# Pre-burst events of gamma-ray bursts with light speed variation

Jie Zhu<sup>a</sup>, Bo-Qiang Ma<sup>a,b,c,\*</sup>

<sup>a</sup>School of Physics, Peking University, Beijing 100871, China
<sup>b</sup>Center for High Energy Physics, Peking University, Beijing 100871, China
<sup>c</sup>Collaborative Innovation Center of Quantum Matter, Beijing, China

#### Abstract

Previous researches on high-energy photon events from gamma-ray bursts (GRBs) suggest a light speed variation  $v(E) = c(1 - E/E_{LV})$  with  $E_{LV} = 3.6 \times 10^{17}$  GeV, together with a pre-burst scenario that hight-energy photons come out about 10 seconds earlier than low-energy photons at the GRB source. However, in the Lorentz invariance violating scenario with an energy dependent light speed considered here, high-energy photons travel slower than low-energy photons due to the light speed variation, so that they are usually detected after low-energy photons in observed GRB data. Here we find four high-energy photon events which were observed earlier than low-energy photons from Fermi Gamma-ray Space Telescope (FGST), and analysis on these photon events supports the pre-burst scenario of high energy photons from GRBs and the energy dependence of light speed listed above.

Keywords: light speed variation, gamma ray burst, pre-burst, Lorentz invariance violation

# 1. Introduction

According to Einstein's relativity, light speed is a constant *c* in free space. However, it is speculated from quantum gravity that the Lorentz invariance might be broken at the Planck scale ( $E_{Pl} \simeq 1.22 \times 10^{19}$  GeV), and that the light speed may have a variation with the energy of the photon. Amelino-Camelia *et al.* [1, 2] first suggested testing Lorentz violation by comparing the arrival times between high energy and low energy photons from gamma-ray bursts (GRBs). For energy  $E \ll E_{Pl}$ , the modified dispersion relation of the photon can be expressed in leading order as

$$E^{2} = p^{2}c^{2} \left[ 1 - s_{n} \left( \frac{pc}{E_{\text{LV},n}} \right)^{n} \right].$$
<sup>(1)</sup>

Assuming that the traditional relation  $v = \partial E / \partial p$  holds, we have the following speed relation

$$v(E) = c \left[ 1 - s_n \frac{n+1}{2} \left( \frac{pc}{E_{\text{LV},n}} \right)^n \right],\tag{2}$$

where n = 1 or n = 2 as usually assumed,  $s_n = \pm 1$  indicates whether high-energy photons travel faster ( $s_n = -1$ ) or slower ( $s_n = +1$ ) than low-energy photons, and  $E_{LV,n}$  represents the nth-order Lorentz violation scale. From Eq. (2) we can derive a time lag between two photons with different energies in flat universe, however we need to consider the expansion of the Universe [3], and the result shows as

$$\Delta t_{\rm LV} = s_n \frac{1+n}{2H_0} \frac{E_{\rm h}^n - E_1^n}{E_{\rm LV,n}^n} \int_0^z \frac{(1+z')^n {\rm d}z'}{\sqrt{\Omega_{\rm m}(1+z')^3 + \Omega_\Lambda}},\tag{3}$$

where  $E_h$  and  $E_l$  correspond to the energies of the observed high-energy and low-energy photons, z is the redshift of the source GRB,  $H_0$ ,  $\Omega_m$  and  $\Omega_{\Lambda}$  are cosmological constants. Here we adopt the present day Hubble constant

\*Corresponding author

Email address: mabq@pku.edu.cn (Bo-Qiang Ma)

Preprint submitted to Phys. Lett. B 820 (2021) 136518, https://doi.org/10.1016/j.physletb.2021.136518

 $H_0 = 67.3 \pm 1.2 \text{ km s}^{-1} \text{Mpc}^{-1}$  [4], the pressureless matter density  $\Omega_m = 0.315^{+0.016}_{-0.017}$  [4] and the dark energy density  $\Omega_{\Lambda} = 0.685^{+0.017}_{-0.016}$  [4].

