Observational constraints on inflection point quintessence with a cubic potential

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We examine the simplest inflection point quintessence model, with a potential given by $V(\phi) = V_0 + V_3\phi^3$. This model can produce either asymptotic de Sitter expansion or transient acceleration, and we show that it does not correspond to either pure freezing or thawing behavior. We derive observational constraints on the initial value of the scalar field, ϕ_i, and V_3/V_0 and find that small values of either ϕ_i or V_3/V_0 are favored. While most of the observationally-allowed parameter space yields asymptotic de Sitter evolution, there is a small region, corresponding to large V_3/V_0 and small ϕ_i, for which the current accelerated expansion is transient. The latter behavior is potentially consistent with a cyclic universe.

I.   INTRODUCTION

According to observational data [1–7], the Universe is composed of approximately 70% dark energy, which is a negative-pressure component, and roughly 30% nonrelativistic matter, which includes baryons and dark matter. Although current observations are consistent with a cosmological constant and cold dark matter (ΛCDM), a dynamical equation of state cannot be ruled out. One widely-studied set of dynamical models is quintessence, which employs a time-dependent scalar field, ϕ, with an associated potential $V(\phi)$ [8–14]. (See Ref. [15] for a review).

Dark energy is characterized by its equation of state parameter, w, the ratio of dark energy pressure to density:

$$w = p/\rho. \tag{1}$$

For a cosmological constant Λ, we have $\rho = constant$ and $w = -1$. For a scalar field ϕ evolving in a potential $V(\phi)$, the pressure and density that determine w are given by

$$p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi), \tag{2}$$

and

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \tag{3}$$

where the dot denotes the time derivative throughout. Observations indicate that $w \approx -1$ today. In order to achieve this with quintessence, we require that $\dot{\phi}^2/2 \ll V(\phi)$. One way to achieve this result is to begin with $\dot{\phi} \approx 0$ and take the present day to correspond to the time immediately after the field begins to roll downhill in the potential; in these "thawing" models, w is initially equal to -1 and increases slightly up to the present. Alternately, one can allow the field to roll downhill with $w > -1$, but then to reach a sufficiently flat region of the potential that w approaches -1 asymptotically. Such models have been dubbed "freezing" models. (See Ref. [16] for a discussion). We will see that inflection point quintessence, proposed in Ref. [17], does not fit neatly into either category but instead corresponds to a hybrid of the two.

Scalar fields with an inflection point in the potential were first proposed as models for inflation [18–32]. The simplest such models correspond to a cubic potential with an inflection point at $\phi = \phi_0$:

$$V(\phi) = V_0 + V_3(\phi - \phi_0)^3, \tag{4}$$

where V_0 and V_3 are constants. As noted in Ref. [23], this model arises naturally in the context of string theory. Note that ϕ can be translated by a constant without changing any physically-observable quantities, so it is simplest to take the inflection point to be at $\phi = 0$, giving

$$V(\phi) = V_0 + V_3\phi^3. \tag{5}$$

In both inflation and quintessence, a value of $w \approx -1$ is achieved as ϕ reaches the inflection point.

An interesting feature of the evolution of ϕ in this potential is that the late-time behavior of ϕ can vary widely depending on the values of V_0, V_3, and ϕ_i (the initial value of ϕ). For some values of these parameters, ϕ evolves smoothly to zero as $t \to \infty$, yielding an asymptotic de Sitter evolution. However, for other values, ϕ evolves through the inflection point at $\phi = 0$, corresponding to a transient period of acceleration. The latter possibility is particularly

interesting because an eternally accelerating universe presents a problem for string theory, inasmuch as the S-matrix in this case is ill-defined [33, 34]. Consequently, a great deal of effort has gone into the development of models in which the observed acceleration is a transient phenomenon [35–44]. When ϕ evolves past 0, our model represents another example of this sort of transient acceleration.

The goal of this investigation is to first determine the model parameters that are allowed by present-day observations, and then to investigate whether both types of future evolution (de Sitter versus transient acceleration) are consistent with the allowed parameter range. In the next section, we discuss the general features of the evolution of ϕ in this model. In Sec. III, we derive observational limits on the model parameters and explore whether these limits are consistent with both types of asymptotic evolution for ϕ. Our main results are summarized in Sec. IV.

