On Joint Detection and Decoding in Short-Packet Communications

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Abstract—We consider a communication problem in which the receiver must first detect the presence of an information packet and, if detected, decode the message carried within it. We present general nonasymptotic upper and lower bounds on the maximum coding rate that depend on the blocklength, the probability of false alarm, the probability of misdetection, and the packet error probability. The bounds, which are expressed in terms of binary-hypothesis-testing performance metrics, generalize finite-blocklength bounds derived previously for the scenario when a genie informs the receiver whether a packet is present. The bounds apply to detection performed either jointly with decoding on the entire data packet, or separately on a dedicated preamble. The results presented in this paper can be used to determine the blocklength values at which the performance of a communication system is limited by its ability to perform packet detection satisfactorily, and to assess the difference in performance between preamble-based detection, and joint detection and decoding. Numerical results pertaining to the binary-input AWGN channel are provided.

I. INTRODUCTION

The design of short-packet communications—an integral component of delay-sensitive information-exchange protocols—is subject to nontrivial tradeoffs between rate, delay, and error probability [1]. During the last decade, these tradeoffs have been characterized for many channels of interest, including the additive white Gaussian noise (AWGN) channel [1], and Rayleigh and Rician fading channels [2]–[6]. These works all rely on the assumption that the receiver has correctly decoded the presence of a data packet, and do not account for the cost of packet detection. However, there exists a plethora of practical applications, including, e.g., sensor networks, event-triggered communications, and random-access protocols, in which the cost of packet detection is not negligible [7]–[9].

A common approach to perform detection in such systems is to incorporate within the data packet a deterministic preamble, known to the receiver. In short-packet applications, however, adding such a preamble may be highly suboptimal, due to the limited size of the data packet. Naturally, the following question arises: how much can be gained from a design in which detection and decoding are performed jointly over the entire data packet, without the insertion of a dedicated preamble?

Prior Art: Consider a frame-synchronous system, in which each frame may be empty or may contain a data packet, and

where the receiver is assumed to have acquired perfect frame synchronization. The task of the receiver is to detect whether a packet is present in a frame, and if so, to decode it. Three types of error events can be defined: a false alarm (FA), i.e., the event that the receiver detects the presence of a packet even though the transmitter was idle; a misdetection (MD), i.e., the event that the receiver erroneously decides that the transmitter was idle; an inclusive error (IE), i.e., the event that the receiver does not decode correctly a transmitted codeword.

An error-exponent analysis of joint detection and decoding was first presented in [7], where the random-coding error exponents $(E_{FA}(R), E_{MD}(R), E_{IE}(R))$ of the aforementioned errors were analyzed for a given rate R over a discrete memoryless channel (DMC). Specifically, the region $(E_{FA}(R), 0, 0)$ was characterized exactly and the region $(E_{FA}(R), E_{MD}(R), 0)$ was characterized in terms of inner and outer bounds [7, Ch. 3–4]. It was also shown that separate detection and decoding strategies are strictly suboptimal for all rates and that the gap from optimality grows with the rate. In [10], the optimal joint detection and decoding rule for a given code and given FA and MD constraints was derived. Furthermore, the authors provide an exact characterization of the corresponding $(E_{FA}(R), E_{MD}(R), E_{IE}(R))$-region for the case of constant-composition codes. The results in [10] were further extended in [11] to account for nonuniformly distributed message sets.

The joint detection and decoding problem is related to the unequal error protection (UEP) problem, in which messages belong to different classes with different reliability requirements [12]. A nonasymptotic analysis of the UEP problem was presented in [13], where several results from [1] are extended to this setting. In particular, the dependence-testing (DT) achievability bound (maximum error probability) [1, Th. 21] and the metaconverse bound [1, Th. 27] are extended to both joint and separate message classification and decoding, and particularized to the binary-symmetric channel (BSC) and the binary-erasure channel (BEC). However, these bounds cannot be applied directly to the detection and decoding problem considered in this paper, since the MD probability cannot be accounted for straightforwardly.

In summary, although prior art is available for the problem of joint detection and decoding, in none of the existing works nonasymptotic bounds are computed for channel models of practical interest in wireless communications. Furthermore, the performance gap between joint detection and decoding and con-

ventional preamble-based detection followed by decoding is not quantified.

Contributions: We present finite-blocklength achievability and converse bounds for joint detection and decoding strategies over general point-to-point channels. The bounds are explicit in the Neyman-Pearson β function—a key performance metric in binary hypothesis testing. Specifically, the achievability bound builds upon the $\beta\beta$ achievability bound presented in [14, Th.1], while the converse bound is based on the $\beta\beta$ converse bound [15, Th. 15], a tightening of the metaconverse bound [1, Th. 27]. We also discuss how to adapt these bounds to the case of detection performed on a dedicated preamble.

