Production of axions during scattering of Alfvén waves by fast-moving Schwarzschild black holes

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Abstract

We discuss a novel mechanism of axion production during scattering of Alfvén waves by a fast moving Schwarzschild black hole. The process couples classical macroscopic objects, and effectively large amplitude electromagnetic (EM) waves, to microscopic axions. The key ingredient is that the motion of a black hole (BH) across magnetic field creates classical non-zero second Poincare invariant, the electromagnetic anomaly (Lyutikov 2011). In the case of magnetized plasma supporting Alfvén wave, it is the fluctuating component of the magnetic field that contributes to the anomaly: for sufficiency small BH moving with the super-Alfvenic velocity the plasma does not have enough time to screen the parallel electric field. This creates time-dependent $\mathbf{E} \cdot \mathbf{B} \neq 0$, and production of axions via the axion-EM coupling.

I. INTRODUCTION

Axions are hypothetical particle invoked to resolve the strong CP problem in quantum chromodynamics [1]. They are also invoked as dark matter candidates [2].

The present model of axion production is based on the observation that a Schwarzschild BH moving in vacuum across magnetic field generates non-zero second Poincare invariant $\mathbf{E} \cdot \mathbf{B} \neq 0$, Ref. [3]. This effect, combined with the idea of axion-photon mixing [4], particularly in the presence of external magnetic fields [5], may lead to axion production via the EM anomaly.

Consider an ideal plasma with density n in external magnetic field $\mathbf{B} = B_0 \mathbf{e}_z$. The plasma supports Alfvén waves, low frequency oscillations of the magnetic field, with frequency of the waves $\omega_A = v_A k$ smaller than the plasma frequency ω_p, $\omega_A \leq \omega_p$. Here

$$v_A = \frac{B_0}{\sqrt{4\pi n m_p}} \tag{1}$$

is Alfvén velocity, B_0 is the magnetic field, m_p is proton mass. Alfvén waves create fluctuating transverse components of the magnetic field $\delta\mathbf{B}$.

Next, let a BH move along the magnetic field with velocity $\beta_{BH} c$. Let $\beta_{BH} c \gg v_A$, so that the BH moves through nearly stationary wiggled magnetic field. In the frame of the BH the Alfvén wave is seen as a propagating wave with frequency $\omega = \beta_{BH} k c$. For sufficiently high velocity of the BH the frequency ω may be larger than the electron plasma frequency

$$\omega_p = \sqrt{\frac{4\pi n e^2}{m_e}} \tag{2}$$

(the corresponding conditions are discussed in §III). As a result, the plasma does not have time to adjust to the $\mathbf{E} \cdot \mathbf{B} = 0$ condition: the BH sees a nearly vacuum electromagnetic wave, but propagating with $\beta_{BH} \ll 1$. In the wave the fluctuations of the electric field are much smaller than of the magnetic field, and can be neglected (in the wave $\delta E/\delta B \sim v_A/c \ll 1$ for non-relativistic Alfvén velocity)

Thus, a BH moves through a spatially varying, nearly vacuum magnetic field $\delta\mathbf{B}$, directed perpendicular to the direction of BH motion. Motion of the BH through magnetic field in vacuum generates non-zero second Poincare invariant $\mathbf{E} \cdot \mathbf{B} \neq 0$, Ref. [3]. In the frame of the BH it is varying with frequency ω. Axions with mass $m_A \sim (\hbar/c^2)\omega$ are then produced by coupling to the electromagnetic anomaly.

II. THE MODEL

A. Static anomaly: motion of BH across magnetic field in vacuum

First we highlight the basic ingredient of the model, that motion of a BH through magnetic field in vacuum generates non-zero second Poincare invariant $\mathbf{E}\cdot\mathbf{B}\neq 0$, Ref. [3].