II.   THE EVOLUTION OF INFLECTION POINT QUINTESSENCE

The equation of motion for a quintessence scalar field is

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \tag{6}$$

where the Hubble parameter H is

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_\phi + \rho_M}{3}. \tag{7}$$

Here, a is the scale factor, which we normalize to $a = 1$ at the present, ρ_ϕ is the scalar field energy density given by Eq. (3), and ρ_M is the matter density. We take $\hbar = c = 8\pi G = 1$ throughout. The matter density is given by

$$\rho_M = \rho_{M0}a^{-3}, \tag{8}$$

where ρ_{M0} is the present-day matter density. We use the cubic potential given by Eq. (5) for $V(\phi)$. While other potentials can correspond to inflection point quintessence (see Sec. IV), the cubic potential is the simplest, and it has the interesting property of yielding both asymptotic de Sitter or transient acceleration depending on the model parameters.

In constraining quintessence models, the observable quantities of interest depend only on the evolution of w as a function of $\Omega_\phi \equiv \rho_\phi/(\rho_\phi + \rho_M)$. This evolution is given by [45]

$$\frac{dw}{d\Omega_\phi} = \frac{3(1+w)(1-w) + \lambda(1-w)\sqrt{3(1+w)\Omega_\phi}}{3w\Omega_\phi(1 - \Omega_\phi)}, \tag{9}$$

with $\lambda \equiv (1/V)(dV/d\phi)$. Hence, the observable quantities depend on the potential only through λ. In our cubic inflection point model,

$$\lambda = \frac{3\phi^2}{(V_0/V_3) + \phi^3}. \tag{10}$$

Therefore, the observable quantities in this model depend only on V_3/V_0, rather than on V_3 and V_0 independently.

We will assume that Hubble friction drives $\dot{\phi}$ to zero in the early universe, so that ϕ begins initially at some value ϕ_i with $\dot{\phi}_i = 0$. With these assumptions, the cubic inflection point quintessence model has only two free parameters: V_3/V_0 and ϕ_i. For a fixed value of V_3/V_0, the asymptotic evolution of ϕ depends on the value of ϕ_i: for a sufficiently large value of ϕ_i, the scalar field evolves through the inflection point at $\phi = 0$, while for smaller values of ϕ_i, the field evolves asymptotically to $\phi = 0$ as $t \to \infty$. However, there exists a critical value of $V_3/V_0 \approx 0.77$; for values of V_3/V_0 below this value, ϕ always evolves asymptotically to zero regardless of the value of ϕ_i. This behavior was first noted for inflation by Itzhaki and Kovetz [26] and later for quintessence by Chang and Scherrer [17].

III.   OBSERVATIONAL LIMITS

To derive observational limits on inflection point quintessence, we numerically integrated the evolution of ϕ for a range of values of V_3/V_0 and ϕ_i using CAMB [46, 47]. Consistency with observational data (the temperature, polarization, and lensing of the Planck 2018 likelihood data [48, 49], baryon acoustic oscillations (BAO) data from 6dF, SDSS DR7, and SDSS DR12 galaxy surveys [50–52], and the Type Ia Supernovae Pantheon data [53]) was

Parameter	68% limits	95% limits
$\log\left(10^{10}A_\mathrm{s}\right)$	3.048 ± 0.014	$3.048^{+0.028}_{-0.026}$
n_s	0.9674 ± 0.0035	$0.9674^{+0.0069}_{-0.0068}$
$100\theta_\mathrm{MC}$	1.04103 ± 0.00028	$1.04103^{+0.00055}_{-0.00054}$
τ_reio	$0.0570^{+0.0064}_{-0.0072}$	$0.057^{+0.014}_{-0.013}$
H_0	$67.42^{+0.63}_{-0.39}$	$67.4^{+1.1}_{-1.2}$
Ω_m	$0.3129^{+0.0052}_{-0.0070}$	$0.313^{+0.013}_{-0.013}$
σ_8	$0.8069^{+0.0075}_{-0.0059}$	$0.807^{+0.014}_{-0.015}$
χ^2	3832 ± 43	$3832\,(\nu\!:906.5)$

TABLE I: Results for the marginalized means, 68% limits, and 95% limits for the initial super-horizon amplitude of curvature perturbations A_s, the scalar spectral index n_s, the approximation to the observed angular size of the sound horizon at recombination θ_{MC}, the optical depth to reionization τ_{reio}, the Hubble constant H_0, the matter density parameter Ω_m, the late-time clustering amplitude σ_8, and the χ^2 in the inflection point quintessence model.

determined using Cobaya [54] along with a Monte Carlo Markov Chain sampler [55, 56], which used the fast-dragging procedure described in [57].

