On the Occurrence of Finite-time Singularities in Swampland-related Quintessence Dark Energy Models

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In this work we focus on the phase space singularities of interactive quintessence model in the presence of matter fluid. This model is related to swampland studies, that the outcomes affect all these Swampland related models with the same dynamical system. We shall form the dynamical system corresponding to the cosmological system, which is eventually autonomous, and by using the dominant balances technique we shall investigate the occurrence or not of finite-time singularities. Our results indicate that the dynamical system of the model may develop finite-time singularities, but these are not general singularities, like in the case that the matter fluids were absent, in which case singularities occurred for general initial conditions. Hence, the presence of matter fluids affects the dynamical system of the cosmological system, making the singularities to depend on the initial conditions, instead of occurring for general initial conditions.

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I. INTRODUCTION

Undoubtedly string theory in its various M-theory forms seems to be the perfect candidate for describing the UVcompletion of the Standard Model of particle physics and gravity. However the predictions of string theory are highly unlikely to be ever observed experimentally in terrestrial experiments. Therefore, although a beautiful theory, it seems that for the time being remains just a candidate UV-completion theory. Nevertheless, string theory may constrain the high energy limits of the classical theory. One such example are the Swampland criteria, firstly introduced in Refs. [\[1,](#page-7-0) [2](#page-7-1)] and were further developed in Refs. [\[3](#page-7-2)[–45,](#page-8-0) [47](#page-8-1), [48](#page-8-2), [52\]](#page-9-0), see also [\[49](#page-9-1)[–51\]](#page-9-2) for the Swampland criteria implications on the H_0 tension problem. The Swampland criteria basically constrain several scalar field parameters at low-energies compared to the mother M-theory. In a previous work [\[52](#page-9-0)] we studied the singularity structure of the dynamical system corresponding to scalar field theories used for the Swampland models in vacuum. In this paper we extend the work [\[52](#page-9-0)] to include perfect matter fluids, and specifically dark matter and radiation fluids. The corresponding dynamical system is perfectly studied in [\[8](#page-7-3)]. Our aim is to see whether the matter fluids affect the finite-time singularity structure of the phase space of the scalar field model. As we show, indeed the matter fluids affect the finite-time structure of the phase space of the model, and in the present case scenario, singularities typically exist, but only for limited sets of initial conditions in the phase space. This is in contrast with the results obtained in our previous work [\[52](#page-9-0)], where finite-time singularities occurred for a general set of initial conditions.

In the present work, we use natural units $c = \hbar = 1$ and the metric at use is the flat Friedmann Robertson Walker (FRW) metric whose line element is given by

$$
ds^{2} = -dt^{2} + a(t) \sum_{i=1}^{3} dx_{i}^{2},
$$
\n(1)

where $a(t)$ is the scale factor. The Ricci scalar for the FRW metric is

$$
R = 6\dot{H} + 12H^2,\tag{2}
$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. We also adopt reduced Planck units, that is $\kappa = \frac{1}{M}$ $\frac{1}{M_p}=1.$

II. ESSENTIAL FEATURES OF DOMINANT BALANCES METHODOLOGY

Our aim is to study the finite-time singularities structure of the dynamical system developed in [\[8\]](#page-7-3). To this end we shall use the method of dominant balances, see Refs. [\[52\]](#page-9-0) and [\[53](#page-9-3)] and references therein for more details. We brief it here for reasons of completeness.

• Consider a dynamical system of n differential equations of the form

$$
\dot{x}_i = f_i(x),\tag{3}
$$

where $i = 1, 2, \ldots, n$. Approaching the region of the singularity, we can extract from f_i the part that becomes considerable, we shall call it from now on dominant part. This dominant part constitutes a mathematically consistent truncation of the system and denote it as \hat{f}_i . Now, [\(3\)](#page-0-0) has become

$$
\dot{x}_i = \hat{f}_i(x). \tag{4}
$$

We should note that the dot denotes differentiation with respect to time t . In our case, in spite of t the e-foldings number N is used, but one shall not falter since the applied process is totally similar.

