

# Gravitational Wave Distances in Horndeski Cosmology

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Gravitational wave propagation encounters a spacetime friction from a running Planck mass in modified gravity, causing the luminosity distance to deviate from that in general relativity (or given by the photon luminosity distance to the source), thus making it a valuable cosmological probe. We present the exact expression for the cosmological distance deviation in Horndeski gravity including theories that have a  $G_5$  term yet propagate at the speed of light. An especially simple result ensues for coupled Gauss-Bonnet gravity, which we use to show it does not give a viable cosmology. We also generalize such coupling, and review the important connection of gravitational wave cosmological distance deviations to growth of cosmic structure measured by redshift space distortions.

## I. INTRODUCTION

Gravitational waves (GW) provide a new probe of cosmology as well as of gravitation in the strong field regime. For cosmology, standard siren distances from GW are a new type of distance measure, crosschecking those from photon luminosity distance (e.g. Type Ia supernovae) and angular distance (e.g. baryon acoustic oscillations). Thus they can map out the cosmic expansion history. However, the GW distance depends on the propagation of gravitational waves – if this differs from that in general relativity then one must account for this in interpretation of the distance.

GW propagation can differ from general relativity in its speed of propagation and an additional friction beyond the Hubble friction of expanding space. This new spacetime friction is due to a change in the coupling of gravity to spacetime curvature, and can be thought of as an evolving gravitational strength (Newton's constant) or running Planck mass. (Additional effects such as graviton mass and source terms can enter, but do not in the Horndeski class of gravity we consider here.)

Thus GW not only map out cosmic expansion history, but cosmic gravity history, and we will highlight as well a connection to cosmic growth history. Thus a simple expression for the deviation of GW propagation from general relativity is of interest. This is constrained by the implication of near simultaneous GW and electromagnetic bursts from GW170817 [1] indicating that the speed of GW propagation equals the speed of light, within the most direct interpretation (see [2, 3] for other possibilities). This stringently restricts the gravitation theory.

Working within the class of Horndeski theory, the most general scalar-tensor theory with second order equations of motion, the restriction to the speed of light is usually taken to remove one of the four terms in the action and prevent another from depending on the scalar field motion. This is not absolute, however, and can lead to some interesting cases.

In Section II we show how the general expression for the GW distance deviation looks in the usual interpretation and in the extended one. Section III treats a special

case of the latter situation, involving a coupling to the Gauss-Bonnet term, of particular interest since it is a geometric invariant. In Section IV and V we examine a generalization, and another special case, respectively. The extraordinary connection between GW propagation and cosmic structure growth deviations from general relativity in some theories is visited in Sec. VI and we conclude in Sec. VII.

## II. GW DISTANCE DEVIATION

The propagation equation for the GW amplitude  $h$  in a cosmological background is [4–7]

$$\ddot{h} + (3 + \alpha_M)H\dot{h} + k^2h = 0, \quad (1)$$

where an overdot is a derivative with respect to time,  $\alpha_M = d \ln M_\star^2 / d \ln a$  is the Planck mass evolution rate,  $H = \dot{a}/a$  is the Hubble parameter,  $a$  the cosmic scale factor, and  $k$  the wavenumber. As stated, we work within the Horndeski class of gravity and have already set the GW speed of propagation to the speed of light (i.e.  $c_T = 1$ ).

The emitted amplitude is predicted by general relativity based on the detected GW characteristics (and it is assumed general relativity holds in the emission process, as most viable cosmic gravity theories have screening mechanisms that restore to general relativity in regions of much higher density than the cosmic background). Comparing this to the observed amplitude gives a GW distance to the source through the cosmic inverse square distance law (energy  $\sim$  amplitude<sup>2</sup>  $\sim$  1/distance<sup>2</sup> so  $d_L \sim 1/h$ ).