Our main result is shown in Fig. 1, which depicts the observational constraints on our model parameters. For fixed ϕ_i, the observations favor smaller values of V_3/V_0 and vice-versa. This makes sense, since smaller values of V_3/V_0 and ϕ_i drive the model to more closely resemble the observationally-favored ΛCDM model. Indeed, as $\phi_i \to 0$ for fixed V_3/V_0 or $V_3/V_0 \to 0$ for fixed ϕ_i, the model becomes indistinguishable from ΛCDM; this is reflected in the contours shown in Fig. 1.

Upper and lower limits at the 68% and 95% C.L. for various other cosmological parameters using the inflection point quintessence model are given in Table I. The inflection point quintessence model is compared to the standard ΛCDM model in Fig. 2, which also includes the χ^2 values.
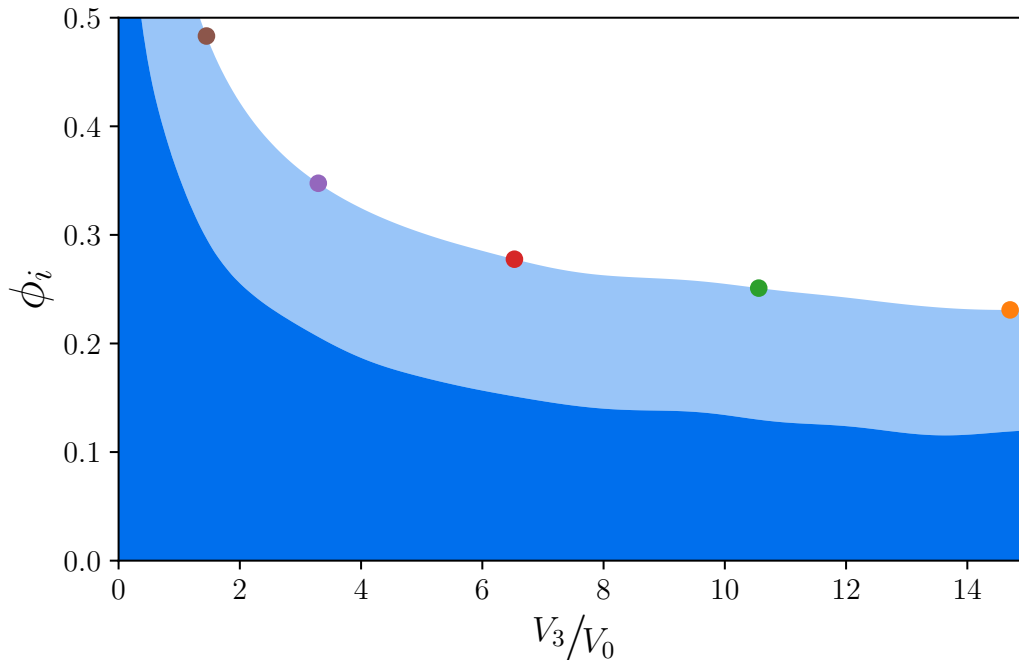