• Without loss of generality, the x_i 's near the singularity assume the form

$$
x_i = a_i(t - t_c)^{p_i},\tag{5}
$$

where t_c is but an integration constant. Substituting [\(5\)](#page-1-0) in [\(4\)](#page-1-1) and equating the exponents, one may find the p_i 's that constitute the vector $\mathbf{p} = (p_1, p_1, \dots, p_n)$. Having the exponents, we return to the system in order to calculate the a_i 's and similarly form the vector $\mathbf{a} = (a_1, a_2, \dots, a_n)$. If a does contain only real entries, it may give rise only to finite-time singularities while if it has complex entries, it may give rise only to non-finite-time singularities. It should be underlined that a cannot assume zero entries. Taking that into account, every set (a, p) is called a *dominant balance of the system*.

• The next thing to do is calculate the Kovalevskaya matrix, which is of the form

$$
R = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} - \begin{pmatrix} p_1 & 0 & \cdots & 0 \\ 0 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & p_n \end{pmatrix},
$$
(6)

evaluate it in one of the dominant balances found in the previous step and find its eigenvalues. The said eigenvalues have to be of the form $(-1, r_2, \ldots r_n)$. If $r_2, r_3, \ldots r_n > 0$, the singularity is general, that is independent of the initial conditions. On the other hand, even if one of these eigenvalues is negative, the singularity is local, that is dependent on the initial conditions.

III. ANALYSIS OF THE DYNAMICAL SYSTEM VIA DOMINANT BALANCES

We start with the presentation of the relevant scalar field dynamical system, just as presented in [\[8\]](#page-7-3), which is characterized by the equations

$$
3H^2 = \rho_{\phi} + \rho_{dm} + \rho_r,
$$

\n
$$
\dot{\rho}_{\phi} + 3H(\rho_{\phi} + P_{\phi}) = -\rho_{dm} - 3H\rho_{dm},
$$

\n
$$
\dot{\rho}_{\phi} + 3H(\rho_{\phi} + P_{\phi}) = -Q\rho_{dm}\dot{\phi},
$$
\n(7)

 $\rho_{dm} + 3H\rho_{dm} = Q\rho_{dm}\dot{\phi},$

where ρ_{ϕ} , ρ_{dm} , ρ_r are the energy densities of the scalar field, dark matter and radiation, respectively, P_{ϕ} is the pressure of the scalar field and Q a constant expressing the interaction between dark matter and dark energy. Considering an inflationary potential of the form $V(\phi) \sim e^{\lambda \phi}$ and introducing the dimensionless variables

$$
x_1 = \frac{\dot{\phi}}{\sqrt{6}H},\tag{8}
$$

$$
x_2 = \frac{\sqrt{V}}{\sqrt{3}H},
$$

$$
x_3 = \frac{\sqrt{\rho_{dm}}}{\sqrt{3}H},
$$

$$
x_4 = \frac{\sqrt{\rho_r}}{\sqrt{3}H},
$$

eqs. [\(7\)](#page-1-2) become

$$
\frac{dx_1}{dN} = -3x_1 - \frac{\sqrt{6}}{2}\lambda x_2^2 + \frac{1}{2}(3x_1^3 - 3x_1x_2^2 - 3x_1x_3^2 + x_1x_4^2 + 3x_1) - \frac{\sqrt{6}}{2}Q(1 - x_1^2 - x_2^2 - x_3^2 - x_4^4),
$$
(9)

$$
\frac{dx_2}{dN} = \frac{\sqrt{6}}{2}\lambda x_1 x_2 + \frac{1}{2}(3x_1^2 x_2 - 3x_2^3 - 3x_2x_3^2 + x_2x_4^2 + 3x_2),
$$

$$
\frac{dx_3}{dN} = -\frac{3}{2}x_3 + \frac{1}{2}(3x_1^2 x_3 - 3x_2^2 x_3 - 3x_3^3 + x_3x_4^2 + 3x_3),
$$

$$
\frac{dx_4}{dN} = -2x_4 + \frac{1}{2}(3x_1^2 x_4 - 3x_2^2 x_4 - 3x_3^2 x_4 + x_4^3 + 3x_4),
$$

with the Friedmann constraint being $x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1$ and $\lambda = |V'|/V$ a constant of order unity arising from the Swampland (the prime denotes differentiation with respect to the scalar field). Our next task, in order to apply the dominant balances method, is to figure out all the possible distinct mathematically consistent truncations of [\(9\)](#page-2-0); that is vectors whose entries are dominant terms from each one of the differential equations. It should be noted that since we are working on a 4-manifold, constant and linear terms cannot be dominant and are neglected whatsoever.