The observed time difference between high energy and low energy photons should not be only the time lag due to Lorentz violation, i.e., Eq.(3), but also an intrinsic time lag  $\Delta t_{in}$  at the source GRB [5, 6, 7, 8, 9], which means that in the source reference system high-energy photon events and low-energy photon events have an intrinsic time difference  $\Delta t_{in}$ . Considering the expansion of the Universe, we have

$$\Delta t_{\rm obs} = \Delta t_{\rm LV} + (1+z)\Delta t_{\rm in},\tag{4}$$

where  $\Delta t_{obs}$  is the difference of observed arrival times between high-energy and low-energy photons, *z* is the redshift of the source GRB and  $\Delta t_{LV}$  is the time lag caused by Lorentz violation as expressed in Eq. (3). In fact, with cosmic photons from one single source, one has difficult to make clear distinction between the Lorentz violation effect and the intrinsic source effect from the observed time difference  $\Delta t_{obs}$ , see Eq. (4). The combination of multi-GeV photons from GRBs with different redshifts renders it feasible to make distinction between Lorentz violation effect and intrinsic source effect.

Previous studies [8, 9] on high energy photon events from GRBs detected by Fermi Gamma-ray Space Telescope (FGST) [10, 11] suggest a regularity of high energy photon events with a conclusion that s = +1, n = 1,  $E_{LV,1} = (3.60 \pm 0.26) \times 10^{17}$  GeV and  $\Delta t_{in} = (-10.7 \pm 1.5)$  s. In this physics picture, it is suggested that high-energy photons come out about 10 seconds earlier than low-energy photons at the GRB source, and because of light speed variation, high-energy photons travel slower than low-energy photons, and the light speed difference and the long cosmological distances lead to an expectation that one usually observes low-energy photons earlier than high-energy photons, as is indeed the case in the earlier observations of GRB data.

Here we want to search for high-energy photon events which are observed earlier than low-energy ones, and we call these events as **observed pre-burst events**. In traditional point of view one may consider these events as just background noises without any significance, but in the picture of light speed variation, these events should come from pre-burst emission of high energy photons from GRBs so that they are novel signals to support the observed regularity as an indication for the light speed variation [8, 9]. However to find observed pre-burst events is not easy. If the energy of the photon is too high, the speed of the photon may cause it fell behind low-energy photons. Just take the conclusion of Refs. [8, 9] and do a simple calculation, if we want to find observed pre-burst events, we have

$$\Delta t_{\rm LV} + (1+z)\Delta t_{\rm in} < 0. \tag{5}$$

Usually  $E_h \sim \text{GeV}$  while  $E_1 \sim \text{keV}$ , so it is reasonable to take  $E_1$  as 0. Combining Eq. (5) with Eq. (3) and let n = 1, we have

$$E_{\rm h} < -E_{\rm LV,1} H_0 \Delta t_{\rm in} (1+z) / \int_0^z \frac{(1+z') dz'}{\sqrt{\Omega_{\rm m} (1+z')^3 + \Omega_{\Lambda}}}.$$
 (6)

As shown in Fig. 1, we can not expect that we could find observed pre-burst events with too high energy. For example, if the redshift of a GRB is 2, we can only expect observed pre-burst high-energy photons with energy less than 10.9 GeV.

## 2. Data Acquirement

We search for the data from the Fermi Gamma-ray Space Telescope (FGST). FGST consists of the Fermi Large Area Telescope (LAT) [10] and the Gamma-Ray Burst Monitor (GBM) [11]. LAT aims to collect high-energy events while GBM aims to collect low-energy events. The GBM data can be downloaded from the Fermi website [12] while the LAT data need to be retrieved and downloaded from this website [13]. The trigger time  $t_{trig}$  of GBM is usually assumed as the onset time of GRBs, so we search for high-energy events before  $t_{trig}$ . The retrieval scope of time is set from  $t_{trig} - 50$  s to  $t_{trig} - 0$  s, and the retrieval radius is set to 2 degrees. The energy range is set from 100 MeV to 100 GeV, and the lower limit is chosen to reject events with poorly reconstructed directions and energies. We have searched 48 GRBs, from GRB080916C to GRB210204A, which are not only detected by FGST but also have redshifts recorded. Although our retrieval scope of time and energy is too much larger than we need, we find only 3 GRBs which have observed pre-burst events recorded, and they are **GRB 201020A**, **GRB 201020B** and **GRB 201021C**.