FIG. 1: Planck 2018 CMB + SN + BAO 1σ (dark blue) and 2σ (light blue) contours for the inflection point quintessence model with the cubic potential of Eq. (5), as a function of V_3/V_0 and ϕ_i. The colored circles label the curves in Fig. 3 showing the evolution of w for the corresponding values of V_3/V_0 and ϕ_i.
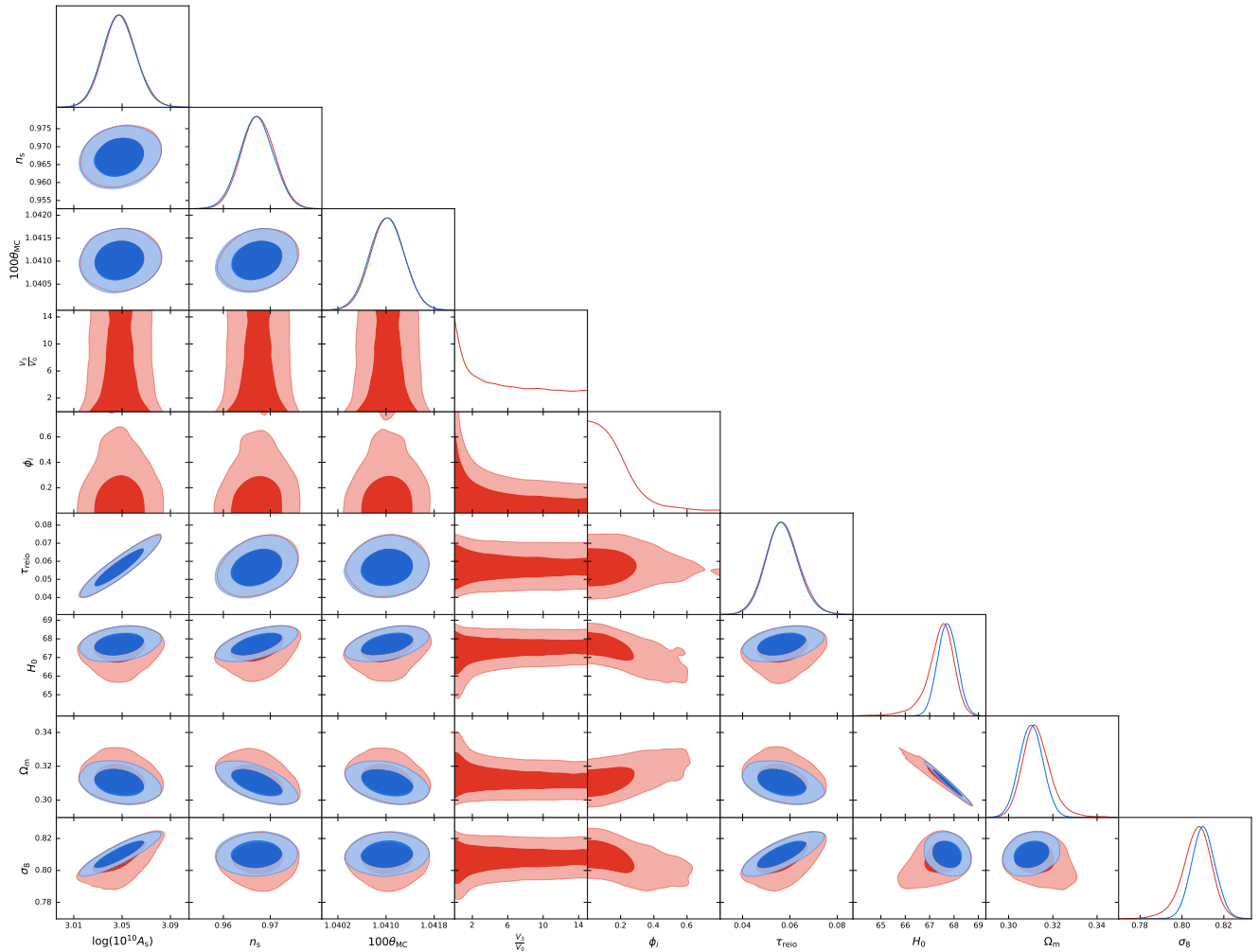
FIG. 2: Planck 2018 CMB + SN + BAO 1σ (dark region) and 2σ (light region) contours for the ΛCDM model (blue) and the inflection point quintessence model (red) with the cubic potential of Eq. (5), as a function of V_3/V_0 and ϕ_i.

Fig. 3 depicts the evolution of w as a function of the scale factor a for different parameter values along the 2σ contour. These curves illustrate the fact that inflection point quintessence cannot be treated as either a purely freezing or purely thawing model. Instead, the scalar field begins with $w = -1$. As the field rolls downhill, w increases (thawing behavior) eventually reaching a maximum value. Then as the field approaches the inflection point, w decreases toward -1 (freezing behavior).