A. 1st mathematically consistent truncation

The first mathematically consistent truncation of [\(9\)](#page-2-0) is

$$
\hat{f}_1 = \begin{pmatrix} \frac{\sqrt{6}}{2} Q x_4^4 \\ -\frac{3}{2} x_2^3 \\ -\frac{3}{2} x_3^3 \\ \frac{1}{2} x_4^3 \end{pmatrix} \tag{10}
$$

and applying the method presented in the previous section, we easily find

$$
\mathbf{p} = \left(-\frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right) \tag{11}
$$

and the following 8 dominant balances:

$$
\mathbf{a_1} = \left(-3\sqrt{\frac{3}{2}}Q, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -i\right),
$$

\n
$$
\mathbf{a_2} = \left(-3\sqrt{\frac{3}{2}}Q, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -i\right),
$$

\n
$$
\mathbf{a_3} = \left(-3\sqrt{\frac{3}{2}}Q, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -i\right),
$$

\n
$$
\mathbf{a_4} = \left(-3\sqrt{\frac{3}{2}}Q, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -i\right),
$$
\n(12)

$$
\mathbf{a}_5 = \left(-3\sqrt{\frac{3}{2}}Q, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, i\right),
$$

$$
\mathbf{a}_6 = \left(-3\sqrt{\frac{3}{2}}Q, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, i\right),
$$

$$
\mathbf{a}_7 = \left(-3\sqrt{\frac{3}{2}}Q, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, i\right),
$$

$$
\mathbf{a}_8 = \left(-3\sqrt{\frac{3}{2}}Q, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, i\right).
$$

The vectors a_i have complex entries, so finite-time singularities cannot occur. The Kovalevskaya matrix of (6) assumes the form,

$$
R = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 2\sqrt{6}Qx_4^3\\ 0 & \frac{1}{2} - \frac{9}{2}x_2^2 & 0 & 0\\ 0 & 0 & \frac{1}{2} - \frac{9}{2}x_3^2 & 0\\ 0 & 0 & 0 & \frac{1}{2} + \frac{3}{2}x_4^2 \end{pmatrix}.
$$
 (13)

Substituting the dominant balances $(a_1, p), (a_2, p), (a_3, p)$ and (a_4, p) in [\(13\)](#page-3-0), we obtain

$$
R = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 2i\sqrt{6}Q \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},
$$
(14)

while for $(a_5, p), (a_6, p), (a_7, p)$ and (a_8, p) in [\(13\)](#page-3-0), we get

$$
R = \begin{pmatrix} \frac{1}{3} & 0 & 0 & -2i\sqrt{6}Q \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.
$$
 (15)

Both [\(14\)](#page-3-1) and [\(15\)](#page-3-2) have the same set of eigenvalues:

$$
r = \left(-1, -1, -1, \frac{1}{3}\right) \tag{16}
$$

and, since $r_2, r_3 < 0$, we conclude that only local singularities may occur.

B. 2nd mathematically consistent truncation

The second mathematically consistent truncation of [\(9\)](#page-2-0) is

$$
\hat{f}_2 = \begin{pmatrix} -\frac{3}{2}x_1x_2^2\\ \frac{\sqrt{6}}{2}\lambda x_1x_2\\ -\frac{3}{2}x_3^3\\ \frac{1}{2}x_4^3 \end{pmatrix}
$$
\n(17)

and applying the aforementioned method we easily find

$$
\mathbf{p} = \left(-1, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right) \tag{18}
$$

and the following 8 dominant balances:

$$
a_1 = \left(-\frac{1}{\sqrt{6}\lambda}, \sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}, -i\right),
$$
\n
$$
a_2 = \left(-\frac{1}{\sqrt{6}\lambda}, \sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}, i\right),
$$
\n
$$
a_3 = \left(-\frac{1}{\sqrt{6}\lambda}, \sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}, -i\right),
$$
\n
$$
a_4 = \left(-\frac{1}{\sqrt{6}\lambda}, \sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}, i\right),
$$
\n
$$
a_5 = \left(-\frac{1}{\sqrt{6}\lambda}, -\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}, -i\right),
$$
\n
$$
a_6 = \left(-\frac{1}{\sqrt{6}\lambda}, -\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}, i\right),
$$
\n
$$
a_7 = \left(-\frac{1}{\sqrt{6}\lambda}, -\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}, -i\right),
$$
\n
$$
a_8 = \left(-\frac{1}{\sqrt{6}\lambda}, -\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}, -i\right),
$$
\nsignularity of (6) assumes the form