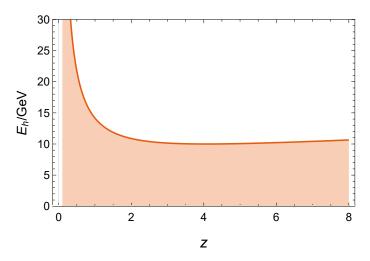


Figure 1: Plot of the condition Eq. (6) to observe pre-burst events. z is the redshift of the GRB source, and  $E_h$  is the observed energy of high-energy photon event. The shaded part represents the area of Eq. (6)

Here we discuss  $\Delta t_{obs}$ , the difference of observed arrival time between high-energy photon and low energy photon. In the following discussion, we set  $t_{trig}$  as the origin of the time coordinate. As defined

$$\Delta t_{\rm obs} = t_{\rm high} - t_{\rm low},\tag{7}$$

where  $t_{high}$  is the arrival time of high-energy photon and  $t_{low}$  represents the arrival time of low-energy photon. For  $t_{high}$  of single photons, we can get it from the retrieved LAT data, and for  $t_{low}$ , we adopt the first significant peak criteria discussed by the work of Ref. [14]. For every GRB in this work, the first significant peak time is close to the trigger time. For example the first significant peak for GRB201020B indicates that  $t_{low} = -0.45$  s as shown in Fig. 2. We list all of the observed pre-burst events of the two GRBs in Table 1.

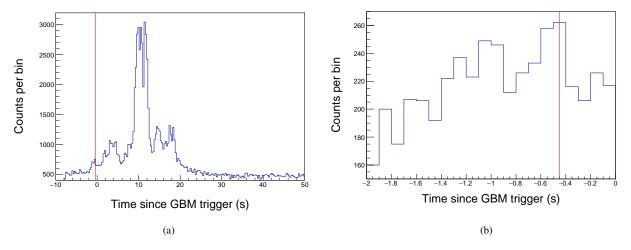


Figure 2: Light curves of the two brightest trigger detectors combined (GBM NaI-n7 and NaI-nb, 7.9 ~ 258 keV) for GRB 201020B. In the left panel (a), photon events are binned in 0.3 second intervals. In the right panel (b), photon events are binned in 0.1 second intervals to determine the peak of the first significant peak as  $t_{low} = -0.45$  s.

Table 1: The data of observed pre-burst events

GRB	z	t <sub>high</sub> (s)	$t_{\rm low}$ (s)	$E_{\rm obs}~({\rm GeV})$	$E_{\text{source}}$ (GeV)	RA(°)	Dec(°)	p
201020A	2.903	-22.955	-1.875	0.229	0.892	-22.955	260.264	0.79
201020B	0.804	-17.405	-0.45	1.227	2.214	67.360	77.323	0.64
201021C(1)	1.07	-12.839	-0.625	0.469	0.972	11.485	-54.462	0.84
201021C(2)	1.07	-37.173	-0.625	0.155	0.322	15.070	-54.820	0.76

Data of the observed pre-burst events of the three GRBs. *z* is the redshift of the GRB,  $t_{high}$  is the arrival time of high-energy photon and  $t_{low}$  represents the first significant peak time of low-energy photons.  $E_{obs}$  is the energy observed by LAT while  $E_{source} = (1 + z)E_{obs}$  is the corresponding energy at the source. (RA, Dec) is the position of the events (J2000). *p* is the probability that the event is associated with the source and is generated by the *Fermi* ScienceTool *gtsrcprob* [15]. Redshift of GRB 201020A is from Ref. [16], redshift of GRB 201020B is from Ref. [17] and redshift of GRB 201021C is from Ref. [18].

#### 3. Data Analysis and Result

Here we use the similar analyse method introduced in Ref. [8]. As a brief introduction, combining Eq. (3) and Eq. (4), we have

$$\frac{\Delta t_{\rm obs}}{1+z} = s_n \frac{K_n}{E_{\rm LV,n}^n} + \Delta t_{\rm in},\tag{8}$$

where  $K_n$  is the Lorentz violation factor

$$K_n = \frac{1+n}{2H_0} \frac{E_h^n}{1+z} \int_0^z \frac{(1+z')^n dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}}.$$
(9)

