

The interior of hairy black holes in standard model physics

Theo M. Nieuwenhuizen

*Institute for Theoretical Physics, University of Amsterdam
PO Box 94485, 1090 GL Amsterdam, The Netherlands*

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A large class of stationary, non-rotating black hole metrics is proposed, in which the interior is regular with a core consisting of a condensate of Higgs and Z bosons generated from the nuclear binding energy of the initial H atoms. Gravitational collapse is prevented by negative pressures from the Higgs condensate and a small imbalance in the distribution of electric charges. Non-condensed, thermal particles are present as well. The approach holds for masses exceeding $0.75 \cdot 10^{-4} M_\odot$. The inner horizon sets an inner core of 11 cm, while the characteristic radius of the full core is $270 (M/M_\odot)^{1/3}$ cm. For increasing charge, the core expands; in the extremal case, it fills the interior. While the net charge is easily shielded, the build up of horizons may prevent this in the interior, and consequently avoid a singularity. In black hole merging, the core of a nearly extremal one may be exposed, forming a new class of events. The approach is a stepping stone towards rotating black holes.

I. INTRODUCTION

A black hole (BH) is said to have an event horizon but no hair[1], being described only by its mass, charge and angular momentum. Nothing can escape from it, except for Hawking radiation[2], a feeble quantum effect. The 2020 Nobel prize was awarded to Roger Penrose “for the discovery that black hole formation is a robust prediction of the general theory of relativity”, and to Reinhard Genzel and Andrea Ghez “for the discovery of a supermassive compact object at the centre of our galaxy”.

A BH is a trapped surface, often described by a metric. The Schwarzschild metric[3] involves only the mass; the Reissner–Nordström (RN) metric[4–7] adds a central charge, and the Kerr–Newman (KN) metric[8] models its rotation. When charge free, the rotating BH is described by the Kerr metric[9], widely employed in astrophysics. These metrics are source free, that is, the matter density vanishes everywhere, except at the singularity. The Schwarzschild and Reissner–Nordström metrics are singular at the origin. It is widely believed that the singularity must be described by quantum gravity[10, 11].

Regularized metrics have been proposed, see, e.g., [12–17], and the reviews [18, 19]. They were presented in an ad hoc manner, but the Bardeen black hole (and its generalizations) can be built self-consistently by considering a coupling to non-linear electrodynamics [20].

While these attempts involve hypothetical forms of matter, we propose, as next step forward, a large class of regular BH metrics involving the standard model (SM) of particle physics. The SM is our best theory of Nature; it has been vindicated for dozens of reaction channels at the LHC[21]. Employing the SM for BH interiors would vindicate it the more and save us from having to open “Pandora’s box” of paradoxes related with the BH singularity[22]. Last but not least, it would even do away with the need for quantum gravity in astrophysical BHs.

In the present work we consider models for charged, non-rotating BHs. In section II we lay out the basic approach, while we treat the full approach in section III. In section IV we give a summary and in section V we

discuss the setup and speculate on possible merits.

II. THE BASIC APPROACH

The Schwarzschild metric describes a point mass in GR (general relativity). With $S(r) = 2GM/r$, it reads[34]

$$g_{\mu\nu} = \text{diag}(1 - S, \frac{1}{S - 1}, -r^2, -r^2 s_\theta^2), \quad s_\theta = \sin \theta, \quad (1)$$

where $r^\mu = (t, r, \theta, \phi)$ with $\mu = 0, 1, 2, 3$ denote areal-like coordinates. Signals get an infinite redshift at the event horizon $r = R_S = 2GM$, where $g_{00} \rightarrow 0$; this astonished Einstein so much, that he initially dismissed the case.

The RN BH with charge Q is covered by $M \rightarrow \mu(r) = M - Q^2/2r$. The Kretschmann curvature invariant,

$$\mathcal{K} = R^{\kappa\lambda\mu\nu} R_{\kappa\lambda\mu\nu} = (4G^2/r^6) \times (12\mu^2 - 16r\mu\mu' + r^2\mu\mu'' + 8r^3\mu'\mu'' + r^4\mu''^2), \quad (2)$$

diverges for $\mu(r) = M$ as $1/r^6$ for $r \rightarrow 0$, and in the RN case even as $1/r^8$, which points at a physical singularity at the origin. This has led to the opinion that the BH singularity can only be described by quantum gravity.