Just like in the previous case, finite-time singularities cannot occur. The Kovalevskaya matrix of [\(6\)](#page-1-3) assumes the form

$$
R = \begin{pmatrix} 1 - \frac{3}{2}x_2^2 & -3x_1x_2 & 0 & 0 \\ \sqrt{\frac{3}{2}}\lambda x_2 & \frac{1}{2} + \sqrt{\frac{3}{2}}\lambda x_2 & 0 & 0 \\ 0 & 0 & \frac{1}{2} - \frac{9}{2}x_3^2 & 0 \\ 0 & 0 & 0 & \frac{1}{2} + \frac{3}{2}x_4^2 \end{pmatrix}.
$$
 (20)

Substituting the dominant balances $(a_1, p), (a_2, p), (a_3, p)$ and (a_4, p) in [\(20\)](#page-4-0), we obtain

$$
R = \begin{pmatrix} 0 & \frac{1}{\lambda} & 0 & 0 \\ \lambda & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},
$$
\n(21)

while for $(a_5, p), (a_6, p), (a_7, p)$ and (a_8, p) in [\(20\)](#page-4-0), we obtain

$$
R = \begin{pmatrix} 0 & -\frac{1}{\lambda} & 0 & 0 \\ -\lambda & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.
$$
 (22)

Both [\(21\)](#page-4-1) and [\(22\)](#page-4-2) have the same set of eigenvalues:

$$
r = (-1, -1, -1, 1) \tag{23}
$$

and, because $r_2, r_3 < 0$, non-general solutions are the only plausible scenario.

C. 3rd mathematically consistent truncation

The third mathematically consistent truncation of [\(9\)](#page-2-0) is the following:

$$
\hat{f}_3 = \begin{pmatrix} \frac{\sqrt{6}}{2} Q x_1^2 \\ -\frac{3x_2^3}{2} \\ \frac{x_3 x_4^2}{2} \\ -\frac{3x_3^2 x_4}{2} \end{pmatrix} . \tag{24}
$$

Using the dominant balances method, the following solution for the vector p is obtained:

$$
\mathbf{p} = \left(-1, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right). \tag{25}
$$

Accordingly, for p being the above, the following vector-solutions a_i are found:

$$
\mathbf{a}_{1} = \left(-\frac{2}{\sqrt{6}Q}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, i\right),
$$
\n
$$
\mathbf{a}_{2} = \left(-\frac{2}{\sqrt{6}Q}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -i\right),
$$
\n
$$
\mathbf{a}_{3} = \left(-\frac{2}{\sqrt{6}Q}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -i\right),
$$
\n
$$
\mathbf{a}_{4} = \left(-\frac{2}{\sqrt{6}Q}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, i\right),
$$
\n
$$
\mathbf{a}_{5} = \left(-\frac{2}{\sqrt{6}Q}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, i\right),
$$
\n
$$
\mathbf{a}_{6} = \left(-\frac{2}{\sqrt{6}Q}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -i\right),
$$
\n
$$
\mathbf{a}_{7} = \left(-\frac{2}{\sqrt{6}Q}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -i\right),
$$
\n
$$
\mathbf{a}_{8} = \left(-\frac{2}{\sqrt{6}Q}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -i\right),
$$

By calculating the Kovalevskaya matrix,

$$
R = \begin{pmatrix} \sqrt{6}Qx_1 + 1 & 0 & 0 & 0\\ 0 & -\frac{9x_2^2}{2} + \frac{1}{2} & 0 & 0\\ 0 & 0 & \frac{x_4^2}{2} + \frac{1}{2} & x_3x_4\\ 0 & 0 & -3x_3x_4 & -\frac{3x_3^2}{2} + \frac{1}{2} \end{pmatrix}
$$
(27)

we find its form for each a_i :

$$
R(\mathbf{a_1}) = R(\mathbf{a_2}) = R(\mathbf{a_3}) = R(\mathbf{a_4}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{\sqrt{3}} \\ 0 & 0 & -\sqrt{3}i & 0 \end{pmatrix},
$$
(28)

$$
R(\mathbf{a_5}) = R(\mathbf{a_6}) = R(\mathbf{a_7}) = R(\mathbf{a_8}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\ 0 & 0 & \sqrt{3}i & 0 \end{pmatrix}.
$$