These results further illustrate the extent to which current observations drive this model toward ΛCDM. Even at the 2σ level, the value of w never increases beyond -0.95. These curves also illustrate the existence of a "pivot redshift" $z_p = 0.37$ noted by Alam et al. [52]. This redshift corresponds to $a_p = 0.73$. The supernova data are most strongly constraining at this value of a, causing all of our curves to pass through the same value of w near this point.

In Fig. 4, we address the key question of interest for this model: do current observations allow both types of asymptotic behavior for the scalar field? The red region in this figure corresponds to the parameter values for which ϕ evolves through the inflection point, rather than evolving asymptotically to $\phi = 0$. The observations clearly favor the latter behavior; over most of the allowed parameter range, $\phi \to 0$ asymptotically. However, for large values of V_3/V_0, there is an observationally-allowed range of values for ϕ_i for which ϕ evolves through the inflection point. Thus, the cubic inflection point model can support a transient accelerated expansion that is compatible with current observations.
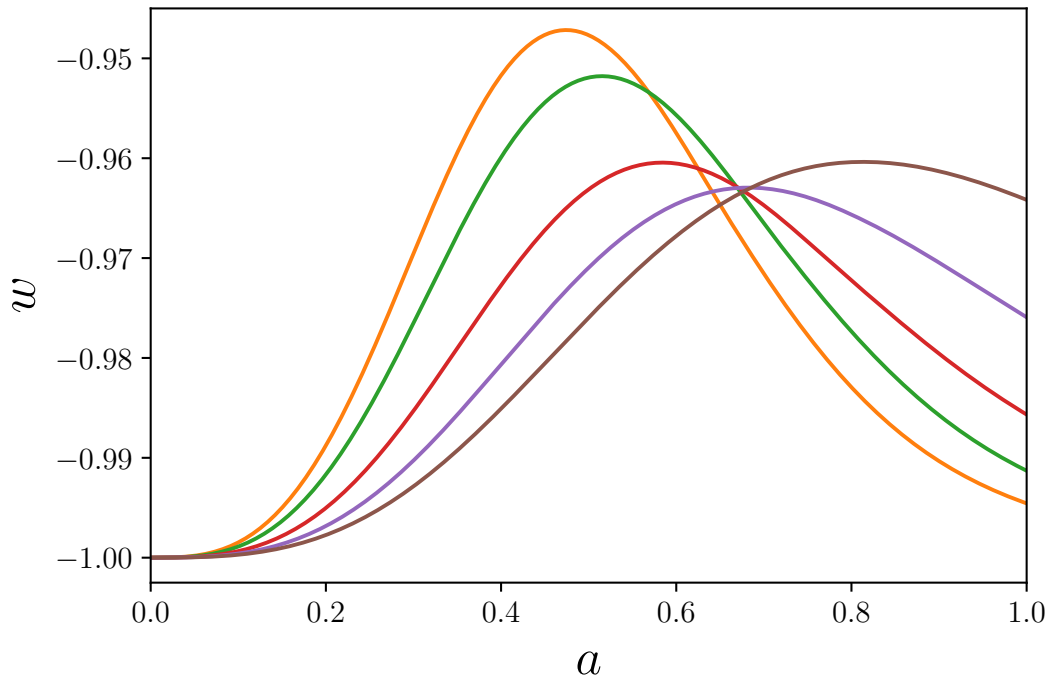
FIG. 3: Evolution of the inflection point quintessence equation of state parameter w as a function of the scale factor a, normalized to $a = 1$ at present, for five representative values of the model parameters. The color of each curve corresponds to the color of the corresponding point in parameter space indicated in Fig. 1.
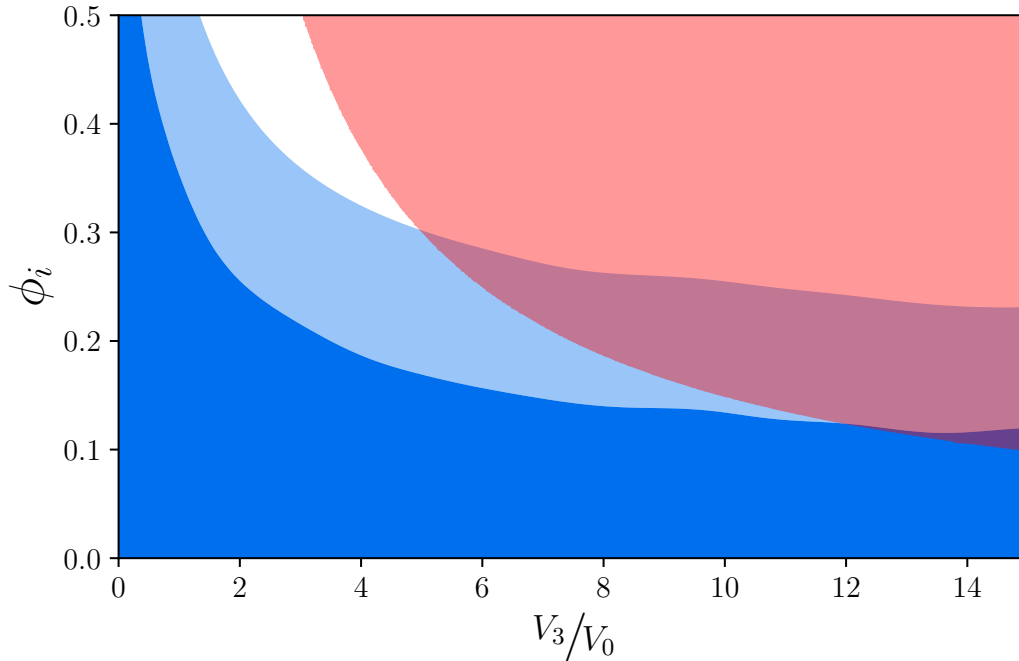

