at the case where conflicting links are not scheduled simultaneously. $\mathcal{I}_i + \mathcal{I}_j \leq 2$ when $E_{(i,j)} = 0$ (no edge between s_i and s_j), i.e., both of \mathcal{I}_i and \mathcal{I}_j can be 1. On the other hand, $\mathcal{I}_i + \mathcal{I}_j \leq 1$ when $E_{(i,j)} = 1$, i.e., both of \mathcal{I}_i and \mathcal{I}_j can not be 1. Thus, one of them will be selected or both of them will not be selected.

III. PRELIMINARIES OF QUANTUM OPTIMIZATION

This section presents the preliminaries of quantum optimization, i.e., bra-ket notation (refer to Sec. III-A), quantum gates (refer to Sec. III-B), and quantum approximate optimization algorithm (QAOA) (refer to Sec. III-C).

A. Bra-Ket Notation

In quantum computing research, a bra-ket notation is widely and generally used for mathematically presenting quantum states or qubit states [17]. The *ket* and *bra* in this bra-ket notation can be represented as column vectors and row vectors. As a result, single qubit states (i.e., $|0\rangle$ and $|1\rangle$), can be mathematically presented as,

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (6)$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad (7)$$

and therefore,

$$|0\rangle = \langle 0|^\dagger = [1 \ 0]^\dagger, \quad (8)$$

$$|1\rangle = \langle 1|^\dagger = [0 \ 1]^\dagger. \quad (9)$$

where \dagger stands for Hermitian transpose. Therefore, the superposition state of a single qubit state can be presented as,

$$c_1 |0\rangle + c_2 |1\rangle = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}, \quad (10)$$

where c_1 and c_2 are probability amplitudes, and note that the c_1 and c_2 are complex numbers [23].

B. Quantum Gates

This section presents quantum gates or operators which mathematically represent single-qubit or 2-qubit operations [23]. First, Hadamard gate H , Pauli- X gate X , Pauli- Y gate Y , and Pauli- Z gate Z can be formulated as,

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad (11)$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad (12)$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \text{ and} \quad (13)$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad (14)$$

Based on this, the rotation- X gate $R_X(\theta)$, the rotation- Y gate $R_Y(\theta)$, and the rotation- Z gate $R_Z(\theta)$ are as,

$$R_X(\theta) = \begin{bmatrix} \cos \frac{\theta}{2} & -i \sin \frac{\theta}{2} \\ -i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{bmatrix}, \quad (15)$$

$$R_Y(\theta) = \begin{bmatrix} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \\ \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{bmatrix}, \text{ and} \quad (16)$$

$$R_Z(\theta) = \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}, \quad (17)$$

where θ is an angular value.

C. Quantum Approximate Optimization Algorithm (QAOA)

QAOA is one of the widely known noisy intermediate-scale quantum (NISQ) optimization algorithms for approximating combinatorial problems [16], [24], [25]. This QAOA is used for formulating H_P (i.e., problem Hamiltonian) and H_M (i.e., mixing Hamiltonian) based on the objective function $f(y)$. Then, the QAOA generates the parameterized states $|\gamma, \beta\rangle$ by alternately and iteratively applying the H_P and H_M on initial state $|s\rangle$. Here, $f(y)$, $H_P|y\rangle$, H_M , and $|\gamma, \beta\rangle$ are defined as (18), (19), (20), and (21), where $n \in \mathbb{Z}^+$, $p \in \mathbb{Z}^+$, and X_k is the Pauli- X operator applying on the k -th qubit; γ and β are the hyper-parameters those can be computed via approximation. Here, H_P encodes $f(y)$ in (19), operating diagonally in n -qubit quantum basis states [26]. In the computation procedure of QAOA, via the iterative measurement of $|\gamma, \beta\rangle$, the expectation of H_P should be obtained. Finally, the samples of $f(y)$ can be obtained as [16],

$$\langle f(y) \rangle_{\gamma, \beta} = \langle \gamma, \beta | H_P | \gamma, \beta \rangle. \quad (22)$$

The near-optimal or optimal approximation values of the hyper-parameters γ and β are obtained using conventional optimization, e.g., stochastic gradient descent [27], [28]. Thus, the solution can be computed from (22) via the obtained hyper-parameters γ and β . Finally, it can be shown that the QAOA-based approximation is one of widely known hybrid quantum-classical optimization algorithms where the efficient Hamiltonian design (for quantum approach) and the approximation of efficient hyper-parameters (for conventional optimization approach) are correlated [29], [30].