A clear derivation of the solution to the GW propagation Eq. (1) was given by [7, 8], and related directly to GW distances for Horndeski gravity by [9, 10]:

$$d_{L,GW}(a) = d_L^{GR}(a) \left[ \frac{M_\star^2(a=1)}{M_\star^2(a)} \right]^{1/2}. \quad (2)$$

Thus the key quantity of interest from modified gravity is the Planck mass evolution.

In a general Horndeski theory,

$$M_\star^2 = 2 \left( G_4 - 2XG_{4X} + XG_{5\phi} - H\dot{\phi}XG_{5X} \right), \quad (3)$$

where  $G_4(\phi, X)$  and  $G_5(\phi, X)$  are two of the Horndeski Lagrangian terms (the others do not enter GW propagation),  $X = \dot{\phi}^2/2$ ,  $\phi$  is a scalar field, and subscripts  $\phi$  or  $X$  denote derivatives with respect to that quantity [11]. Conventionally when the GW propagation speed  $c_T$  is the speed of light,  $G_5 = 0$  and  $G_{4X} = 0$ , leaving just  $M_\star^2 = 2G_4(\phi)$ . For general relativity,  $G_4 = M_{pl}^2/2$  so  $M_\star^2 = M_{pl}^2$ , a constant; we will work in units such that  $M_{pl}^2 = 1$ .

Thus in the conventional case,

$$d_{L,GW}(a) = d_L^{GR}(a) \left[ \frac{G_4(\phi(a=1))}{G_4(\phi(a))} \right]^{1/2}. \quad (4)$$

However, one can also have the GW propagation speed  $c_T \equiv 1 + \alpha_T$  equal to the speed of light when

$$\alpha_T \sim 2G_{4X} - 2G_{5\phi} - (\ddot{\phi} - H\dot{\phi})G_{5X} = 0. \quad (5)$$

Substituting this into Eq. (3) gives

$$M_\star^2 = 2G_4(\phi, X) - \dot{\phi}\dot{G}_5(\phi, X), \quad (6)$$

where  $\dot{G}_i = \dot{\phi}G_{i\phi} + \dot{X}G_{iX} = \dot{\phi}\dot{G}_{i\phi} + \dot{\phi}\ddot{\phi}G_{iX}$ .

The general expression for distance deviations then becomes

$$d_{L,GW}(a) = d_L^{GR}(a) \left( \frac{[2G_4 - \dot{\phi}\dot{G}_5](a=1)}{[2G_4 - \dot{\phi}\dot{G}_5](a)} \right)^{1/2}. \quad (7)$$

Given a Horndeski theory, one solves the equations of motion (including for  $\phi(a)$ ), and can determine the GW distance deviation from general relativity.

### III. COUPLED GAUSS-BONNET GRAVITY

For certain theories within the Horndeski class, the above expressions work out particularly simply. An interesting case is coupled Gauss-Bonnet gravity, demonstrated to be part of the Horndeski class in [12]. The Gauss-Bonnet invariant

$$\mathcal{G} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}, \quad (8)$$

is a particular scalar combination of the Riemann, Ricci tensor, and Ricci scalar curvatures, and so an important geometric object. Being a topological term, by itself it does not alter the equations of motion from general relativity, however promoting it to a function  $f(\mathcal{G})$  or coupling it to a scalar field as  $f(\phi)\mathcal{G}$  in the action gives it dynamics and hence modifies gravity.

We consider the latter case, with the action being the usual Ricci scalar plus a  $f(\phi)\mathcal{G}$  term. There can be kinetic and potential terms of the scalar field as well, but

they will not affect GW propagation. As coupled Gauss-Bonnet (CGB) gravity possesses a  $G_5$  term and a  $G_4$  term depending on  $X$ , normally it has a GW speed of propagation different from the speed of light. However, [13, 14] demonstrated the condition such that a restricted case survives: the CGB speed of GW propagation will be the speed of light if the coupling satisfies

$$\ddot{f} = H\dot{f}. \quad (9)$$

If we then evaluate Eq. (3) or (6) for the CGB terms

$$G_4 = \frac{1}{2} + 4f_{\phi\phi}X(2 - \ln X), \quad G_5 = -4f_\phi \ln X, \quad (10)$$