For given requirements on the FA, MD, and IE probabilities, we use the bounds to characterize the maximum coding rate as a function of the packet length both for joint detection and decoding and for detection performed on a dedicated preamble, followed by decoding. Numerical results for the binary-input AWGN channel indicate that packet detection, even when performed jointly with decoding, is the performance bottleneck when the packets are very short (less than 190 channel uses), and the SNR is low (e.g., $0\,\mathrm{dB}$). In this regime, it turns out useful to consider input distributions for which the binary input symbols are transmitted with different probabilities, since this is beneficial for packet detection. As the packet length grows and/or the SNR increases, packet decoding becomes the performance bottleneck. Performing detection solely on the basis of a dedicated preamble turns out to be highly suboptimal in the short-packet regime, even when the length of the preamble is optimized.

Notation: Uppercase letters such as X and $\boldsymbol{X}$ are used to denote scalar random variables and vectors, respectively; their realizations are written in lowercase, e.g., x and $\mathbf{x}$. We denote the expectation operator by $\mathbb{E}[\cdot]$, and, for a given event $\mathcal{E}$, we use $P_X[\mathcal{E}]$ to denote the probability that $\mathcal{E}$ occurs when the underlying probability measure is P_X. We write $\log(\cdot)$ and $\log_2(\cdot)$ to denote the natural logarithm and the logarithm to the base 2, respectively, and $\mathbb{1}[\cdot]$ to denote the indicator function. Finally, consider two probability distributions P_X and Q_X on a common measurable space $\mathcal{X}$, and let $P_{Z|X} : \mathcal{X} \to \{0,1\}$ be a randomized binary test, where $Z = 1$ indicates that the test chooses P_X. The Neyman-Pearson β function is defined as

$$\beta_\alpha(P_X, Q_X) \triangleq \min_{P_{Z|X}:P_X[Z=1]\geq\alpha} Q_X[Z=1] \qquad (1)$$

where $\alpha \in [0, 1]$. The test achieving (1) is the Neyman-Pearson test [16], which involves thresholding the log-likelihood ratio $\log dP_X/dQ_X$, where dP_X/dQ_X stands for the Radon-Nykodim derivative of P_X with respect to Q_X. We shall also use for convenience the following equivalent function

$$\alpha_\beta(P_X, Q_X) \triangleq \min_{P_{Z|X}:Q_X[Z=1]\leq\beta} P_X[Z=0] \qquad (2)$$

where $\beta \in [0, 1]$. This function turns out to be easier to relate to error probabilities than (1).

II. SYSTEM MODEL

We consider frame-synchronous transmissions where the packet duration is equal to the frame size and encompasses n

channel uses. The receiver is assumed to be oblivious to the presence of any packet and must, therefore, perform both detection and decoding. The channel law within each frame is denoted by $P_{\boldsymbol{Y}|\boldsymbol{X}}(\mathbf{y}|\mathbf{x})$, where $\mathbf{x} \in \mathcal{X}^n$ and $\mathbf{y} \in \mathcal{Y}^n$. Throughout, we assume that $\mathcal{X}$ contains a special symbol, which we denote by $\varnothing$, used to indicate that the transmitter is idle. We shall use $\boldsymbol{\varnothing}$ to indicate the vector containing n copies of $\varnothing$.

When the transmitter is active, it encodes a message W, uniformly distributed in the set $\{1, \ldots, M\}$, to a codeword in the codebook $\{\mathbf{c}_i\}_{i=1}^M \subset \mathcal{X}^n$. To perform joint detection and decoding, the receiver partitions the set $\mathcal{Y}^n$ of possible received signals $\mathbf{y}$ into $M + 1$ disjoint regions $\mathcal{R}_0, \mathcal{R}_1, \ldots, \mathcal{R}_M$ whose union covers $\mathcal{Y}^n$. If $\mathbf{y} \in \mathcal{R}_m$, where $m = 1, \ldots, M$, the decoder returns the estimate $\widehat{W} = m$. If $\mathbf{y} \in \mathcal{R}_0$, the decoder declares that the transmitter was idle.