Consider magnetic field along z direction and a Schwarzschild BH moving along x direction. Using standard relations for electromagnetic fields in general relativity [6–8] and choosing the four-potential in Schwarzschild coordinates

$$
\begin{aligned}
A_0 &= \alpha\beta_{BH} r\sin\theta\cos\phi B_0\\
A_\phi &= \frac{1}{2} r\sin\theta B_0\\
\alpha &= \sqrt{1-2M_{BH}/r}
\end{aligned}
\tag{3}
$$

(in unites $c=G=1$; θ is the polar angle, ϕ is the azimuthal angle), we find the EM tensor

$$
\begin{aligned}
F^{\mu\nu} &= \begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & -\cos(\theta)\cos(\phi)\beta_0 & \alpha\sin(\theta)+\sin(\phi)\beta_0\\ 0 & \cos(\theta)\cos(\phi)\beta_0 & 0 & \cos(\theta)\\ 0 & -\alpha\sin(\theta)-\sin(\phi)\beta_0 & -\cos(\theta) & 0 \end{pmatrix} B_0\\
F^{\mu\nu}_{;\nu} &= 0\\
\mathrm{Det}F &= \frac{4\beta_0^2 B_0^4 M^2\sin^2(\theta)\cos^2(\theta)\cos^2(\phi)}{r^2} = (\mathbf{E}\cdot\mathbf{B})^2
\end{aligned}
\tag{4}
$$

Explicitly,

$$
\begin{aligned}
\mathbf{E} &= \{\sin(\theta)\cos(\phi), \alpha\cos(\theta)\cos(\phi), -\alpha\sin(\phi)\}\,\beta_{BH}B_0\\
\mathbf{B} &= \{-\cos(\theta), \alpha\sin(\theta), 0\}\,B_0\\
\mathbf{E}\cdot\mathbf{B} &= -\frac{M\sin(2\theta)\cos(\phi)}{r}\beta_{BH}B_0^2 \propto \beta_{BH}B_0^2\frac{M}{r}
\end{aligned}
\tag{5}
$$

Note that the anomaly $\mathbf{E}\cdot\mathbf{B}\neq 0$ is highly non-local, $\propto 1/r$. We also comment that $\mathbf{E}\cdot\mathbf{B}\neq 0$ does not appear during scattering of an electromagnetic wave by the black hole, Appendix A. It is important that the Alfvén waves have non-vacuum dispersion.

There is a non-zero divergence of the electromagnetic topological current J_ν

$$
\begin{aligned}
&J_\nu = A^\mu ({}^*F_{\mu\nu}) \\
&J_0 = \mathbf{A} \cdot \mathbf{B} = 0 \\
&J_i = \mathbf{E} \times \mathbf{A} + \frac{A_0}{\alpha}\mathbf{B} \\
&J_{\mu;\mu} = -\frac{7}{4}\sin 2\theta \cos\phi B_0 E_0 \frac{M}{r} = \frac{7}{4}\mathbf{E}\cdot\mathbf{B}
\end{aligned} \tag{6}
$$

Thus, the non-zero second Poincare electromagnetic invariant leads to the appearance of sources of topological axial vector currents. This can lead to the local violation of the baryon and lepton numbers through the triangle anomaly [9, 10].

B. Motion of black hole through Alfvén wave creates EM anomaly

Consider an Alfvén wave with the wave number k and fluctuating magnetic field $\delta\mathbf{B}$. In the frame of the BH moving non-relativistically with $\beta_{BH}c \gg v_A$, the transverse component of the magnetic field varies as

$$
\delta B(t) = \delta B \cos(k(\beta_{BH} t - z)) \tag{7}
$$

In the expression for the anomaly (5) we can then put $B \to \delta B(t)$.

$$
\mathbf{E}\cdot\mathbf{B} \propto \beta_{BH}(\delta B)^2 \cos^2(k(\beta_{BH} t - z))\frac{M}{r} \tag{8}
$$

The sign of $\mathbf{E}\cdot\mathbf{B}$ depends on the location, $\propto \sin(2\theta)\cos(\phi)$, Eq (5).

C. Coupling to axions

Axions interact only minimally with ordinary matter. Axions couple to EM fields via the anomaly

$$
\begin{aligned}
&\mathcal{L}_{a\gamma} = g_{a\gamma} a(\mathbf{E}\cdot\mathbf{B}) \\
&g_{a\gamma} = \xi \times 2 \times 10^{-10} GeV^{-1} \frac{m_a}{1eV}
\end{aligned} \tag{9}
$$

a stands for axion field, m_a is axion mass (expected in the range $10^{-6} - 1$ eV, Ref [11]), ξ is some parameter.

Using (8) we find

$$\mathcal{L}_{a\gamma} = g_{a\gamma} a \beta_{BH} (\delta B)^2 \cos^2(k(\beta_{BH} t - z)) \frac{M}{r} \tag{10}$$

Thus, we have a *classical* configuration with time dependent $\mathbf{E} \cdot \mathbf{B}$ anomaly. In this case the axion production is possible and can be large (it is proportional to large classical field, rather than small quantum fluctuations).

