

# Robust Beamforming Design for Rate Splitting Multiple Access-Aided MISO Visible Light Communications

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**Abstract**—In this paper, we focus on the optimal beamformer design for rate splitting multiple access (RSMA)-aided multiple-input single-output (MISO) visible light communication (VLC) networks. First, we derive the closed-form lower bounds of the achievable rate of each user, which are the first theoretical bound of achievable rate for RSMA-aided VLC networks. Second, we investigate the optimal beamformer design for RSMA-aided VLC networks to maximize the sum rate under the optical and electrical power constraints. In addition, we show that the proposed RSMA-aided networks can achieve superior performance compared with space-division multiple access (SDMA) and non-orthogonal multiple access (NOMA).

**Index Terms**—Visible light communication, beamformer design.

## I. INTRODUCTION

By utilizing the deployed light emitting diode (LED) as the transmitter, VLC can provide illumination and high-speed communication simultaneously, without introducing interference to radio-frequency (RF) communications [1], [2]. Rate-splitting multiple access (RSMA) was proposed [3], which performs linearly precoded rate splitting at the transmitter and successive interference cancellation (SIC) at the receivers. RSMA is a promising solution for VLC networks. Two existing works have initiated the study of the RSMA design in VLC networks [4], [5]. However, the achievable rate of RSMA-aided VLC networks is still unknown due to the distinct characteristics of VLC. To be specific, both peak and average optical power in VLC networks should be limited for eye safety and practical illumination considerations [6], [7].

Motivated by the limitations of existing works on RSMA-aided VLC networks [3]–[7], we focus on the two key fundamental issues of RSMA-aided VLC networks: identifying achievable rates and developing both optimal and robust beamformer design. Specifically, by considering practical power constraints, i.e., peak optical power constraint, average optical power constraint, and average electrical power constraint, we derive the lower bound of achievable rates of users in RSMA-aided VLC networks. Based on the derived lower bound of achievable rates, we investigate the optimal beamformer design for RSMA-aided VLC networks to maximize the sum rate under the optical and electrical power constraints of LEDs. Simulation results show that the proposed algorithms of

RSMA-aided VLC networks can achieve superior performance compared with several baseline schemes.

## II. SYSTEM MODEL OF DOWNLINK RSMA-AIDED VLC NETWORK

For the considered downlink RSMA-aided VLC network, the VLC base station (BS) equipped with  $N$  LEDs simultaneously serves  $K$  single-access point users by adopting RSMA scheme. Specifically, at BS, the message  $M_k$  intended to user- $k$  is split into a common message  $M_{k,0}$  and a private message  $M_k^p$ ,  $\forall k \in \mathcal{K}$ . Then, all the common messages of  $K$  users  $\{M_{k,0}\}_{k=1}^K$  are combined into a super common message  $M_0$ , i.e.,  $M_0 \triangleq \{M_{1,0}, \dots, M_{K,0}\}$ . In addition, these  $K+1$  signals  $\{s_i\}_{i=0}^K$  satisfy  $|s_i| \leq A_i$ ,  $\mathbb{E}\{s_i\} = 0$  and  $\mathbb{E}\{s_i^2\} = \varepsilon_i$ ,  $\forall i \in \mathcal{I}$ , where  $A_i > 0$  denotes the signal amplitude. Therefore, the transmitted signal vector  $\mathbf{x} = [x_1, \dots, x_N]^T$  of VLC BS is given by

$$\mathbf{x} = \sum_{i=0}^K \mathbf{w}_i s_i + \mathbf{b}. \quad (1)$$

The average electrical power and optical power of the transmitted signal  $\mathbf{x}$  are respectively given by

$$P_e = \mathbb{E}\{\|\mathbf{x}\|^2\} = \sum_{i=0}^K \varepsilon_i \|\mathbf{w}_i\|^2 + Nb^2, \quad (2a)$$

$$P_o^{\text{ave}} = \mathbb{E}\{\mathbf{x}\} = Nb. \quad (2b)$$

Let  $I_H$  denotes the maximum permissible current of LEDs, the beamformer  $\mathbf{w}_i$  should satisfy

$$\sum_{i=0}^K A_i \mathbf{w}_i^T \mathbf{e}_n + b \leq I_H, \forall n \in \mathcal{N}, \quad (3)$$

where  $\mathbf{e}_n$  is a unit vector with the  $n$ th element equal to 1.