Then we make a  $\Delta t_{obs}/(1+z)$  versus  $K_1$  plot and try to find linear relation between different events. These photons with a same intrinsic time lag would fall on an inclined straight line in the  $\Delta t_{obs}/(1+z) - K_n$  plot, and we can determine  $\Delta t_{in}$ of them as the intercept of the line with the Y axis. The slope of the mainline is  $1/E_{LV,1}$ , from which one can determine the Lorentz violation scale  $E_{LV,1}$ . However, it is not reasonable to assume that all of the high-energy photons emit at exactly a same time and that the intrinsic time lag  $\Delta t_{in}$  for every GRB is the same, and there may be a distribution for the intrinsic time lag  $\Delta t_{in}$ . Since we know little about the intrinsic emission mechanism of GRBs, here we just assume that  $\Delta t_{in}$  follows a normal distribution with mean  $\mu$  and standard deviation  $\sigma$ . Here we use Bayesian analysis and maximum likelihood estimation (MLE) to fit the data [5, 21]. For a linear model y = kx + b and given n groups of data  $(x_i, y_i)$  with errors  $\sigma_{x_i}$  and  $\sigma_{y_i}$ , the likelihood function for an individual point can be expressed as

$$p(y_i|x_i, \sigma_{x_i}, \sigma_{y_i}, k, b) = \frac{1}{\sqrt{2\pi(\sigma_{x_i}^2 k^2 + \sigma_{y_i}^2)}} \exp\left(-\frac{(kx_i + b - y_i)^2}{2(\sigma_{x_i}^2 k^2 + \sigma_{y_i}^2)}\right).$$
(10)

If the parameter b follows a Gaussian distribution with mean  $\mu$  and standard deviation  $\sigma$ , we can derive the likelihood function for an individual point as

$$p(y_{i}|x_{i},\sigma_{x_{i}},\sigma_{y_{i}},k,\mu,\sigma) = \int_{-\infty}^{\infty} p(y_{i}|x_{i},\sigma_{x_{i}},\sigma_{y_{i}},k,b)p(b|\mu,\sigma)db$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi(\sigma_{x_{i}}^{2}k^{2}+\sigma_{y_{i}}^{2})}} \exp\left(-\frac{(kx_{i}+b-y_{i})^{2}}{2(\sigma_{x_{i}}^{2}k^{2}+\sigma_{y_{i}}^{2})}\right) \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(b-\mu)^{2}}{2\sigma^{2}}\right)db \qquad (11)$$

$$= \frac{1}{\sqrt{2\pi(\sigma_{x_{i}}^{2}k^{2}+\sigma_{y_{i}}^{2}+\sigma^{2})}} \exp\left(-\frac{(kx_{i}+\mu-y_{i})^{2}}{2(\sigma_{x_{i}}^{2}k^{2}+\sigma_{y_{i}}^{2}+\sigma^{2})}\right),$$

and we can write the data likelihood as

$$p(\{y_i\}|\{x_i,\sigma_{x_i},\sigma_{y_i}\},k,\mu,\sigma) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi(\sigma_{x_i}^2k^2 + \sigma_{y_i}^2 + \sigma^2)}} \exp\left(-\frac{(kx_i+\mu-y_i)^2}{2(\sigma_{x_i}^2k^2 + \sigma_{y_i}^2 + \sigma^2)}\right).$$
 (12)

For the high-energy events with positive arrival time, we use the data of Refs. [8, 9]. Then we add the highenergy events with negative arrival time listed in Table 1. Considering the energy resolution of LAT [10] (within 10% uncertainty) and the uncertainties of the cosmological parameters,  $\Delta t_{obs}/(1 + z)$  and  $K_1$  can be calculated as shown in Table 2. Since the time resolution for GBM and LAT is smaller than 10  $\mu s$  [19, 20] and it is much more smaller than the time scale in our data, we assume that there is no error in  $\Delta t_{obs}$ . Thus we can write the likelihood function as

$$L = C \exp\left[-\frac{1}{2} \sum_{i=1}^{n} \left(\frac{\left(\frac{\Delta t_{obs_i}}{1+z_i} - a_{\rm LV}K_i - \mu\right)^2}{\sigma^2 + a_{\rm LV}^2 \sigma_{k_i}^2} + \ln(\sigma^2 + a_{\rm LV}^2 \sigma_{k_i}^2)\right)\right],\tag{13}$$