The resonance condition, $\omega_a = m_a$, requires that the Compton length of axions

$$\lambda_a = \frac{\hbar}{m_a c} \tag{11}$$

matches temporal variations of the anomaly,

$$k_{res} = \frac{1}{\beta_{BH} \lambda_a} \tag{12}$$

III. ASTROPHYSICAL APPLICABILITY

The process discussed in §II is theoretically possible, but how realistic is it? Conceptually, the main problem is that the model invokes macroscopic effects (which are qualitatively large in value) to produce microscopic particles. Since the axion production is resonant, the corresponding scales must match: this is the main limitation/uncertainty.

Many astrophysical setting are possible, from stellar mass black holes moving through magnetized interstellar medium (ISM), to primordial black holes, to the processes in the Early Universe. As a basic example (which will be shown to be hard to satisfy), let us consider a BH moving trough an ISM. The first requirement is that the BH moves faster than Alfvén waves. In a typical Galactic weakly magnetized plasmas the Alfvén velocity (1) is sub-relativistic; typical values in the interstellar medium are $\sim 10 - 100$ km/sec [12]. If velocity of the BH is smaller than v_A, then variations of the EM field on the scale of the BH will occur on time scale R_g/v_A, where $R_g = 2GM/c^2$ is the Schwarzschild radius. On the other hand if velocity of the BH is lager than v_A, variations of the EM field on the scale of the BH then will occur on time scale R_g/β_{BH}. Assuming large Alfvén Mach number $M_a = \beta_{BH} c/v_A \gg 1$, and equating $\beta_{BH} c/R_g = \omega_p$ we find a mass of the BH so that the plasma time $1/\omega_p$ equals light travel time over the horizon.

$$M_{BH} = \frac{c^3 \sqrt{m_e}}{4\sqrt{\pi} e G \sqrt{n}} \beta_{BH} = 3.6 \times 10^{33} \beta_{BH} n^{-1/2} \,\text{gramm} \tag{13}$$

Thus in plasma of density $n = 1$ cm^{-3}, a $\sim$ Solar mass BH moving subrelativistically has light travel time of the order of plasma time. Smaller BH will induce $E_\parallel$ that will not be screened by plasma.

High spacial velocity of stellar-mass BHs may come from merger. It is expected that BH kick during mergers can be as high as $\beta_{BH} \sim 10^{-2} \sim$ few 10^3 km/s [13]. Faster moving BH produces shorter time scale variations; this eases constraints on the condition that plasma effects do not short out $E_\parallel$. Thus, stellar-mass BHs moving with Mach number $M_A \geq 1$ can produce variations of EM fields on time-scale shorter that plasma scale, and thus, parallel electric field, and the EM anomaly.

Next, let us estimate the Compton length of axions, and compare it to the astrophysical expectations (this is needed for the resonant axion production). For the expected axion mass of $\sim 10^{-6}$ eV, the Compton length of axions (11) evaluates to

$$\lambda_a = 20 \left(\frac{m_a}{10^{-6}\text{eV}}\right)^{-1} \text{cm} \tag{14}$$

The BH with Schwarzschild radius that equals λ_a would have a mass

$$M_{BH,a} = \frac{c\hbar}{2Gm_a} = 6 \times 10^{-5} M_\odot \left(\frac{m_a}{10^{-6}\text{eV}} \text{cm}\right)^{-1}, \tag{15}$$

just somewhat larger than the mass of the Earth. We arrive at an important point: macroscopic objects (of the order of the mass of the Earth) *can* couple to the microscopic axions.

The resonant frequency of the anomaly's oscillation in the frame of the BH, $\omega = \beta_{BH} c k_{res}$, with k_{res} given by (12), should be larger than ω_p. This requires

$$n \leq \frac{m_a^2 m_e c^4}{4\pi e^2 \hbar^2} = 7 \times 10^8 \left(\frac{m_a}{10^{-6}\text{eV}}\right)^2 \text{cm}^{-3}, \tag{16}$$

an easily satisfiable condition.