Since \mathcal{K} is regular at R_S , no physical singularity occurs there. Indeed, an observer freely falling into the BH is described by the Painlevé Gullstrand (PG) metric[23, 24], which is regular for $r > 0$, viz. $g_{\mu\nu} = \gamma_{\mu\nu} - S k_\mu k_\nu$, where $\gamma_{\mu\nu} = \text{diag}(1, -1, -r^2, -r^2 s_\theta^2)$ is the Minkowski metric and $k_\mu = (1, 1, 0, 0)$ a null vector, viz. $g^{\mu\nu} k_\mu k_\nu = 0$.

To study the BH interior, we consider a general shape of $\mu(r)$. The Einstein equations then do not deal with a vacuum situation but demand an energy momentum tensor (EMT)

$$T^\mu_\nu = \mathcal{A} \delta^\mu_\nu + \mathcal{B} C^\mu_\nu, \quad C^\mu_\nu = \text{diag}(1, 1, -1, -1), \\ \mathcal{A} = \frac{2\mu' + r\mu''}{16\pi r^2}, \quad \mathcal{B} = \frac{2\mu' - r\mu''}{16\pi r^2}. \quad (3)$$

This result holds for both metrics, since they differ by their time variable while both are stationary.

A. The mass in the field theoretic approach

The *mass* of the metric is often derived by invoking the far field only. In the field theoretic description of gravitation, a bulk description is achieved. Hereto one assumes an underlying Minkowski space with metric $\gamma_{\mu\nu}$ and one introduces $k^{\mu\nu} = \sqrt{g/\gamma} g^{\mu\nu}$ with $g = \det g_{\mu\nu}$ and $\gamma = \det \gamma_{\mu\nu}$. The Einstein equations can then be formulated in the Minkowski space [25, 26]

$$A^{\mu\nu} = 8\pi G\Theta^{\mu\nu}, \quad A^{\mu\nu} = \frac{1}{2}(k^{\mu\nu}k^{\alpha\beta} - k^{\mu\alpha}k^{\nu\beta})_{;\alpha\beta}, \quad (4)$$

where the ‘‘acceleration tensor’’ [27] $A^{\mu\nu}$ involves covariant derivatives in Minkowski space, indicated by the column. The total energy momentum tensor is $\Theta^{\mu\nu} = (g/\gamma)(T^{\mu\nu} + t^{\mu\nu})$, with $t^{\mu\nu}$ representing the EMT of the gravitational field; in Cartesian coordinates it is the Landau-Lifshitz pseudo tensor. In our case: $k^{\mu\nu} = g^{\mu\nu}$, since $g = \gamma = -r^4 s_\theta^2$. The mass (energy) of the metric is

$$E = \int_{R^3} d^3r \sqrt{-\gamma} \Theta^{00}, \quad d^3r = dr d\theta d\phi. \quad (5)$$

For the PG metric this results in $E = \mu(r)|_0^\infty$. The general approach to get $E = M$ is to take the value at $r \rightarrow \infty$ and neglect the value at $r = 0$. For the metric (1) the equivalent is $E = \mu(r)/(1 - 2G\mu(r)/r)|_0^\infty$, which, instead, exposes a peculiarity at R_S . So a fundamental approach has to deal with the interior, in particular with $r \rightarrow 0$.

B. Employing the standard model

We shall investigate whether the standard model *can* describe the BH interior. Clearly, \mathcal{K} remains finite when $\mu \sim r^3$ for $r \rightarrow 0$, in which case $E = M$, as desired. In the dynamical BH formation process, this is the worst case before a singularity develops. From (3) it follows that $\int_{R^3} d^3r T_0^0 = \int_0^{R_S} dr 4\pi r^2 (\mathcal{A} + \mathcal{B})$ indeed equals $\mu(R_S) = M$, since $\mu(0) = 0$ grace to the regularization.

As in the RN case, the condition $S = 1$ implies an event horizon (at R_S) and an inner horizon at some $R_i > 0$.