FIG. 4: Planck 2018 CMB + SN + BAO 1σ (dark blue) and 2σ (light blue) contours for the inflection point quintessence model with the region (red) overlaid atop the parameter space where ϕ will eventually pass through the inflection point.

IV. RESULTS

Our results indicate that the inflection point model with the cubic potential given by Eq. (5) can be made consistent with present observational constraints. This model is poorly described as either a thawing or freezing model; the equation of state parameter w increases from -1 to a maximum value and then decreases back toward -1. However, even for models that are nearly ruled out at the 2σ level, w never evolves above a value of -0.95, as current observations continue to drive acceptable models closer to ΛCDM. While most of the observationally-allowed parameter space corresponds to a value of ϕ that asymptotes to $\phi = 0$, there is a small region of parameter space for which ϕ can evolve through the inflection point, corresponding to the interesting case of transient acceleration.

These two cases result in very different predictions for the future evolution of the Universe. When ϕ evolves asymptotically to 0, the future evolution of the Universe becomes indistinguishable from ΛCDM. Alternately, when ϕ evolves through the inflection point, the future evolution depends on the shape of the potential for $\phi < 0$. If we simply extrapolate Eq. (5) to negatives values of ϕ, we see that $V(\phi)$ eventually becomes negative. The evolution of the universe with negative quintessence potentials has long been studied [58, 59]. When the total energy density (matter plus quintessence) reaches zero, the universe stops expanding and begins a contracting phase. As noted in Refs. [60–62], this provides a natural way to construct a cyclic model for the Universe.

The cubic potential is by no means the only model that can correspond to inflection point quintessence. Any model of the form

$$V(\phi) \;=\; V_0 + V_n \, \mathrm{sgn}(\phi)\phi^n, \quad n \text{ even}, \tag{11}$$

$$V(\phi) \;=\; V_0 + V_n\phi^n, \quad n \text{ odd}, \tag{12}$$

with $n \geq 2$ has an inflection point at $\phi = 0$ and can therefore serve as a model for inflection point quintessence. While the cubic model is the simplest and most natural inflection point quintessence model, these other models can also yield interesting behavior, resulting in either asymptotic de Sitter evolution or transient acceleration, depending on the value of n and the particular model parameters [17]. We have not attempted a detailed analysis of these other models, but we expect qualitatively similar results for the allowed parameter ranges; current observations will tend to favor small values of either ϕ_i or V_n/V_0.

Acknowledgments

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