Another constraint comes from the condition that before the BH comes, the plasma must support fairly short wavelength Alfvén oscillation (14). This requires (at least) that λ_a be larger than the Debye length r_D,

$$\begin{aligned}
&\frac{\lambda_a}{r_D} = \beta_{BH}^{-1} \frac{2\sqrt{\pi} e \hbar \sqrt{n}}{c m_a \sqrt{k_B T}} = 30 \times \left(\frac{m_a}{10^{-6}\text{eV}}\right) n^{-1/2} \geq 1 \\
&n \leq \frac{(m_a c)^2 k_B T}{4\pi e^2 \hbar^2 \beta_{BH}^2} = 10^3 \left(\frac{m_a}{10^{-6}\text{eV}}\right)^2 \beta_{BH}^{-2} \,\text{cm}^{-3} \\
&r_D = \frac{v_T}{\omega_p}
\end{aligned} \tag{17}$$

where T is the temperature of the ISM plasma. Also a satisfiable condition.

The most stringent constraint comes from the fact that resistive effects in the ISM dissipate short wavelength Alfvén waves. For a BH moving with β_{BH} through an Alfvén wave of wavelength λ_A the frequency of oscillations seen in its frame, $\beta_{BH} c/\lambda$, should match the axion mass (Eqns (11-12):

$$\lambda_A = \frac{2\pi}{k_{res}} = 10^3 \, \beta_{BH} \left(\frac{m_a}{10^{-6}\text{eV}} \, \text{cm} \right)^{-1} \text{cm} \tag{18}$$

This wavelength is substantially smaller that the expected inner scale of Kolmogorov turbulence in the ISM, $l_{min} \sim 10^{10}$ cm, Ref. [14]. Thus, a single stellar mass BHs moving through ISM are not like to encounter Alfvén waves of the required properties (see below a comment on the beat oscillations of the anomaly between two BHs).

IV. DISCUSSION

The proposed mechanism of axion production involves combined effects of several physics disciplines: electromagnetism and plasma physics (Alfvén waves), General Theory of Relativity (Black holes), and particle physics (axions). The proposed mechanism couples large classical quantities to the weakly interacting axions, hence can be highly efficient. The mechanism is somewhat related to axion-photon mixing in magnetic fields [5].

The proposed mechanism of axion production has a number of specific points/advantaged:

- it involves interaction of macroscopic classical fields and matter (hence could be more powerful than typically small quantum effects).

- it involves macroscopic fluctuating electromagnetic fields (whose values are typically much larger than that of the photon fields). As an example, consider an Alfvén wave in magnetic field B_0 with relative amplitude $a_H \equiv \delta B/B_0 \leq 1$. The laser non-linearity parameter [15] then estimates to

$$a \equiv \frac{e(\delta B)}{m_e c \omega} = a_H \frac{eB_0}{m_e c^2 k \beta_{BH}} = 2 a_H \frac{eGM_{BH}B_0}{m_e c^4 \beta_{BH}} = 10^2 a_H \beta_{BH}^{-1} B_0 \left(\frac{M_{BH}}{M_\odot} \right) \tag{19}$$

  where in the last relation we estimated $k \sim 1/R_g$; B_0 is in Gauss. This is incredibly intense EM wave, far beyond what is reachable in the laboratory experiments.

- it is non-local (so that different regions produce axions incoherently, this eliminates strong cancellation). One further complication involves interference between newly produced axions. If axions are produced with typical velocity $v_a \ll c$, their de Broglie wavelength (coherence scale) $\lambda_{D,a} \sim 1/(m_a v_a)$ is much larger than the Compton length λ_a, Eq. (11), Refs [16–18]. The coherence scale $\lambda_{D,a}$ depends on environment and variations of the surrounding fields,

Moving Kerr BH adds another level of complexity [19]. First, Wald's solution [20] also produces (stationary) $\mathbf{E} \cdot \mathbf{B} \neq 0$. In an Alfvén wave (*e.g.*, polarized orthogonally to the BH's spin) the anomaly will be time-dependent and thus can couple to axions. Fast motion of a BH along the magnetic field then can ensure that the corresponding variations are sufficiently fast and not screened by plasma.

Our estimates demonstrate that a single (sub)stellar mass BHs moving in an ISM is not likely to produce axions, due to the lack of resonant Alfvén waves (Alfvén waves of the required wavelength of $\sim$ a meter can marginally propagate in the ISM plasma, but suffer quick, on astronomical time scales, resistive decay).