The \mathcal{B} term in (3) may arise from an electric charge distribution. Consider a potential $A_\mu = A_0(r)\delta_\mu^0$. It derives from a charge density [35]

$$J_\gamma^\mu = J_\gamma^0 \delta_\mu^0, \quad J_\gamma^0 = -\frac{1}{\mu_0} (A_0'' + \frac{2}{r} A_0'). \quad (6)$$

It has charge $Q = -R_S^2 A_0'(R_S)$ and Coulombian EMT,

$$T_{\gamma\nu}^\mu = \frac{A_0'^2}{8\pi} C_{\nu}^\mu, \quad C_{\nu}^\mu = \text{diag}(1, 1, -1, -1). \quad (7)$$

For the RN metric: $A_0 = Q/r$, $\mathcal{A} = 0$ and $\mathcal{B} = Q^2/8\pi r^4$. In general, matching with (3) imposes

$$-A_0' = \sqrt{\frac{2\mu' - r\mu''}{2r^2}} = \frac{m_P}{2r} \sqrt{2S - r^2 S''}, \quad S \equiv \frac{2G\mu}{r}, \quad (8)$$

with $m_P = 1/\sqrt{G}$ the Planck mass. The small- r refinement $\mu \sim r^3 - r^5$ implies regularity, $A_0' \sim -r$ and $\mathcal{B} \sim r^2$. The charge is zero when $r\mu'' = 2\mu'$ ($= 0$ typically) at R_S .

Also the \mathcal{A} term in (3) may result from SM physics. Inside the BH, the matter, mostly H-atoms, is dissolved into electrons and quarks, with the nuclear binding energy converted, in principle, into Higgs, Z and W bosons, photons and gluons. To start, we assume that the interior involves a condensate of the Higgs field[35]. At the macroscopic scale, *the kinetic terms of the massive fields can be neglected*, as they bring a factor $1/R_S^2 = m_P^4/M^2$ rather than $M_{H,Z,W}^2 \sim v^2$. In the absence of W and Z fields, the EW Lagrangians reads, in the unitary gauge,

$$\mathcal{L}_H = -\frac{\lambda}{4}(\varphi^2 - v^2)^2 - \frac{\varphi}{v}\rho_F, \quad \mathcal{L}_\gamma = \frac{F^{\mu\nu}F_{\nu\mu}}{4\mu_0}, \quad (9)$$

with $v = 246$ GeV the vacuum expectation value of $\varphi = v + h$, with h the Higgs field, and $\lambda = 0.129$ the self-coupling. Next, ρ_F is the rest mass density of the fermions (electrons, and up and down quarks), with their Yukawa coupling to φ made explicit. Neglecting the variation of J_γ^μ with the metric, the EMTs have elements [14]

$$T_{H\nu}^\mu = -\delta_\nu^\mu \mathcal{L}_H, \quad T_{\gamma\nu}^\mu = \frac{1}{\mu_0} F^{\mu\rho} F_{\rho\nu} - \delta_\nu^\mu \mathcal{L}_\gamma. \quad (10)$$

It yields $\mathcal{A}_H = \frac{1}{4} \text{tr} T_H = \frac{\lambda}{4}(\varphi^2 - v^2)^2 + \frac{\varphi}{v}\rho_F$. Upon matching it with \mathcal{A} , the Higgs field gets pushed away from its vacuum value $h = 0$. Since its derivatives are negligible, φ is subject to the equation of motion

$$\lambda(\varphi^2 - v^2)\varphi = -\frac{\rho_F}{v}, \quad (11)$$

the equivalent of the Gross-Pitaevskii equation[28] for the condensate wavefunction in Bose-Einstein condensation.