The optical channel between LED and user is dominated by line-of-sight (LOS) link, while diffuse links can be neglected [8], [9]. Specifically, let  $\mathbf{g}_k \triangleq [g_{k,1}, \dots, g_{k,N}]^T$  denote the channel gain vector of user- $k$ , where  $g_{k,n}$  is the channel gain between the  $n$ th LED and the user- $k$ . Therefore, at user- $k$ , the

received signal  $y_k$  is given as

$$y_k = \underbrace{\mathbf{g}_k^T \mathbf{w}_0 s_0}_{\text{common signal}} + \underbrace{\mathbf{g}_k^T \mathbf{w}_k s_k}_{\text{private signal}} + \underbrace{\sum_{i=1, i \neq k}^K \mathbf{g}_k^T \mathbf{w}_i s_i + \mathbf{g}_k^T \mathbf{b}}_{\text{interference}} + z_k, \quad (4)$$

where  $z_k$  is the received noise which follows the Gaussian distribution with mean zero and covariance  $\sigma_k^2$ . The term  $\mathbf{g}_k^T \mathbf{b}$  denotes the DC component, which can be removed by the capacitor.

Thus, the residual received signal of user- $k$  after SIC process can be represented

$$y_k^{\text{SIC}} = \underbrace{\mathbf{g}_k^T \mathbf{w}_k s_k}_{\text{desired private signal}} + \underbrace{\sum_{i=1, i \neq k}^K \mathbf{g}_k^T \mathbf{w}_i s_i + \mathbf{g}_k^T \mathbf{b}}_{\text{interference}} + z_k. \quad (5)$$

### III. DOWNLINK RSMA-AIDED VLC NETWORK

Let  $R_{k,c}$  denotes the achievable rate of decoding common signal  $s_0$ , and its lower bound is given by

$$R_{k,c} \geq \frac{1}{2} \log_2 \left( \frac{2\pi\sigma_k^2 + \sum_{i=0}^K |\mathbf{g}_k^T \mathbf{w}_i|^2 e^{1+2(\alpha_i + \gamma_i \varepsilon_i)}}{2\pi\sigma_k^2 + 2\pi \sum_{j=1}^K |\mathbf{g}_k^T \mathbf{w}_j|^2 \varepsilon_j} \right). \quad (6)$$

Let  $R_{k,p}$  denote the achievable rate of decoding private signal  $s_k$ , and its lower bound is given by

$$R_{k,p} \geq \frac{1}{2} \log_2 \left( \frac{2\pi\sigma_k^2 + \sum_{i=1}^K |\mathbf{g}_k^T \mathbf{w}_i|^2 e^{1+2(\alpha_i + \gamma_i \varepsilon_i)}}{2\pi\sigma_k^2 + 2\pi \sum_{j=1, j \neq k}^K |\mathbf{g}_k^T \mathbf{w}_j|^2 \varepsilon_j} \right). \quad (7)$$

To ensure that  $s_0$  is successfully decoded by all users, the transmission rate of common message should not exceed  $R_c = \min \{R_{1,c}, \dots, R_{K,c}\}$ . Let  $c_k \geq 0$  denotes one portion of achievable rate of common message of user- $k$ , where  $\sum_{k=1}^K c_k = R_c$  [3]. Therefore, the achievable rate of user- $k$ , denoted as  $R_k$ , can be expressed by

$$R_k = c_k + R_{k,p}, \quad \forall k \in \mathcal{K}. \quad (8)$$

#### A. Optimal Beamformer Design

Mathematically, the sum rate maximization problem of RSMA-aided VLC networks can be formulated as the follow-

ing optimization problem

$$\max_{\{\mathbf{w}_i\}_{i=0}^K, \{c_k\}_{k=1}^K} \sum_{k=1}^K R_k \quad (9a)$$

$$\text{s.t.} \sum_{k=1}^K c_k \leq R_{k,c}, \quad c_k \geq 0, \quad \forall k \in \mathcal{K}, \quad (9b)$$

$$\sum_{i=0}^K \|\mathbf{w}_i\|^2 \varepsilon_i \leq P_t, \quad (9c)$$

$$\sum_{i=0}^K A_i \mathbf{w}_i^T \mathbf{e}_n \leq \min \{b, I_H - b\}, \quad \forall n \in \mathcal{N}. \quad (9d)$$