then we obtain

$$M_\star^2 = 1 + 8H\dot{f} = 1 + 8H\dot{\phi}f_\phi. \quad (11)$$

Eq. (9) can be readily solved to give  $\dot{f} = ca$ , where  $c$  is a constant. Note that we never need to solve the scalar field equation of motion – this is a model independent result! (If we are given  $f(\phi)$ , then we can find  $\phi(a)$  through  $f_\phi\dot{\phi} = ca$ .)

Thus, independent of the specific coupling,

$$d_{L,GW}(a) = d_L^{GR}(a) \left( \frac{1 + 8cH_0}{1 + 8caH} \right)^{1/2}. \quad (\text{CGB}) \quad (12)$$

Given a cosmological background expansion, i.e. Hubble parameter  $H(a)$ , which may depend on the coupling form, as well as kinetic and potential terms, one has an exact prediction for the GW distance deviation.

Note that since  $M_\star^2 = 1 + 8caH = 1 + 8c\dot{a}$ , then  $M_\star^2$  blows up as  $a \rightarrow 0$ . We explore this further below. CGB is also sometimes used as an inflation theory, and note that for  $H$  constant the solution to Eq. (9) is  $\dot{f} = ce^{Ht}$  and  $M_\star^2 = 1 + 8cH_0e^{H_0t}$ .

Let us examine some of the other gravity theory quantities, such as the Planck mass evolution rate  $\alpha_M$ . We have

$$\alpha_M \equiv \frac{\dot{M}_\star^2}{HM_\star^2} = \frac{8c\dot{a}}{H(1 + 8c\dot{a})}. \quad (13)$$

Note that

$$\alpha_M \rightarrow \frac{a\ddot{a}}{\dot{a}^2} = -q, \quad (a \rightarrow 0) \quad (14)$$

where  $q$  is the cosmic deceleration parameter. Thus, in CGB  $\alpha_M$  directly measures the acceleration of the universe. (And since we do not have  $\alpha_M \rightarrow 0$ , general relativity is never fully restored in the early universe.) For the inflation case we indeed find  $\alpha_M = 1$ .

Since in our universe  $\ddot{a}$  changes sign as the expansion moves from matter domination to our present accelerated epoch, then there will be a time when  $\alpha_M$  crosses zero. Recall that  $\alpha_M = 0$  is referred to as No Run Gravity [15] (when it is a persistent condition) and gives zero

gravitational slip. That is an interesting, if momentary, property of CGB.

The property functions such as  $\alpha_M$  and  $\alpha_T$  give a useful perspective on observational effects of modified gravity [11]. Computing the function  $\alpha_B$  that describes the braiding, or mixing between the scalar kinetic term and the metric,

$$\alpha_B = \frac{-8H\dot{f}}{M_\star^2} = \frac{-8caH}{1+8caH}. \quad (15)$$

We see that this has  $\alpha_B \rightarrow -1$  at early times, and this holds for the inflationary solution as well. This means that in early matter domination  $\alpha_B = 2\alpha_M = -1$ , in radiation domination  $\alpha_B = \alpha_M = -1$ , and in inflation  $\alpha_B = -\alpha_M = -1$ . The last relation is characteristic of  $f(R)$  gravity as well. The remaining property function  $\alpha_K$ , the kineticity, does not have much observable impact, affecting the spatial clustering of the scalar field. We find it depends on  $f_{\phi\phi\phi\phi}$ , and hence is not model independent.