where *C* is a constant,  $a_{LV} = 1/E_{LV,1}$ ,  $K_i$  is  $K_1$  of the *i*th data,  $\mu$  and  $\sigma$  are the mean and the standard deviation of  $\Delta t_{in}$ . The MLE result of the likelihood function for the data from Table 2 and Refs. [8, 9] is  $a_{LV} = 2.35 \times 10^{-18} \text{ GeV}^{-1}$ ,  $\mu = -8.57 \text{ s}$  and  $\sigma = 5.43 \text{ s}$ , and the 95% CL range for the slope is  $a_{LV} = 2.35_{-0.73}^{+0.83} \times 10^{-18} \text{ GeV}^{-1}$ . The likelihood function for the slope parameter and the plot for the fit are shown in Fig. 3. The main contribution to uncertainties of  $K_1$  is the energy resolution of LAT. The energies of the observed pre-burst events are too small compared with those with positive arrival time, so we can hardly see the uncertainties in this figure. This result gives a limit on  $E_{LV,1}$  that  $E_{LV,1} = 4.2_{-1.1}^{+1.9} \times 10^{17} \text{ GeV}$ . From Fig. 3 we can see clearly that the three of the points in this work are near the main line of Refs. [8, 9], and we also choose the data near the mainline and fit them again. The MLE result of the likelihood function for the slope is  $a_{LV} = 2.36_{-0.27}^{+0.36} \times 10^{-18} \text{ GeV}^{-1}$ . The likelihood function for the slope is  $a_{LV} = 2.36_{-0.27}^{+0.36} \times 10^{-18} \text{ GeV}^{-1}$ ,  $\mu = -7.73$  s and  $\sigma = 1.45$  s, and the 95% CL range for the slope is  $a_{LV} = 2.36_{-0.27}^{+0.36} \times 10^{-18} \text{ GeV}^{-1}$ . The likelihood function for the slope parameter and the plot for the fit are shown in Fig. 4. Fig. 4 suggests strong correlation between the data in Table 2 and Refs. [8, 9], and the result suggests that  $E_{LV,1} = 4.23_{-0.56}^{+0.36} \times 10^{17} \text{ GeV}$ .

Table 2: Values of  $\Delta t_{obs}/(1 + z)$  and  $K_1$  of observed pre-burst events

GRB	$\Delta t_{\rm obs}({ m s})$	$\Delta t_{\rm obs}/(1+z)(s)$	$K_1(\times 10^{17} \mathrm{s} \cdot \mathrm{GeV})$
201020A	-21.08	-5.40	$0.81 \pm 0.08$
201020B	-16.95	-9.40	$2.77 \pm 0.28$
201021C(1)	-12.21	-5.90	$1.23 \pm 0.13$
201021C(2)	-36.55	-17.66	$0.41 \pm 0.04$

We need to consider the probability of a large uncertainty around 5 seconds in  $\Delta t_{obs}$  due to the determination of the first significant peak  $t_{low}$  of low energy photons. With an err bar of 5-seconds introduced for each  $\Delta t_{obs}$  in the analysis, the MLE result corresponding to Fig. 4 suggests that  $a_{LV} = 2.30 \times 10^{-18} \text{ GeV}^{-1}$ ,  $\mu = -7.05 \text{ s}$  and  $\sigma = 0.00 \text{ s}$ . We can understand the result of  $\sigma = 0.00 \text{ s}$  in this way: the main difference between whether we introduce an error  $\sigma_{y_i}$  for every point or not is equivalent to adopt an effective  $\tilde{\sigma} = \sqrt{\sigma^2 + \sigma_{y_i}^2}$  or just  $\sigma$  in Eq. (13). If the best fit for  $\sigma$  of the data without  $\sigma_{y_i}$  is bigger than  $\sigma_{y_i}$ , we can absorb the error of data into the parameter  $\sigma$  of the normal distribution and get the result  $\sigma = \sqrt{\tilde{\sigma}^2 - \sigma_{y_i}^2}$ . However in our previous fitting the result gives us  $\sigma = 1.45$  s and almost every error for the data  $\sigma_{y_i} = 5/(1 + z_i)$  seconds is bigger than it, so it is reasonable to get the best estimation of  $\sigma = 0.00$  s. The 95% CL range for the parameters are  $a_{LV} = 2.30^{+0.49}_{-0.30} \times 10^{-18} \text{ GeV}^{-1}$  and  $\mu = -7.05^{+2.01}_{-2.87}$  s, and thus we get  $E_{LV,1} = 4.34^{+0.65}_{-0.76} \times 10^{17} \text{ GeV}$ .