Following [10], we next define the FA, MD, and IE probabilities as follows:

$$P_{\mathrm{FA}} \triangleq \sum_{m=1}^M P_{\boldsymbol{Y}|\boldsymbol{X}=\boldsymbol{\varnothing}}[\boldsymbol{Y} \in \mathcal{R}_m] \qquad (3)$$

$$P_{\mathrm{MD}} \triangleq \frac{1}{M} \sum_{m=1}^M P_{\boldsymbol{Y}|\boldsymbol{X}=\mathbf{c}_m}[\boldsymbol{Y} \in \mathcal{R}_0] \qquad (4)$$

$$P_{\mathrm{IE}} \triangleq \frac{1}{M} \sum_{m=1}^M \sum_{\substack{k=0 \\ k\neq m}}^M P_{\boldsymbol{Y}|\boldsymbol{X}=\mathbf{c}_m}[\boldsymbol{Y} \in \mathcal{R}_k]. \qquad (5)$$

Observe that P_{IE} comprises the probability of MD and the probability of decoding a wrong codeword.

An $(M, n, \epsilon_{\mathrm{FA}}, \epsilon_{\mathrm{MD}}, \epsilon_{\mathrm{IE}})$–code is a code with blocklength n, M codewords, and $P_{\mathrm{FA}} \leq \epsilon_{\mathrm{FA}}$, $P_{\mathrm{MD}} \leq \epsilon_{\mathrm{MD}}$, and $P_{\mathrm{IE}} \leq \epsilon_{\mathrm{IE}}$. Similar to [17], it will turn out convenient to consider a more general notion of *randomized code* in which both transmitter and receiver are equipped with a common randomness, which allows them to time-share between deterministic $(M, n, \epsilon_{\mathrm{FA}}, \epsilon_{\mathrm{MD}}, \epsilon_{\mathrm{IE}})$–codes. As usual, the maximum coding rate is defined as

$$\begin{aligned} &R^*(n, \epsilon_{\mathrm{FA}}, \epsilon_{\mathrm{MD}}, \epsilon_{\mathrm{IE}}) \\ &\triangleq \sup\left\{\frac{\log_2(M)}{n} : \exists(M, n, \epsilon_{\mathrm{FA}}, \epsilon_{\mathrm{MD}}, \epsilon_{\mathrm{IE}})\text{–code}\right\}. \end{aligned} \qquad (6)$$

III. NONASYMPTOTIC BOUNDS

A. Joint Detection and Decoding

In Theorem 1 below, we present a joint-detection-and-decoding achievability bound that generalizes the $\beta\beta$ bound for the case of genie-aided detection presented in [14, Th.1].

Theorem 1: Let $P_{\boldsymbol{X}}$ be an arbitrary distribution on $\mathcal{X}^n$, and let $P_{\boldsymbol{Y}}$ be the corresponding output distribution. Fix an arbitrary auxiliary distribution $Q_{\boldsymbol{Y}}$ on $\mathcal{Y}^n$, and two parameters $\delta^{(1)} \in [0, 1]$ and $\delta^{(2)} \in [0, 1]$. For all $M \leq 2/\delta^{(2)} + 1$, there exists a randomized $(M, n, \epsilon_{\mathrm{FA}}, \epsilon_{\mathrm{MD}}, \epsilon_{\mathrm{IE}})$-code involving time sharing

between four deterministic codes, that satisfies

$$\epsilon_{\mathrm{FA}} \leq \delta^{(1)} \tag{7}$$

$$\epsilon_{\mathrm{MD}} \leq \alpha_{\delta^{(1)}}(P_{\boldsymbol{Y}}, P_{\boldsymbol{Y}|\boldsymbol{X}=\varnothing}) \tag{8}$$

$$\epsilon_{\mathrm{IE}} \leq \alpha_{\delta^{(1)}}(P_{\boldsymbol{Y}}, P_{\boldsymbol{Y}|\boldsymbol{X}=\varnothing})$$
$$+ 1 - \alpha_{\frac{M-1}{2}\delta^{(2)}}(P_{\boldsymbol{Y}}, Q_{\boldsymbol{Y}}) + \alpha_{\delta^{(2)}}(P_{\boldsymbol{X}\boldsymbol{Y}}, P_{\boldsymbol{X}}Q_{\boldsymbol{Y}}). \tag{9}$$

Proof: See Appendix A. ∎

Remark 1: Theorem 1 recovers the $\beta\beta$ achievability bound [14, Th. 1] when genie-aided detection is considered and, hence, $P_{\mathrm{FA}} = P_{\mathrm{MD}} = 0$. Furthermore, for the case of genie-aided detection, by setting $Q_{\boldsymbol{Y}} = P_{\boldsymbol{Y}}$ and using the definition of the Neyman-Pearson test, one recovers from Theorem 1 the DT achievability bound [1, Th.17].

Next, we present a converse bound that draws inspiration from the $\beta\beta$ converse bound introduced in [15, Th. 15] to characterize the empirical output distribution of good channel codes.