The gravitational strength, relative to Newton's constant in general relativity, entering the growth of structure is denoted  $G_{\text{matter}}$ , and that for light propagation is  $G_{\text{light}}$ ; the difference between them is referred to as the gravitational slip. Both involve combinations of  $\alpha_M$  and  $\alpha_B$ ; their expressions can be found in, for example, [16]. Evaluating for CGB,

$$G_{\text{matter}} = \frac{2H\ddot{a} - H^2\dot{a} - 8c(H^2\dot{a}^2 - 2\ddot{a}^2)}{H(1+8c\dot{a})[2\ddot{a} - H\dot{a} - 4c(3H\dot{a}^2 - 4\ddot{a}\dot{a})]} \quad (16)$$

$$G_{\text{light}} = \frac{2H\ddot{a} - H^2\dot{a} + 4cH\dot{a}\ddot{a} - 8c(H^2\dot{a}^2 - \ddot{a}^2)}{H(1+8c\dot{a})[2\ddot{a} - H\dot{a} - 4c(3H\dot{a}^2 - 4\ddot{a}\dot{a})]} \quad (17)$$

Note that when  $c \rightarrow 0$ , and the coupling vanishes, then  $G_{\text{matter}} \rightarrow 1$ ,  $G_{\text{light}} \rightarrow 1$ , i.e. we recover general relativity.

In the matter dominated or radiation dominated eras (or any with background equation of state  $w_b > -1/3$ ),  $G_{\text{matter}}$  and  $G_{\text{light}}$  approach zero going into the past. This is not surprising since we found that  $\alpha_M$  and  $\alpha_B$  go to constants, and  $M_\star^2$  in the denominator blows up. This vanishing of gravity does not make for a viable cosmology. If the dominant component has  $w_b < -1/3$  then  $G_{\text{matter}}$  and  $G_{\text{light}}$  approach one going into the past. For inflation, with  $w_b = -1$ , going back into the past  $G_{\text{matter}}$  and  $G_{\text{light}}$  will be one as in general relativity, but when inflation lasts more than a few e-folds, i.e.  $ce^{H_i t}$  gets large, then again gravity vanishes.

Thus there is no valid inflation nor late time cosmology for coupled Gauss-Bonnet gravity with GW propagation at the speed of light, independent of the model, i.e. coupling function. (One could use it for inflation with GW speed  $c_T \neq 1$ , but the gravitation theory must somehow change by the late universe.)

#### IV. GENERALIZING THE COUPLING

While coupled Gauss-Bonnet gravity has some attractive features, such as use of the geometric invariant, it

did not give rise to a viable cosmology with GW speed  $c_T = 1$ . Let us explore whether we can keep some useful features to find the GW distance deviation in a viable theory. In setting  $\alpha_T = 0$ , CGB led to the constraint on the coupling  $f = H\dot{f}$ , which has the advantage of a model independent form  $\dot{f} = ca$ , and one does not have to know the dependence  $f(\phi)$  to compute the GW distance deviation and property functions.

We can achieve this generally by writing

$$G_4 = f_{\phi\phi}(\phi)g_4(X), \quad G_5 = f_{\phi}(\phi)g_5(X). \quad (18)$$

One obtains  $\alpha_T = 0$  with  $\ddot{f} = H\dot{f}$  when

$$g_5(X) = -X \int dx \frac{g_{4x}}{x^2}. \quad (19)$$

Solutions include

$$g_5 = c + d \ln X, \quad g_4 = X[(c - d - 1) + d \ln X] \quad (20)$$

$$g_5 = cX^n, \quad g_4 = \frac{c(1-n)}{1+n} X^{n+1} \quad (21)$$

$$g_5 = cX^{-1}, \quad g_4 = 2c \ln X, \quad (22)$$

leading respectively to

$$M_\star^2 = -2dH\dot{f} + 2(d-1)Xf_{\phi\phi}, \quad (23)$$

$$M_\star^2 = -2ncX^nH\dot{f} + 2[n^2c/(1+n)]X^{n+1}f_{\phi\phi} \quad (24)$$

$$M_\star^2 = 2cX^{-1}H\dot{f} + 2c(-3+2 \ln X)f_{\phi\phi}. \quad (25)$$

One could proceed with all the observables for these theories, however the presence of  $f_{\phi\phi}$  in  $M_\star^2$  means that we have lost model independence. Only in the first case can we remove  $f_{\phi\phi}$ , by choosing  $d = 1$ . Eqs. (20) and (23) generalize the CGB case, which has  $c = 0$  (and note that  $f_{\text{here}} = -4f_{\text{CGB}}$ ), but note this makes no difference for  $M_\star^2$ . On the other hand, the  $f_{\phi\phi}$  term in  $M_\star^2$  offers the hope that  $M_\star^2$  will not blow up in the past, allowing for a viable cosmology. Since one would have to compute this model by model, we do not pursue it further.