If there are many BHs moving uncorrelatedly, then, on the one hand, the $\mathbf{E} \cdot \mathbf{B}$ will tend to average out on scales much larger than the typical separation between BHs (but locally it will still be dominated by a single one, since $\mathbf{E} \cdot \mathbf{B} \propto 1/r$). On the other hand, two BHs with somewhat different velocities will produce the anomaly varying on the beat frequencies. The minus-beat frequency can be much smaller, and couple to longer wavelengths Alfvén waves. We leave investigation of other possible astrophysical sites (*e.g.* Alfvén waves propagating in magnetospheres of compact objected with strong gravity) to a future work.

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[1] J. E. Kim and G. Carosi, Reviews of Modern Physics **82**, 557 (2010), 0807.3125.

[2] K. A. Olive and Particle Data Group, Chinese Physics C **38**, 090001 (2014).

[3] M. Lyutikov, Phys. Rev. D **83**, 064001 (2011), 1101.0639.

[4] P. Sikivie, Phys. Rev. Lett. **51**, 1415 (1983).

[5] G. Raffelt and L. Stodolsky, Phys. Rev. D **37**, 1237 (1988).

[6] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (San Francisco: W.H. Freeman and Co., 1973, 1973).

[7] L. D. Landau and E. M. Lifshitz, *The classical theory of fields* (1975).

[8] I. D. Novikov and V. P. Frolov, *Physics of black holes* (1989).

[9] G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976).

[10] V. A. Rubakov and M. E. Shaposhnikov, Physics Uspekhi **39**, 461 (1996), hep-ph/9603208.

[11] G. G. Raffelt, *Astrophysical Axion Bounds* (2008), vol. 741, p. 51.

[12] C. F. McKee and J. P. Ostriker, Astrophys. J. **218**, 148 (1977).

[13] M. Campanelli, C. Lousto, Y. Zlochower, and D. Merritt, ApJ Lett. **659**, L5 (2007), gr-qc/0701164.

[14] J. W. Armstrong, B. J. Rickett, and S. R. Spangler, Astrophys. J. **443**, 209 (1995).

[15] A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N. Stepanov, Oxford Pergamon Press International Series on Natural Philosophy **1** (1975).

[16] P. Sikivie, D. B. Tanner, and K. van Bibber, Phys. Rev. Lett. **98**, 172002 (2007), hep-ph/0701198.

[17] X. Liang and A. Zhitnitsky, Phys. Rev. D **99**, 023015 (2019).

[18] M. C. D. Marsh, J. H. Matthews, C. Reynolds, and P. Carenza, arXiv e-prints arXiv:2107.08040 (2021), 2107.08040.

[19] V. S. Morozova, L. Rezzolla, and B. J. Ahmedov, Phys. Rev. D **89**, 104030 (2014), 1310.3575.

[20] R. M. Wald, Phys. Rev. D **10**, 1680 (1974).

Appendix A: EM waves in Schwarzschild spacetime

Using spherical wave functions, we can express the EM four-vector [6]

$$A_\mu = \left\{0, \frac{f(r)}{r^2} Y_{lm}, \frac{g(r)}{l(l+1)} \frac{\partial_r f}{r} \partial_\theta Y_{lm}, \frac{g(r)}{l(l+1)} \frac{\partial_r f}{r \sin\theta} \partial_\phi Y_{lm} \right\} \tag{A1}$$

We find equations for f and g,

$$\begin{aligned} &2r\alpha(\alpha-g)f'-(1-\alpha^2)f=0 \\ &r^3\alpha^3\partial_r(gf')-\left(l(l+1)\alpha^2-r^2\omega^2\right)f=0 \end{aligned} \tag{A2}$$

Or

$$\begin{aligned} g &= \alpha-\frac{(1-\alpha^2)f}{2r\alpha f} \\ f'' &= \left(\frac{l(l+1)}{\alpha^2 r^2}-\frac{-3\alpha^4+2\alpha^2+4r^2\omega^2+1}{4\alpha^4 r^2}\right)f \end{aligned} \tag{A3}$$

In flat space Eq. (A2) give $g(1)=1$, $f=j_l(\omega r)=\sqrt{r}J_{l+1/2}(\omega r)$, where j_l are spherical Bessel functions.

Importantly, $\mathrm{Det}F_{\mu\nu}=0$ in this case: there is no EM anomaly.