Since the objective function is non-convex, problem (9) is computationally intractable. To transform problem (9) into tractable and equivalent problem, we first introduce a number of auxiliary variables as follows

$$\hat{\mathbf{w}} \triangleq [\mathbf{w}_0^T, \dots, \mathbf{w}_K^T]^T, \quad (10a)$$

$$\hat{\mathbf{d}} \triangleq [\varepsilon_0^{1/2}, \dots, \varepsilon_K^{1/2}]^T \otimes \mathbf{1}_N, \quad (10b)$$

$$\mathbf{a}_n \triangleq [A_0, \dots, A_K]^T \otimes \mathbf{e}_n, \quad (10c)$$

$$\mathbf{G}_{c,k} \triangleq \text{diag} \{\tau_0, \dots, \tau_K\} \otimes (\mathbf{g}_k \mathbf{g}_k^T), \quad (10d)$$

$$\overline{\mathbf{G}}_{c,k} \triangleq 2\pi \text{diag} \{0, \varepsilon_1, \dots, \varepsilon_K\} \otimes (\mathbf{g}_k \mathbf{g}_k^T), \quad (10e)$$

$$\hat{\mathbf{G}}_{c,k} \triangleq \text{diag} \{0, \tau_1, \dots, \tau_K\} \otimes (\mathbf{g}_k \mathbf{g}_k^T), \quad (10f)$$

$$\hat{\mathbf{G}}_{p,k} \triangleq 2\pi \text{diag} \{0, \varepsilon_1, \dots, \varepsilon_{i-1}, 0, \varepsilon_{i+1}, \dots, \varepsilon_K\} \otimes (\mathbf{g}_k \mathbf{g}_k^T), \quad (10g)$$

where  $\mathbf{1}_N$  is a  $N \times 1$  vector with all the elements equal 1, and  $\tau_i = e^{1+2(\alpha_i + \gamma_i \varepsilon_i)}$ ,  $\forall i \in \mathcal{I}$ .

Based on the definitions (10), problem (9) can be equivalently reformulated as follows

$$\max_{\{\hat{\mathbf{w}}, c_k\}_{k=1}^K} \sum_{k=1}^K t_k \quad (11a)$$

$$\text{s.t.} \frac{1}{2} \log_2 \left( 2\pi\sigma_k^2 + \hat{\mathbf{w}}^T \hat{\mathbf{G}}_{c,k} \hat{\mathbf{w}} \right) - \frac{1}{2} \log_2 \left( 2\pi\sigma_k^2 + \hat{\mathbf{w}}^T \hat{\mathbf{G}}_{p,k} \hat{\mathbf{w}} \right) \geq t_k - c_k, \quad (11b)$$

$$\frac{1}{2} \log_2 \left( 2\pi\sigma_k^2 + \hat{\mathbf{w}}^T \mathbf{G}_{c,k} \hat{\mathbf{w}} \right) - \frac{1}{2} \log_2 \left( 2\pi\sigma_k^2 + \hat{\mathbf{w}}^T \overline{\mathbf{G}}_{c,k} \hat{\mathbf{w}} \right) \geq \sum_{k=1}^K c_k, \quad (11c)$$

$$c_k \geq 0, \quad \forall k \in \mathcal{K}, \quad (11d)$$

$$\|\hat{\mathbf{w}} \odot (\hat{\mathbf{d}} \hat{\mathbf{d}}^T)\|^2 \leq P_t, \quad (11e)$$

$$\hat{\mathbf{w}}^T \mathbf{a}_n \mathbf{a}_n^T \hat{\mathbf{w}} \leq \min \{b^2, (I_H - b)^2\}, \quad \forall n \in \mathcal{N}, \quad (11f)$$

where  $t_k$  is an auxiliary variable,  $\forall k \in \mathcal{K}$ .