C. A specific example

The positive energy conditions $\mathcal{A}, \mathcal{B} \geq 0$, with the conditions at 0 and ∞ , constrain the unspecified function μ . Nevertheless, realistic cases exist; an example is

$$\mu = \frac{M_1 r^3}{(\sqrt{r_s^2 + r^2} + r_u)^3}, \quad M_1 = M \frac{(\sqrt{r_s^2 + R_S^2} + r_u)^3}{R_S^3}, \quad (12)$$

where $r_u = ur_s$ with u a c -number. This yields

$$\mathcal{A} = \frac{3M_1 x^5}{16\pi r_s^3 u^7} \frac{4u^4 - u^2 + 7u^2 x + 5x^2 + x^3}{(x+1)^5}, \quad (13)$$

$$\mathcal{B} = \frac{3M_1 x^6}{16\pi r_s^3 u^7} \frac{r^2 4u^2 + 5x + x^2}{(x+1)^5}, \quad x = \frac{r_u}{\sqrt{r_s^2 + r^2}}.$$

While $\mathcal{A}(0)$ and $\mathcal{B}(r)$ are positive, the positivity of $\mathcal{A}(R_S)$ is secure when $u \geq \frac{1}{2}$. The Kretschmannian,

$$\mathcal{K} = \frac{24G^2 M_1^2 x^6}{r_s^6 u^{14} (x+1)^{10}} \left[4u^8(2x^4 + 7x^2 - 2x + 1) + 4u^6 x^2(7x^3 - 11x^2 + 23x - 7) + 6u^2 x^6(3x^2 + 10x - 25) + u^4 x^4(4x^3 + 43x^2 - 142x + 107) + 3x^8(x+5)^2 \right], \quad (14)$$

is finite and maximal at $r = 0$. The EM potential reads

$$A_0 = \text{const.} - \frac{4}{3} \sqrt{\frac{M_1}{r_s u^3}} \left[\frac{\sqrt{6(4u^2 + 5x + x^2)}}{4(1+x)^{3/2}} \right. \\ \left. + \frac{E\left(i\sqrt{\frac{3(1+\bar{u})}{2(1+x)}}; \sqrt{\frac{1-\bar{u}}{1+\bar{u}}}\right) - \bar{u} F\left(i\sqrt{\frac{3(1+\bar{u})}{2(1+x)}}; \sqrt{\frac{1-\bar{u}}{1+\bar{u}}}\right)}{i(1-\bar{u})\sqrt{1+\bar{u}}}\right], \quad (15)$$

with the const. such that $A_0(0) \equiv 0$ and with the incomplete elliptic integrals of the first and second kind,

$$E(iw; k) - \bar{u} F(iw; k) = i \int_0^w \frac{dt(1+k^2 t^2 - \bar{u})}{\sqrt{(1+k^2 t^2)(1+t^2)}}, \quad (16)$$

and $\bar{u} = \sqrt{25 - 16u^2}/3$. The charge density,

$$J^0 = \frac{\sqrt{6M_1}}{16\pi u^{5/2}} x^4 \times \\ \frac{5u^2(4u^2 - 1) + u^2(7x^2 + 42x) + x^2(3x^2 + 18x + 35)}{u^3(x+1)^{7/2}\sqrt{4u^2 + 5x + x^2}}, \quad (17)$$

is nonnegative. The charge within radius r is

$$Q(r) = \sqrt{6M_1 r u} \frac{(u^2 - x^2)^{3/2} \sqrt{4u^2 + 5x + x^2}}{2u^4(x+1)^{5/2}}, \quad (18)$$

and the total BH charge is $Q = Q(R_S)$.

D. Physical estimates

Equating the average mass density M/R_S^3 to the EW one, λv^4 , defines, surprisingly, a Neptunian mass scale

$$M_* = \frac{m_P^3}{\sqrt{\lambda} v^2} = 0.75 \times 10^{-4} M_\odot = 25 M_\oplus = 1.5 M_\Psi. \quad (19)$$

For $M \gg M_*$, let $\mu(r)$ go from 0 to M at a typical core scale $R_c \ll R_S$. An EW mass density up to R_c then sets

$$R_c \sim \frac{M^{1/3}}{\lambda^{1/3} v^{4/3}} = R_* \left(\frac{M}{M_*}\right)^{1/3} = 260 \left(\frac{M}{M_\odot}\right)^{1/3} \text{ cm.} \quad (20)$$

With $\mu \sim M(r/R_c)^3$, the inner horizon $R_i = 2G\mu(R_i)$ corresponds to a fixed characteristic scale $R_i \approx R_*$, viz.

$$R_* = GM_* = \frac{m_P}{\sqrt{\lambda} v^2} = 11 \text{ cm.} \quad (21)$$

The condition $R_c \gg R_*$ again implies $M \gg M_*$.