Their eigenvalues are the same for all a_i :

$$
(r_1, r_2, r_3, r_4) = (-1, -1, -1, 1), \tag{29}
$$

and we see that $r_2, r_3 < 0$. Hence, there is a limited set of initial conditions which leads the dynamical system to no finite-time singularities.

D. 4th mathematically consistent truncation

The fourth mathematically consistent truncation of [\(9\)](#page-2-0) that can be obtained is

$$
\hat{f}_4 = \begin{pmatrix} \frac{\sqrt{6}}{2} Q x_1^2 \\ \frac{x_2 x_4^2}{2} \\ -\frac{3x_2^2 x_3}{2} \\ -\frac{3x_3^2 x_4}{2} \end{pmatrix} . \tag{30}
$$

For (30) , the dominant balances (a, p) are the same as for (24) . The corresponding Kovalevskaya matrix is

$$
R = \begin{pmatrix} \sqrt{6}Qx_1 + 1 & 0 & 0 & 0\\ 0 & \frac{x_4^2}{2} + \frac{1}{2} & 0 & x_2x_4\\ 0 & -3x_2x_3 & -\frac{3x_2^2}{2} + \frac{1}{2} & 0\\ 0 & 0 & -3x_3x_4 & -\frac{3x_3^2}{2} + \frac{1}{2} \end{pmatrix}
$$
(31)

and by substituting each dominant balance in [\(31\)](#page-6-1) we evaluate the corresponding eigenvalues:

$$
R(\mathbf{a_1}) = R(\mathbf{a_2}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{\sqrt{3}} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\sqrt{3}i & 0 \end{pmatrix},
$$
(32)

$$
(r_1, r_2, r_3, r_4) = (-1, -1, (-1)^{1/3}, -(-1)^{2/3}),
$$

$$
R(\mathbf{a_3}) = R(\mathbf{a_4}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{3}i & 0 \end{pmatrix},
$$
(33)

$$
(r_1, r_2, r_3, r_4) = (-1, 1, -(-1)^{1/3}, (-1)^{2/3}),
$$

$$
R(\mathbf{a_5}) = R(\mathbf{a_7}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{\sqrt{3}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\sqrt{3}i & 0 \end{pmatrix},
$$
(34)

$$
(r_1, r_2, r_3, r_4) = (1, -(-1)^{1/3}, (-1)^{2/3}, -1/2),
$$

$$
R(\mathbf{a_6}) = R(\mathbf{a_8}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\sqrt{3}i & 0 \end{pmatrix},
$$
(35)

$$
(r_1, r_2, r_3, r_4) = (-1, 1, -(-1)^{1/3}, (-1)^{2/3}).
$$

For a_1 , a_2 , a_3 , a_4 , a_6 and a_8 , [\(30\)](#page-6-0) leads to mathematically unappealing results, due to the complex entries $(-1)^{1/3}$ and $(-1)^{2/3}$. Concerning a_5 and a_7 , the first Kovalevskaya eigenvalue is $r_1 \neq -1$ and therefore no conclusion can be reached.

IV. CONCLUSIONS

In the present work we studied a Swampland related scalar field theory in the presence of matter and radiation fluids, focusing on the dynamical system that governs the cosmological system. Our aim was to find whether finitetime singularities occur in the dynamical system, and if yes, whether these correspond to general or restricted sets of initial conditions. By employing the method of dominant balances and its accompanying theorems we were able to select the dominant balances of the dynamical system and demonstrate that indeed finite-time singularities may actually occur in the system, however these are limited types of singularities, meaning that these occur for a very narrow range of initial conditions. This is in contrast with the case that the matter fluids were not present, thus the major effect of the matter fluids is to essentially remove the finite-time singularities from the phase space of the scalar field model.

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