Then, we employ the SDR technique to relax the constraints of problem (11). Constraints (11b) and (11c) can be respec-

tively rewritten as follows

$$\underbrace{\frac{1}{2}\log_2\left(2\pi\sigma_k^2 + \text{Tr}(\widehat{\mathbf{W}}\widehat{\mathbf{G}}_{c,k})\right)}_{\text{concave}} - \underbrace{\frac{1}{2}\log_2\left(2\pi\sigma_k^2 + \text{Tr}(\widehat{\mathbf{W}}\widehat{\mathbf{G}}_{p,k})\right)}_{\text{concave}} \geq t_k - c_k, \quad (12a)$$

$$\underbrace{\frac{1}{2}\log_2\left(2\pi\sigma_k^2 + \text{Tr}(\widehat{\mathbf{W}}\widehat{\mathbf{G}}_{c,k})\right)}_{\text{concave}} - \underbrace{\frac{1}{2}\log_2\left(2\pi\sigma_k^2 + \text{Tr}(\widehat{\mathbf{W}}\widehat{\mathbf{G}}_{c,k})\right)}_{\text{concave}} \geq \sum_{k=1}^K c_k. \quad (12b)$$

Then, by using the first-order Taylor series expansion, we have

$$\frac{1}{2}\log_2\left(2\pi\sigma_k^2 + \text{Tr}(\widehat{\mathbf{W}}\widehat{\mathbf{G}}_{c,k})\right) - L_{p,k}\left(\widehat{\mathbf{W}}\right) \geq t_k - c_k, \quad (13a)$$

$$\frac{1}{2}\log_2\left(2\pi\sigma_k^2 + \text{Tr}(\widehat{\mathbf{W}}\widehat{\mathbf{G}}_{c,k})\right) - L_{c,k}\left(\widehat{\mathbf{W}}\right) \geq \sum_{k=1}^K c_k, \quad (13b)$$

where  $L_{p,k}$  and  $L_{c,k}$  are linear functions of variable  $\widehat{\mathbf{W}}$ , which are given by

$$L_{p,k}\left(\widehat{\mathbf{W}}\right) = \frac{1}{2}\log_2\left(2\pi\sigma_k^2 + \text{Tr}(\widehat{\mathbf{W}}^{[m]}\widehat{\mathbf{G}}_{p,k})\right) + \frac{\text{Tr}\left(\widehat{\mathbf{G}}_{p,k}(\widehat{\mathbf{W}} - \widehat{\mathbf{W}}^{[m]})\right)}{\left(2\pi\sigma_k^2 + \text{Tr}(\widehat{\mathbf{W}}^{[m]}\widehat{\mathbf{G}}_{p,k})\right) 2 \ln 2}, \quad (14a)$$

$$L_{c,k}\left(\widehat{\mathbf{W}}\right) = \frac{1}{2}\log_2\left(2\pi\sigma_k^2 + \text{Tr}(\widehat{\mathbf{W}}^{[m]}\widehat{\mathbf{G}}_{c,k})\right) + \frac{\text{Tr}\left(\widehat{\mathbf{G}}_{c,k}(\widehat{\mathbf{W}} - \widehat{\mathbf{W}}^{[m]})\right)}{\left(2\pi\sigma_k^2 + \text{Tr}(\widehat{\mathbf{W}}^{[m]}\widehat{\mathbf{G}}_{c,k})\right) 2 \ln 2}, \quad (14b)$$

where  $\widehat{\mathbf{W}}^{[m]}$  is a feasible point obtained from the  $m$ th iteration.

Therefore, at the  $(m+1)$ th iteration, the convex approximation form of problem (11) is given as

$$\max_{\widehat{\mathbf{W}}} \sum_{k=1}^K t_k \quad (15a)$$

s.t.(13a), (13b)

$$\text{Tr}\left(\widehat{\mathbf{W}} \odot (\widehat{\mathbf{d}}\widehat{\mathbf{d}}^T)\right) \leq P_t, \quad (15b)$$

$$\text{Tr}\left(\widehat{\mathbf{W}}\mathbf{a}_n\mathbf{a}_n^T\right) \leq \min\left\{b^2, (I_H - b)^2\right\}, \forall n \in \mathcal{N}, \quad (15c)$$

$$\widehat{\mathbf{W}} \succeq \mathbf{0}, c_k \geq 0, \forall k \in \mathcal{K}. \quad (15d)$$

#### IV. SIMULATION RESULTS AND DISCUSSION

Consider a RSMA-aided multi-user VLC network deployed in a room with a size of  $7 \times 7 \times 5\text{m}^3$ , where the unit of

distances is meter.