Like for the Earth, we call the sphere $r < R_i$ *the inner core*, the spherical shell $R_i < r < R_c$ *the outer core* and the region $R_c < r < R_S$ *the mantle*. The transition from inner to outer core is sharp, from outer core to mantle it is gradual. Instead of a crust, there is an event horizon.

The dimensionless value of \mathcal{K} in this smeared singularity shows that curvature is strong but finite,

$$R_c^4 \mathcal{K} \sim \frac{R_S^2}{R_c^2} \sim \left(\frac{M}{M_*}\right)^{4/3} \sim 3 \times 10^5 \left(\frac{M}{M_\odot}\right)^{4/3}. \quad (22)$$

The electromagnetic field has energy density $A_0'^2 \sim \lambda v^4$. The typical values of A_0 and the total charge Q are

$$|A_0| \sim \sqrt{\lambda} v^2 R_c = \left(\frac{M}{M_*}\right)^{1/3} m_P, \quad Q \sim \left(\frac{M}{M_*}\right)^{2/3} \frac{m_P^2}{\sqrt{\lambda} v^2}. \quad (23)$$

A RN BH has maximal charge $Q = M/m_P$. The fraction of uncompensated charges that form the BH, $\varepsilon = (Q/e)/(M/m_N)$, has typically the small value $\varepsilon_M = (m_N/e m_P)(M_*/M)^{1/3} = 4 \cdot 10^{-20} (M_\odot/M)^{1/3}$, which is still large enough to prevent collapse into a singularity.

Lastly, the fraction of rest mass in the fermions is $(2m_u + m_d + m_e)/m_N = 0.0079$. This implies that the \mathcal{A}_H below eq. (10) can not match \mathcal{A} numerically.

III. THE FULL SM APPROACH

Having just run into a numerical mismatch, we have to widen our search for an EW description of the BH interior. We do this in 3 ways. 1) Allow also condensed Z bosons (W bosons perform less well); 2) Allow for thermal particles: thermal electrons and quarks, Higgs and Z bosons, and photons and gluons. 3) Dilatate time by setting $dt \rightarrow S_0 dt$, $g_{00} \rightarrow (1-S)S_0^2$ with $S_0(r) \leq 1$. Since the BH is stationary, its mass will not depend on S_0 .

1) *The Z condensate.* We extend the EW Lagrangian and EMT by the Z field contribution

$$\mathcal{L}_Z = \frac{M_Z^2 \varphi^2}{2\mu_0 v^2} Z^\rho Z_\rho, \quad T_{Z\nu}^\mu = \frac{M_Z^2 \varphi^2}{\mu_0 v^2} (Z^\mu Z_\nu - \frac{\delta_\nu^\mu}{2} Z^\rho Z_\rho). \quad (24)$$

2) *Thermal matter.* At finite temperature, a Bose-Einstein condensate does not collect all particles, leaving *thermal* particles in the gas[28]. A thermal EMT has the general form $T_{\vartheta\nu}^\mu = (\rho_\vartheta + p_\vartheta)U^\mu U_\nu - p_\vartheta \delta_\nu^\mu$, with U^μ the velocity vector. In the inner core, time is normal so that $U^\mu = \delta_0^\mu/\sqrt{g_{00}}$ and $T_{\vartheta\nu}^\mu = \text{diag}(\rho_\vartheta, -p_\vartheta, -p_\vartheta, -p_\vartheta)$. In the outer core and mantle, r is the time-like variable, so that $T_{\vartheta\nu}^\mu = \text{diag}(-p_\vartheta, \rho_\vartheta, -p_\vartheta, -p_\vartheta)$. This combines as

$$T_{\vartheta\nu}^\mu = \mathcal{A}_\vartheta \delta_\nu^\mu + \frac{\sigma}{4\pi G} \mathcal{C}_\nu^\mu + \frac{\tau}{2\pi G} (\delta_0^\mu \delta_\nu^0 - \delta_1^\mu \delta_\nu^1), \quad (25) \\ \mathcal{A}_\vartheta = \frac{\rho_\vartheta - 3p_\vartheta}{4}, \quad \sigma = \pi G(\rho_\vartheta + p_\vartheta), \quad \tau = \pm \sigma,$$

with $\tau = \sigma$ in the inner core but $-\sigma$ in the outer core and mantle. Relativistic thermal particles lead to $\mathcal{A}_\vartheta = 0$.