There is another method for removing  $f_{\phi\phi}$  from appearing in  $M_\star^2$ . One takes both the condition  $g_{4X} = g_5 - Xg_{5X}$  that led to Eq. (19), and a further condition  $g_5 = 2g_{4X} - X^{-1}g_4$ . This then gives

$$M_\star^2 = 2H\dot{f} \left( \frac{-g_4}{X} + g_{4X} - 2Xg_{4XX} \right). \quad (26)$$

Having  $X$  in  $M_\star^2$  is also model independent so we want to remove it as well. This is accomplished with

$$g_5 = c + 2d + d \ln X, \quad g_4 = X(c + d \ln X), \quad M_\star^2 = -2dH\dot{f}. \quad (27)$$

This will have the same problems of  $M_\star^2$  blowing up in the past as CGB (which is a special case with  $c = -2$ ,  $d = 1$ ).

Thus, the model independent cases we have examined are not viable, and the potentially viable gravity theories are not independent of the form of the  $\phi$  dependence. The next section explores a middle path.

## V. $G_{5X} = 0$

In Eq. (5) for  $\alpha_T$ , a term with  $\ddot{\phi}$  appears explicitly. We can remove this by setting  $G_{5X} = 0$ , in the hopes of avoiding having to be explicit about the scalar field evolution. To keep the GW speed  $c_T = 1$ , this then implies  $G_{4X} = G_{5\phi}$ . That in turn gives

$$M_\star^2 = 2(G_4 - XG_{4X}). \quad (28)$$

When forming  $\alpha_M$ , we will end up with a term involving  $\dot{X}G_{4XX}$ , which again introduces  $\ddot{\phi}$ , so we also take  $G_{4XX} = 0$ .

This implies

$$G_4 = \frac{1}{2} + Xf_\phi + g(\phi), \quad G_5 = f(\phi) + c, \quad (29)$$

yielding the simple expressions

$$M_\star^2 = 1 + 2g(\phi) \quad (30)$$

$$\alpha_M = \frac{2\dot{\phi}g_\phi}{H[1 + 2g(\phi)]} \quad (31)$$

$$\alpha_B = \frac{2\dot{\phi}(XG_{3X} - 3Xf_{\phi\phi} - g_\phi)}{H[1 + 2g(\phi)]}. \quad (32)$$

Gravitational wave distances go as

$$d_{L,GW}(a) = d_L^{GR}(a) \left[ \frac{1 + 2g(a=1)}{1 + 2g(a)} \right]^{1/2}. \quad (33)$$

It is interesting to note that  $G_5$  does not affect them, i.e.  $f(\phi)$  does not enter (though it appears in  $\alpha_B$ ). Note that  $f = 0$  gives a theory with simple scalar coupling  $g(\phi)$  to the Ricci scalar, equivalent to  $f(R)$  gravity.

While model dependent, this class of theories at least has the virtue of simplicity. In the early universe, we want general relativity to describe cosmology so  $M_\star^2 \rightarrow 1$ , implying  $g \rightarrow 0$  (or a constant) there, and  $\alpha_M, \alpha_B \rightarrow 0$  (neglecting  $G_{3X}$ ) if  $g$  and  $f_\phi$  start out slowly rolling. At late times, if the cosmology goes to a frozen de Sitter state, then the numerators of  $\alpha_M$  and  $\alpha_B$ , which involve  $\dot{g} = \dot{\phi}g_\phi$  and  $\dot{f}_\phi$ , vanish, again restoring general relativity.