Fig. 1 (a) shows the sum rate (bits/sec/Hz) of  $U_1$  and  $U_2$  versus the transmitted power budget  $P_t$  (dB) of the RSMA, NOMA and SDMA schemes. It can be observed that the sum rate of the three schemes increases as  $P_t$  increases, and the sum rate of RSMA is higher than that of NOMA and SDMA schemes. Furthermore, Fig. 1 (b) depicts the sum rate of two users  $U_3$  and  $U_4$  for different transmitted power budget  $P_t$  of the RSMA, NOMA and SDMA schemes. From Fig. 1 (b), we observe that the RSMA scheme always outperforms both the NOMA and SDMA schemes, which is similar to that in Fig. 1 (a).

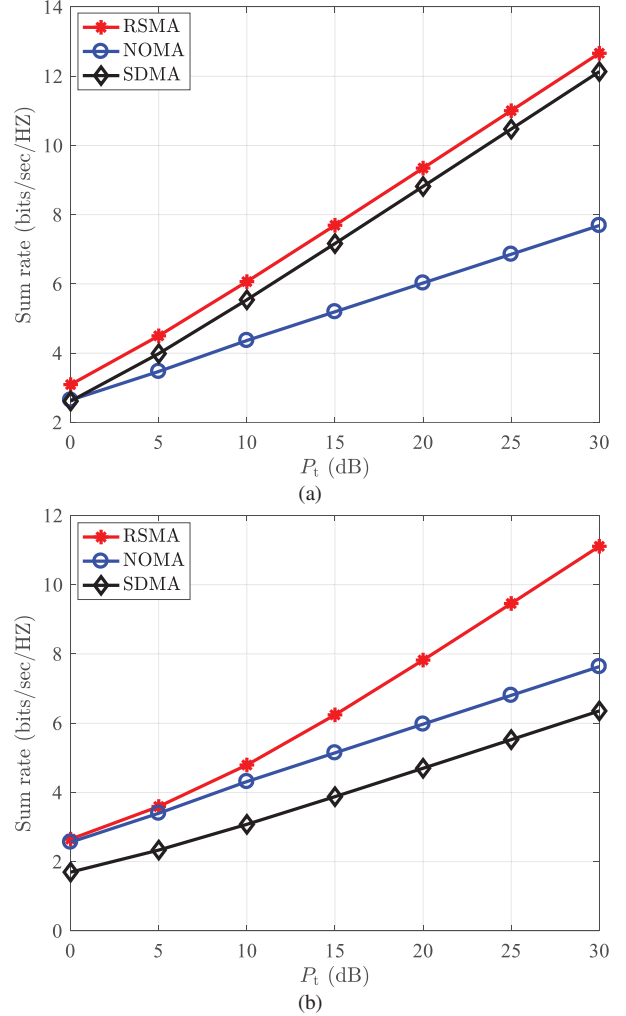


Fig. 1. (a) The sum rate (bits/sec/Hz) versus  $P_t$ (dB) with perfect CSIT, where  $N = 4$ ,  $K = 2$  users are located at  $U_1$  and  $U_2$ ; (b) The sum rate (bit/sec/Hz) versus  $P_t$  (dB) with perfect CSIT, where  $N = 4$ ,  $K = 2$  users are located at  $U_3$  and  $U_4$ ;

#### V. CONCLUSION

In this work, we addressed optimal and robust beamformer design allocation for RSMA-aided VLC networks. Specifically, we derived the lower bound of achievable rates of RSMA-aided VLC networks. Moreover, based on the derived

bound of achievable rates, we investigated the optimal beamformer design to maximize the sum rate under the optical and electrical power constraints of LEDs. Simulation results illustrated the superiority of the proposed beamformer design and provide useful insights on the design of RSMA-aided VLC networks.

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