3) *Time dilatation.* For $S_0(r) \neq 1$, the trace of T brings

$$\mathcal{A} = \frac{2S + 4rS' + r^2S''}{32\pi G r^2} + \frac{3S_0'S'}{32\pi G S_0} + \frac{2S_0'' + rS_0'''}{16\pi G r S_0} \bar{S}, \quad (26)$$

where $\bar{S} = S - 1$. The enclosed charge is $Q = -r^2 A_0'/S_0$.

The equation of motion for the Higgs field (11) becomes

$$\lambda(\varphi^2 - v^2)\varphi = \frac{\varphi}{\mu_0 v^2} M_Z^2 Z^\rho Z_\rho - \frac{\rho_F}{v}. \quad (27)$$

Having pointed out that ρ_F is small, we now neglect it, so that φ is a square root, and determines

$$\mathcal{A}_Z \equiv \frac{1}{4} \text{tr} T_Z = -\frac{1}{4\mu_0} \frac{\varphi^2}{v^2} M_Z^2 Z^\rho Z_\rho. \quad (28)$$

We assume that $Z_\mu = \delta_0^\mu Z_0$, whence \mathcal{A}_Z is negative in the inner core, and positive in outer core and the mantle. Equating \mathcal{A} to $\mathcal{A}_H + \mathcal{A}_Z + \mathcal{A}_\vartheta$ yields solutions when $\mathcal{A}_\vartheta > 0$; this is possible when the temperature T is not too high, so that the Higgs and Z bosons are not fully relativistic.

Matching the traceless parts of T and $T_\gamma + T_Z + T_\vartheta$ yields

$$\frac{A_0'^2}{S_0^2} = \frac{2S - r^2 S''}{4Gr^2} - \frac{3S_0' S'}{4GS_0} + \frac{S_0' - rS_0''}{2GrS_0} \bar{S} - \frac{\sigma - \tau}{2G}, \quad (29)$$

$$\frac{M_Z^2 \varphi^2}{m_P^2 v^2} \frac{Z_0^2}{S_0^2} = \bar{S}^2 \frac{S_0'}{rS_0} + \bar{S} \tau. \quad (30)$$

Since $\bar{S}\tau \leq 0$ both in the core and the mantle, thermal matter is allowed when $S_0' > 0$, increasing from $S_0(0) < 1$ to $S_0(R_S) = 1$. The small- r expansions $S = s_2 r^2 - s_4 r^4$, $S_0 = u_0(1 + u_2 r^2 + u_4 r^4)$ and $\sigma = \tau = \sigma_2 r^2$ yield

$$\begin{aligned} \mathcal{A} &= 3 \frac{s_2 - u_2}{8\pi G} + \frac{12s_2 u_2 - 15s_4 + 6u_2^2 - 20u_4}{16\pi G} r^2, \\ \frac{A_0'^2}{u_0^2} &= \frac{1}{2} (5s_4 + 8u_4 - 6s_2 u_2) r^2, \\ \frac{M_Z^2 \varphi^2}{m_P^2 v^2} \frac{Z_0^2}{u_0^2} &= 2u_2 - (4s_2 u_2 - 2u_2^2 - 4u_4 + \sigma_2) r^2, \end{aligned} \quad (31)$$

which exhibits a large class of proper, positive solutions with nontrivial S_0 , viz. $0 < u_2 < s_2$ or $u_2 = 0$, $u_4 > \frac{1}{4}\sigma_2$.

At finite r , one example is $S = 2G\mu/r$ from (12) and $S_0 = (r^2 + u_0 R_1^2)/(r^2 + R_1^2)$. It is natural to put $\sigma = |\bar{S}|\bar{\sigma}$, $\tau = -\bar{S}\bar{\sigma}$ for some $0 \leq \bar{\sigma}(r) \leq S_0'/rS_0$, which sets Z_0 via (30). Hence τ and σ vanish at R_i , and (29) is continuous.