### 4. Discussion and Conclusion

The result shown in Fig.4 provides novel signals with significance. First, it supports the conclusion of light speed variation first suggested in Ref. [8] and soon supported by a remarkable high energy event of GRB160509A in

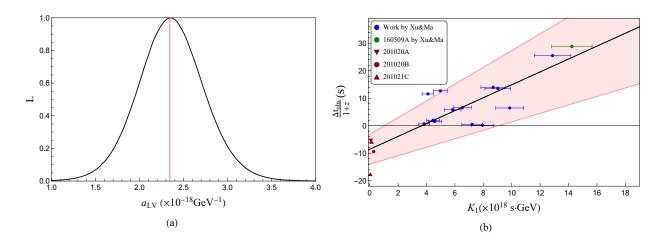


Figure 3: The likelihood function *L* for the slope parameter (left panel) and the plot for the fit of the data from Refs. [8, 9] (Xu&Ma) and Table 1 (right panel). In the left panel (a), the red vertical line represents the maximum likelihood estimation (MLE) result for  $a_{LV}$ . In the right panel (b), the black (thick) straight line represents the fit result, and for the red (thin) lines of the edge of the red (shaded) area, the slopes are the edge of 95% CL range for  $a_{LV}$  and the intercepts are  $\mu + \sigma$  and  $\mu - \sigma$  while  $\mu$  and  $\sigma$  are the best fit parameters.

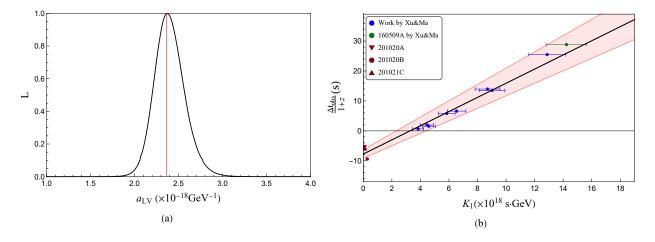


Figure 4: The likelihood function *L* for the slope parameter (left panel) and the plot for the fit of the data near the mainline from Refs. [8, 9] (Xu&Ma) and Table 1 (right panel). In the left panel (a), the red vertical line represents the maximum likelihood estimation (MLE) result for  $a_{LV}$ . In the right panel (b), the black (thick) straight line represents the fit result, and for the red (thin) lines of the edge of the red (shaded) area, the slopes are the edge of 95% CL range for  $a_{LV}$  and the intercepts are  $\mu + \sigma$  and  $\mu - \sigma$  while  $\mu$  and  $\sigma$  are the best fit parameters.

Ref. [9]. Second, it supports the conclusion of a pre-burst stage of GRBs, and as suggested, at about 10 seconds before a gamma-ray burst at the source, there is a pre-burst stage of high-energy photons with energy of multi-GeV scale. Third, although the condition on the observed pre-burst events is too strict, we still find these events and make analysis on them and the result supports the previous conclusion. These events might be regarded as background noises if one lacks of a pre-burst scenario, but analysis on them suggests that they are signals from the pre-burst stage of GRBs. So we should treat observed pre-burst events with important significance as normal high energy photon events.

From another point of view, our analysis of multi-GeV photon events belongs to the catalog of looking for sharp peak structure behind the data. These multi-GeV photons from GRBs are within a very large duration of one hundred seconds with different arrival times, but our analysis indicates that they come from the sources with a same intrinsic time. This implies that these multi-photon events on/near the mainline, if drawing an emission curve with time in the GRB source frame, correspond to a very sharp peak at -7.73 second with a very narrow width of only 2-3 seconds. So our result actually represents the finding of a significant sharp structure of multi-GeV photons from the Fermi GRB data.