## VI. CONNECTING GW DISTANCES TO COSMIC GROWTH

The relation between GW distance deviations and growth of cosmic structure deviations from general relativity is an intriguing connection, developed in [10] and elaborated in [16]. There it was pointed out that in Horndeski theories where  $\alpha_B$  is a function of  $\alpha_M$ , then  $G_{\text{matter}}$  and  $M_\star^2$ , and hence  $d_{L,GW}$  are connected. Not all theories do have a relation  $\alpha_B(\alpha_M)$  since  $G_3, G_4$ , and  $G_5$  are generally independent functions, even if we impose  $c_T = 0$ .

However the class of No Slip Gravity has the very direct  $G_{\text{matter}} = 1/M_\star^2$  (as does the non-Horndeski non-local gravity model in [17]). For the class of standard

scalar-tensor theories,  $G_{\text{light}} = 1/M_\star^2$ . Another interesting case is Only Run Gravity [16], where there is no braiding ( $\alpha_B = 0$ ). In that theory

$$G_{\text{matter}} = \frac{M_\star^2 + (M_\star^2)'}{(M_\star^2)^2}, \quad (34)$$

where a prime denotes  $d/d \ln a$ . Illustrations of the connection between the cosmic structure redshift space distortion quantity  $f\sigma_8(a)$ , basically the growth rate, and  $d_{L,GW}/d_L^{GR}$  are shown in [16] for several theories.

Suppose we ask the inverse question, whether there could be no deviation in GW distances, yet deviation in cosmic growth, and vice versa. If  $d_{L,GW} = d_L^{GR}$  then  $M_\star^2 = 1$  and  $\alpha_M = 0$ . This is called the class of No Run Gravity [15]. Eq. (6) indicates this can occur if either there is  $G_5 = 0, G_4 = 1/2$ , or a balance such that

$$G_4 = \frac{1}{2} \left[ 1 + \dot{\phi}\dot{G}_5 \right] = \frac{1}{2} + XG_{5\phi} + X\ddot{\phi}G_{5X}. \quad (35)$$

This imposes a constraint on the scalar field evolution so it is not very generic. Regardless, since  $G_{\text{matter}}$  also depends on  $\alpha_B$ , which can involve  $G_3$ , we can indeed have deviations in growth.

If growth does not deviate from general relativity, this is called the class of Only Light Gravity [16]. Then there is a differential equation relating  $\alpha_B$  and  $\alpha_M$ . This does give a deviation in  $d_{L,GW}$  that depends on the form adopted for  $\alpha_M$  (or equivalently  $G_4$  and  $G_5$ ).

Thus, GW distance deviations and cosmic growth deviations do serve as complementary probes in general, while in a few interesting classes of gravity they can be critical crosschecks on each other.

## VII. CONCLUSIONS

Gravitational wave propagation has already had a dramatic impact on cosmological gravitation theory, severely restricting models that do not predict propagation at the speed of light. The GW distance deviation from general relativity maps the Planck mass evolution; in the simplest interpretation it traces out the gravitation history of the universe.

There are ways around the usual interpretation within Horndeski gravity that  $c_T = 1$  implies  $G_5 = 0, G_{4X} = 0$ . We give the general expression for the GW distance deviation that does not have this, yet preserves  $c_T = 1$ . A particular interesting example involves coupling to the Gauss-Bonnet geometric invariant, but we demonstrate a no go theorem that coupled Gauss-Bonnet gravity cannot have  $c_T = 1$  and give a viable description of our cosmology, regardless of the exact coupling.

We extend this exploration to further forms of  $G_4$  and  $G_5$  that obey  $c_T = 1$ , giving the forms for the GW distance deviation. Some have attractive properties in being model independent, but are not viable, while others are viable, but one must treat model by model.