A. Z and W sources

While the above solution allows that $Z_0 = 0$, the general case demands that it is properly sourced. Its equation of motion $M_Z^2(\varphi^2/v^2)Z^\mu = -\mu_0 J_Z^\mu$ implies a number of unbalanced particles $\delta N_Z \sim R_c^3 J_Z^0 \sim (M/M_*)^{2/3} M_Z m_P^3 / \lambda v^4 = 2.0 \times 10^{53} (M/M_\odot)^{2/3}$, well less than the $N_H \sim 1.2 \times 10^{57} (M/M_\odot)$ H atoms that make up the BH. For a W boson field $W_\mu = W_\mu^\dagger$ instead of Z_μ , the same would apply; however, this may not extend to the rotating case. Then the quartic terms of the EW Lagrangian contain a term $A^3 A_3 W^0 W_0 \sim A_0^2 W_0^2$ which overwhelms the $M_W^2 W_0^2$ term. As a result, the number of W -sourcing particles $\delta N_W \lesssim (M_Z/A_0) \delta N_Z \sim 10^{-17} (M_*/M)^{1/3} \delta N_Z$ is negligible, while their contribution to the EMT is quadratically small in v/m_P .

The fermion-boson coupling $\mathcal{L} = A_\mu J_\gamma^\mu + Z_\mu^0 J_Z^\mu$ involves the electrodynamic and neutral currents (see, e.g., Peskin

and Schroeder, eq. (20.80) [29]),

$$\begin{aligned} J_\gamma^\mu &= \frac{2e}{3} (n_{u_L}^\mu + n_{u_R}^\mu) - \frac{e}{3} (n_{d_L}^\mu + n_{d_R}^\mu) - e(n_{e_L}^\mu + n_{e_R}^\mu), \\ J_Z^\mu &= \frac{g}{2c_W} (n_{u_L}^\mu - n_{d_L}^\mu - n_{e_L}^\mu + n_{\nu_{eL}}^\mu) - \frac{s_W}{c_W} J_\gamma^\mu. \end{aligned} \quad (32)$$

where $n_x^\mu \equiv \bar{x}\gamma^\mu x$ for up and down quarks, electrons and neutrinos, with left (L) or right (R) handed chirality. Here e is the proton charge, $g = e/s_W$ the weak coupling constant, $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$.

These currents vanish for a star consisting only of ionized H atoms, $n_{u_h}^\mu = 2n_{d_h}^\mu = 2n_{e_h}^\mu$ for $h = L, R$, while $n_{\nu_{eL}}^\mu = 0$. In a supernova explosion more electrons get expelled than protons, which makes J_γ finite; likewise, expulsion of neutrinos (e.g. from the proton-proton reaction $p + p \rightarrow d + e^+ + \nu_e$) makes also the first term of J_Z finite.

IV. SUMMARY

To offer an alternative to the present unresolved structure of the black hole (BH) singularity, we set out to describe the interior of a BH by extended Bose-Einstein condensates in standard model physics. While it is standard to go to a Tolman-Oppenheimer-Volkov equation, in our situation it is more adequate to work with the Einstein equations themselves, where all matter components can be specified separately and we actually made some progress for the rotating case, to be discussed elsewhere.

To start, we recall that the Painlevé Gullstrand metric, which is regular at the event horizon, leads to an ill defined value of the mass and divergent curvature invariant. Replacing the mass M by some smooth function $\mu(r)$ with $\mu \sim r^3$ for $r \rightarrow 0$, heals this, at the cost of a nontrivial energy momentum tensor. We seek to build the latter from the electroweak sector of the standard model of particle physics. This puts forward that the BH interior contains a Higgs and Z condensate with mass densities comparable to the electroweak one, λv^4 .

Gravitational collapse is prevented by negative pressures, an isotropic one from the Higgs and Z condensates and a radial one from the Coulomb repulsion due to a small mismatch of electric charges.

In the thus considered picture, the H atoms that form the BH, dissolve into electrons, and up and down quarks. Elementary reactions like $u + e \leftrightarrow u + W_- + \nu_e \leftrightarrow d + \nu_e$ take place, next to the creation of Higgs and Z bosons, and photons and gluons, from the nuclear binding energy.