In conclusion, we searched for high-energy photon events with earlier arrival time from GRBs with redshift recorded from FGST, and found four pre-burst events of high energy photons from GRBs GRB 201020A, GRB 201020B and GRB 201021C. Analysis on the observed pre-bursts events reveal that three of the observed pre-burst events from these GRBs fall near the mainline that indicates a regularity of high energy photons. The result suggests  $E_{\text{LV},1} = 4.23^{+0.56}_{-0.56} \times 10^{17}$  GeV and the mean value of  $\Delta t_{\text{in}}$  is -7.73 s with a width  $2\sigma = 2.90$  s, which supports the earlier suggestion in Refs.[8, 9] for a light speed variation  $v(E) = c(1 - E/E_{\text{LV}})$  with  $E_{\text{LV}} = 3.6 \times 10^{17}$  GeV and a 10-second earlier pre-burst stage of high energy photons of GRBs.

**Acknowledgements:** We thank the anonymous reviewer for the enlightening suggestions that have helped us to improve the quality of the analysis. This work is supported by National Natural Science Foundation of China (Grant No. 12075003).

#### References

- G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos, Distance measurement and wave dispersion in a Liouville-string approach to quantum gravity. Int. J. Mod. Phys. A 12 (1997) 607.
- [2] G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos, S. Sarkar, Tests of quantum gravity from observations of γ-ray bursts. Nature 393 (1998) 763.
- [3] U. Jacob, T. Piran, Lorentz-violation-induced arrival delays of cosmological particles. JCAP 0801 (2008) 031.
- [4] K. A. Olive et al., (Particle Data Group) Chinese Physics C 38 (2014) 090001.
- [5] J. R. Ellis, N. E. Mavromatos, D. Nanopoulos, A. S. Sakharov, E. K. G. Sarkisyan, Robust limits on Lorentz violation from gamma-ray bursts. Astropart. Phys. 25 (2006) 402-411. [Corrigendum 29 (2008) 158-159].
- [6] L. Shao, Z. Xiao, B.-Q. Ma, Lorentz violation from cosmological objects with very high energy photon emissions. Astropart. Phys. 33 (2010) 312-315.
- [7] S. Zhang, B.-Q. Ma, Lorentz violation from gamma-ray bursts. Astropart. Phys. 61 (2015) 108-112.
- [8] H. Xu,B.-Q. Ma, Light speed variation from gamma-ray bursts. Astropart. Phys. 82 (2016) 72.
- [9] H. Xu,B.-Q. Ma, Light speed variation from gamma ray burst GRB 160509A. Phys. Lett. B 760 (2016) 602.
- [10] W. B. Atwood et al., The Large Area Telescope on the Fermi Gamma-ray Space Telescope mission. Astrophys. J. 697 (2009) 1071.
- [11] C. Meegan, G. Lichti, P. N. Bhat et al., The Fermi Gamma-Ray Burst Monitor. Astrophys. J. 702 (2009) 791.
- [12] https://heasarc.gsfc.nasa.gov/FTP/fermi/data/gbm/triggers/
- [13] https://fermi.gsfc.nasa.gov/cgi-bin/ssc/LAT/LATDataQuery.cgi
- [14] Y. Liu, B.-Q. Ma, Light speed variation from gamma ray bursts: criteria for low energy photons. Euro. Phys. J. C 78 (10) 825.
- [15] https://fermi.gsfc.nasa.gov/ssc/data/analysis/scitools/overview.html
- [16] D. A. Kann, A. de Ugarte Postigo, M. Blazek, et al., GRB 201020A: Redshift from GTC/OSIRIS. GCN Circ. 28717 (2020).
- [17] D. A. Kann, A. de Ugarte Postigo, M. Blazek, et al., GRB 201020B: Redshift from GTC/OSIRIS. GCN Circ. 28765 (2020).
- [18] J.-B. Vielfaure, D. Xu, J. Palmerio, et al., GRB 201021C: VLT/X-shooter redshift. GCN Circ. 28739 (2020).
- [19] https://fermi.gsfc.nasa.gov/ssc/data/analysis/documentation/Cicerone/Cicerone\_Introduction/GBM\_overview.html
- [20] https://fermi.gsfc.nasa.gov/ssc/data/analysis/documentation/Cicerone/Cicerone\_Introduction/LAT\_overview.html
- [21] A. Abramowski, et al. (H.E.S.S.), Search for Lorentz Invariance breaking with a likelihood fit of the PKS 2155-304 flare data taken on MJD 53944. Astropart. Phys. 34 (2011) 738.