$$f(y) \triangleq f(y_1, y_2, \dots, y_n), \quad (18)$$

$$H_P |y\rangle \triangleq f(y) |y\rangle, \quad (19)$$

$$H_M \triangleq \sum_{k=1}^n X_k, \quad (20)$$

$$|\gamma, \beta\rangle \triangleq e^{-i\beta_p H_M} e^{-i\gamma_p H_P} \dots e^{-i\beta_2 H_M} e^{-i\gamma_2 H_P} e^{-i\beta_1 H_M} e^{-i\gamma_1 H_P} |s\rangle, \quad (21)$$

IV. QUANTUM SCHEDULING FOR MWIS-BASED FORMATION IN SATELLITE CONSTELLATION

This section consists of the design of Hamiltonian, i.e., Problem Hamiltonian, i.e., H_P (refer to Sec. IV-A) and Mixing Hamiltonian, i.e., H_M (refer to Sec. IV-B). Lastly, QAOA iterative computation procedure is described in Sec. IV-C.

A. Problem Hamiltonian, H_P

The problem Hamiltonian H_P is designed via the linear combination of the objective Hamiltonian H_O and the constraint Hamiltonian H_C . The objective function and constraints in the mathematical problem for solving the considering MWIS-based scheduling problem are in H_O (refer to Sec. IV-A1) and H_C (refer to Sec. IV-A2).

1) *Hamiltonian for MWIS Objective, H_O* : Suppose that a basic Boolean function $B_1(x) = x$ exists where $x \in \{0, 1\}$. According to the quantum Fourier expansion of this $B_1(x) = x$, it can be mapped to Boolean Hamiltonian H_{B_1} where I and Z are identity operator and Pauli- Z operator [31], i.e.,

$$H_{B_1} = \frac{1}{2}(I - Z), \quad (23)$$

therefore, the objective function (2) can be mapped into the following Hamiltonian,

$$H_{O'} = \sum_{\forall s_k \in S} \frac{1}{2} w_k (I - Z_k), \quad (24)$$

where Z_k is the Pauli- Z operator applied to \mathcal{I}_k . Because the objective function (2) is mapped to $H_{O'}$, it should be maximized via the main objective of MWIS. Thus, it is obvious that this $H_{O'}$ should be maximized as well. Therefore, the objective Hamiltonian H_O should be minimized as,

$$H_O = \sum_{\forall s_k \in S} \frac{1}{2} w_k Z_k. \quad (25)$$

2) *Hamiltonian for MWIS Constraints*: In the MWIS-based scheduling problem, we should avoid the case where both adjacent nodes of the conflict graph are selected. The scheduled and unscheduled nodes have states are denoted as $|1\rangle$ and $|0\rangle$. Here, N_i and N_j are defined as the arbitrary nodes, and $E_A(N_i, N_j)$, $E_B(N_i, N_j)$, and $E_C(N_i, N_j)$ stands for the edge notations for three cases where,

- $E_A(N_i, N_j)$ for *Case A*: s_i and s_j are not scheduled,
- $E_B(N_i, N_j)$ for *Case B*: One of s_i and s_j is scheduled,
- $E_C(N_i, N_j)$ for *Case C*: Both of s_i and s_j are scheduled (impossible situation).

Suppose that the weights of N_i and N_j in *Case C* are defined as W_{N_i} and W_{N_j} . Under this definition, the constraint function $C'(i, j)$, which counts the impossible situations, can be represented as,

$$C'(i, j) = \sum_{i=1}^n \sum_{j=1}^n (W_{N_i} + W_{N_j}) |E_C(N_i, N_j)| \quad (26)$$

where $i > j$; and n and $|E_C(N_i, N_j)|$ are the number of nodes and the number of $E_C(N_i, N_j)$ where $i > j$. This is a primary condition for avoiding impossible situations.

Based on the mathematical program of MWIS problem formulation (i.e., (1)–(5)), $C'(i, j)$ can be re-formed as $C(i, j)$ as,

$$\begin{aligned} C(i, j) &= \sum_{\forall s_i \in S} \sum_{\forall s_j \in S} (w_i + w_j) E_{(i, j)} \\ &= \sum_{\forall s_i \in S} \sum_{\forall s_j \in S} (w_i + w_j) (\mathcal{I}_i \wedge \mathcal{I}_j), \end{aligned} \quad (27)$$