Other probes of cosmological gravity such as the growth rate of structure and light propagation can be connected to GW distance deviation. We show that in some cases this is a direct relation, hence an important crosscheck that deviations from general relativity are real, against systematic effects; in other cases the probes are complementary, working together to reveal the underlying nature of gravitation and the gravity history of

our universe.

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- [1] B. Abbott et al. (LIGO, Virgo Collaborations), Gravitational Waves and Gamma-Rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A, *ApJ Lett.* 848, L13 (2017) [[arXiv:1710.05834](#)]
  - [2] R. A. Battye, F. Pace, D. Trinh, Gravitational wave constraints on dark sector models, *Phys. Rev. D* 98, 023504 (2018) [[arXiv:1802.09447](#)]
  - [3] C. de Rham, S. Melville, Gravitational Rainbows: LIGO and Dark Energy at its Cutoff, *Phys. Rev. Lett.* 121, 221101 (2018) [[arXiv:1806.09417](#)]
  - [4] A. Riazuelo, J-P. Uzan, Quintessence and Gravitational Waves, *Phys. Rev. D* 62, 083506 (2000) [[arXiv:astro-ph/0004156](#)]
  - [5] I.D. Saltas, I. Sawicki, L. Amendola, M. Kunz, Anisotropic stress as signature of non-standard propagation of gravitational waves, *Phys. Rev. Lett.* 113, 191101 (2014) [[arXiv:1406.7139](#)]
  - [6] L. Lombriser, A. Taylor, Breaking a Dark Degeneracy with Gravitational Waves, *JCAP* 1603, 031 (2016) [[arXiv:1509.08458](#)]
  - [7] A. Nishizawa, Generalized framework for testing gravity with gravitational-wave propagation. I. Formulation, *Phys. Rev. D* 97, 104037 (2018) [[arXiv:1710.0845](#)]
  - [8] S. Arai, A. Nishizawa, Generalized framework for testing gravity with gravitational-wave propagation. II. Constraints on Horndeski theory, *Phys. Rev. D* 97, 104038 (2018) [[arXiv:1711.03776](#)]
  - [9] L. Amendola, I. Sawicki, M. Kunz, I.D. Saltas, Direct detection of gravitational waves can measure the time variation of the Planck mass, *JCAP* 1808, 030 (2018) [[arXiv:1712.08623](#)]
  - [10] E.V. Linder, No Slip Gravity, *JCAP* 1803, 005 (2018) [[arXiv:1801.01503](#)]
  - [11] E. Bellini, I. Sawicki, Maximal freedom at minimum cost: linear large-scale structure in general modifications of gravity, *JCAP* 1407, 050 (2014) [[arXiv:1404.3713](#)]
  - [12] T. Kobayashi, M. Yamaguchi, J. Yokoyama, Generalized G-inflation: Inflation with the most general second-order field equations, *Prog. Theor. Phys.* 126, 511 (2011) [[arXiv:1105.5723](#)]
  - [13] S.D. Odintsov, V.K. Oikonomou, Inflationary Phenomenology of Einstein Gauss-Bonnet Gravity Compatible with GW170817, *Phys. Lett. B* 797, 134874 (2019) [[arXiv:1908.07555](#)]
  - [14] S.D. Odintsov, V.K. Oikonomou, F.P. Frominos, Rectifying Einstein-Gauss-Bonnet Inflation in View of GW170817, *Nuc. Phys. B* 958, 115135 (2020) [[arXiv:2003.13724](#)]
  - [15] E.V. Linder, No Run Gravity, *JCAP* 1907, 034 (2019) [[arXiv:1903.02010](#)]
  - [16] E.V. Linder, Limited Modified Gravity, *JCAP* 2010, 042 (2020) [[arXiv:2003.10453](#)]
  - [17] E. Belgacem, Y. Dirian, S. Foffa, M. Maggiore, Gravitational-wave luminosity distance in modified gravity theories, *Phys. Rev. D* 97, 104066 (2018) [[arXiv:1712.08108](#)]