In the simplest approach, the core of the BH involves a Higgs condensate and an electrostatic potential, but it appears that the source for the condensate is too small. This is overcome by also allowing condensed Z bosons and various thermal particles, as well as a local time dilatation coded in the function $S_0(r)$.

A nontrivial value for the $\mathcal{A}_\vartheta = (\rho_\vartheta - 3p_\vartheta)/4$ of the thermal particles acts as a positive energy condition; photons, gluons and relativistic quarks and electrons do not

contribute to this, hence it requires that the temperature be $\lesssim \lambda^{1/4}v$ to prevent that also the thermal Higgs and Z particles are relativistic and \mathcal{A}_θ too small.

V. DISCUSSION AND OUTLOOK

The Planck mass and the microscopic v set a characteristic, Neptunian mass scale, $M_* = m_P^3/\sqrt{\lambda}v^2 = 0.75 \times 10^{-4}M_\odot = 1.5M_\Psi$. Our approach holds for larger masses, which is on the safe side given that BHs of stellar origin are neither expected nor observed with mass less than solar. The related characteristic length scale, $R_* = GM_* = m_P/\sqrt{\lambda}v^2 = 11$ cm, sets the size of the inner core, the region inside the inner horizon, where time has its normal meaning; in the outer core and in the mantle, the radial coordinate has the meaning of time, so that matter can get only ever closer to the origin.

The mass is located in the combined inner and outer core; it has characteristic size $R_c = (M/M_*)^{1/3}R_*$. Beyond it, up to the much larger Schwarzschild radius, there is essentially the vacuum envisioned by Schwarzschild. Our solution provides consistent particle distributions given the μ and S_0 profiles. A criterion for optimizing the profiles is desired.

So far, our charged BH model can act as stepping stone for the case of rotating BHs. But let us consider its own merits in a speculative way. In a supernova explosion, electrons are more easily ejected than protons, making the matter for the BH positively charged. In an imploding stellar core of mass M and charge Q , with profiles $M(r)$ and $Q(r)$ and uniform charge-to-mass ratio, the ratio of forces on a chunk (ch) of matter, $[Q(r)Q_{ch}/r^2]/[GM(r)M_{ch}/r^2] = (Qm_P/M)^2$, allows BH formation just up to the extremal charge M/m_P . For our value $(Qm_P/M)^2 \sim 0.002 (M_\odot/M)^{2/3}$, this back-of-the-envelope estimate offers wiggle room for the onset of charged, few solar mass BH formation. While the net charge is easily undone by pair creation near the horizon

or by accumulation of negative charges from the accretion disk, see e.g., [30], this may not be achieved locally during the collapse, and subsequently made impossible by the establishment of horizons, in which case collapse into a singularity can be avoided. The locked-up internal net positive charge gets compensated by a negatively charged layer just outside the event horizon. If accretion makes the BH extremal (in charge, combined with rotation), the horizons disappear and fireworks occur.

Typically, the mantles are much larger than the cores, so that the cores are shielded in BH-BH merging. If one of the BHs is nearly extremal, ejection of matter may occur due to the exposition of its large core. This predicts a new class of events, alternative to neutron star-BH mergers, with a mass allowed above the neutron star maximum of $\sim 2M_\odot$; a candidate for this is S190426c[31–33].

Another possibility is that the ejecta get a high energy. Indeed, the Coulomb energy of an isolated charge $+e$ at the extremal event horizon radius $R_g = GM$ equals $Qe/R_g = Qm_P^2\sqrt{\alpha}/M \approx 10^{27}(Qm_P/M)$ eV. When this is not screened strongly and cascades, such BH-BH mergers offer an explanation for ultrahigh energy cosmic rays and neutrinos, coincident with the emission of radiation and gravitational waves.

The present work offers a new view on astrophysical BHs and a further application of the standard model of particle physics. Many questions remain, like: Can this class of metrics be reached dynamically? Are they (meta)stable? What is their entropy? Can the approach be generalized to rotating BHs? Are there connections with the Fermi bubbles and/or the X-ray chimneys around Sag A*?

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