where $i > j$; and \wedge stands for an AND gate. In (27), $C(i, j)$ should be 0 because it stands for the number of impossible situations. If making this $C(i, j)$ be not possible, this $C(i, j)$ should be minimized as much as possible. According to the quantum Fourier expansion of AND gate $B_2(x_1, x_2)$, it can be mapped to the following Boolean Hamiltonian H_{B_2} where the Z_1 and Z_2 in this equation are the Pauli- Z operators applying on x_1 and x_2 , respectively [31],

$$B_2(x_1, x_2) = x_1 \wedge x_2 \text{ where } x_1 \in \{0, 1\} \text{ and } x_2 \in \{0, 1\}, \quad (28)$$

$$H_{B_2} = \frac{1}{4}(I - Z_1 - Z_2 + Z_1 Z_2). \quad (29)$$

Based on this result, the constraints (27) can be mapped into following Hamiltonian,

$$H_{C'} = \sum_{\forall s_i \in S} \sum_{\forall s_j \in S} \frac{1}{4} (w_i + w_j) (I - Z_i - Z_j + Z_i Z_j), \quad (30)$$

where $i > j$; and Z_i and Z_j are the Pauli- Z operators applied to \mathcal{I}_i and \mathcal{I}_j , respectively. Because $C(i, j)$ should be zero (or it should be minimized, as explained before), the $H_{C'}$ which is mapped from $C(i, j)$ should be minimized, as well. Thus, the constraint Hamiltonian H_C is as,

$$H_C = \sum_{\forall s_i \in S} \sum_{\forall s_j \in S} -\frac{1}{4} (w_i + w_j) (Z_i + Z_j - Z_i Z_j), \quad (31)$$

where $i > j$.

Based on the obtained H_O and H_C in (25) and (31), the problem Hamiltonian H_P is defined as,

$$\boxed{H_P = H_O + \rho H_C}, \quad (32)$$

where ρ is a hyper-parameter that represents the penalty rate which means the ratio at which H_C (constraints) affects H_P compared to H_O (objective) ($\rho \geq 1$).

B. Mixing Hamiltonian, H_M

The mixing Hamiltonian H_M produces various cases which can appear in the given MWIS-formulated combinatorial problem. Our considering MWIS-based observation scheduling problem can be formulated by a binary bit string which presents a set of nodes. Therefore, various cases can be created by flipping the state of each node, mathematically modelled as $|0\rangle$ or $|1\rangle$. The bit-flip can be handled by the Pauli- X operator. Therefore, H_M can be formed as,

$$\boxed{H_M = \sum_{\forall s_k \in \mathcal{S}} X_k}. \quad (33)$$

C. QAOA Iterative Computation

The application of the designed Hamiltonian to the QAOA iterative optimization computation sequence starts when the design of Hamiltonian functions, i.e., H_P and H_M in (32) and (33), are completed. Then the iterative optimization computation procedure is as follows.

- First of all, the parameterized state $|\gamma, \beta\rangle$ can be generated by applying H_P and H_M to (21), as defined in (25), (31), (32), and (33). Note that the initial state $|s\rangle$ is set to the equivalent superposition state using the Hadamard gates in (11).
- The expectation of H_P can be measured on the generated parameterized state $|\gamma, \beta\rangle$. Here, The parameters γ and β are iteratively updated with traditional optimization computation procedure.
- When the QAOA iterative computation sequence terminates, the optimal (or approximated) parameters γ_{OPT} and β_{OPT} are finally obtained.

Therefore, the MWIS-based monitoring area scheduling solution in observation satellite constellation can be obtained by the measurement of the expectation of H_P on the optimal state $|\gamma_{\text{OPT}}, \beta_{\text{OPT}}\rangle$ as,

$$\boxed{\langle F \rangle = \langle \gamma_{\text{OPT}}, \beta_{\text{OPT}} | H_P | \gamma_{\text{OPT}}, \beta_{\text{OPT}} \rangle}, \quad (34)$$

where $\langle F \rangle$ is the expectation of the MWIS objective function (2) for the obtained solution samples.

V. CONCLUDING REMARKS AND FUTURE WORK

In beyond 5G and 6G communication networks, satellites has been actively used in many applications such as seamless monitoring target areas, even for extreme circumstances, thanks to the use of high-capacity millimeter-wave wireless links in satellites. This paper proposes a scheduling algorithm which aims at the minimization of overlapping monitoring areas among observation satellite constellation. To achieve

this goal, a quantum optimization based algorithm is used because the our considering overlapping formulation can be mathematically modelled via a max-weight independent set (MWIS) problem.

As a future research direction, the proposed algorithm can be evaluated with various realistic satellite scenarios and TensorFlow-Quantum based software implementation [32].

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