Theorem 2: Let $Q_{\boldsymbol{Y}}$ be an arbitrary distribution on $\mathcal{Y}^n$. Then, every $(M, n, \epsilon_{\mathrm{FA}}, \epsilon_{\mathrm{MD}}, \epsilon_{\mathrm{IE}})$–code satisfies

$$M \leq \sup_{P_{\boldsymbol{X}}} \left\{ \frac{1 - \beta_{1-\epsilon_{\mathrm{FA}}}\left(P_{\boldsymbol{Y}|\boldsymbol{X}=\varnothing}, Q_{\boldsymbol{Y}}\right)}{\beta_{1-\epsilon_{\mathrm{IE}}}\left(P_{\boldsymbol{X}}P_{\boldsymbol{Y}|\boldsymbol{X}}, P_{\boldsymbol{X}}Q_{\boldsymbol{Y}}\right)} \right.$$
$$\left. \times \mathbb{1}\left\{ \beta_{1-\epsilon_{\mathrm{MD}}}(P_{\boldsymbol{Y}}, Q_{\boldsymbol{Y}}) \leq 1 - \beta_{1-\epsilon_{\mathrm{FA}}}\left(P_{\boldsymbol{Y}|\boldsymbol{X}=\varnothing}, Q_{\boldsymbol{Y}}\right) \right\} \right\}. \tag{10}$$

Proof: See Appendix B. ∎

Remark 2: By upper-bounding by one both the indicator function and the numerator of the first term, we recover the metaconverse bound [1, Th. 27]—a bound that in our setup would depend only on ϵ_{IE}, and, hence, would not be able to illustrate the impact of the FA and MD requirements on the size of the codebook M.

B. Preamble-Based Detection

We next show how to adapt the achievability bound in Theorem 1 and the converse bound in Theorem 2 to the case of separate detection and decoding. Let $\boldsymbol{X} = [\mathbf{x}_{\mathrm{p}}, \boldsymbol{X}_{\mathrm{d}}]$, where $\mathbf{x}_{\mathrm{p}}$ is a deterministic preamble sequence of length n_{p}, and $\boldsymbol{X}_{\mathrm{d}} \sim P_{\boldsymbol{X}_{\mathrm{d}}}$ on $\mathcal{X}^{\mathrm{d}}$ is the vector containing the remaining $n_{\mathrm{d}} = n - n_{\mathrm{p}}$ information-carrying symbols. Furthermore, let $\boldsymbol{Y} = [\boldsymbol{Y}_{\mathrm{p}}, \boldsymbol{Y}_{\mathrm{d}}]$ be the received signal upon transmission of $\boldsymbol{X} = [\mathbf{x}_{\mathrm{p}}, \boldsymbol{X}_{\mathrm{d}}]$. The decoder uses $\boldsymbol{Y}_{\mathrm{p}}$ to detect if a codeword is present, and if so, decoding is performed over the remaining $\boldsymbol{Y}_{\mathrm{d}}$ symbols. In this setup, packet detection involves a simple binary hypothesis testing problem. Hence, the tradeoff between ϵ_{FA} and ϵ_{MD} can be expressed as

$$\epsilon_{\mathrm{MD}} = \alpha_{\epsilon_{\mathrm{FA}}}(P_{\boldsymbol{Y}_{\mathrm{p}}|\boldsymbol{X}_{\mathrm{p}}=\mathbf{x}_{\mathrm{p}}}, P_{\boldsymbol{Y}_{\mathrm{p}}|\boldsymbol{X}_{\mathrm{p}}=\varnothing}). \tag{11}$$

A bound on ϵ_{IE} can be obtained by following steps similar to the ones reported in Appendix A. Specifically, one can show that for an arbitrary $Q_{\boldsymbol{Y}}$ and $\delta \in [0, 1]$, the following bound holds for all $M \leq 2/\delta + 1$:

$$\epsilon_{\mathrm{IE}} \leq \epsilon_{\mathrm{MD}} + (1 - \epsilon_{\mathrm{MD}})$$
$$\times \left[1 - \alpha_{\frac{M-1}{2}\delta}(P_{\boldsymbol{Y}_{\mathrm{d}}}, Q_{\boldsymbol{Y}_{\mathrm{d}}}) + \alpha_{\delta}(P_{\boldsymbol{X}_{\mathrm{d}}, \boldsymbol{Y}_{\mathrm{d}}}, P_{\boldsymbol{X}_{\mathrm{d}}}Q_{\boldsymbol{Y}_{\mathrm{d}}}) \right]. \tag{12}$$

Here, $P_{\boldsymbol{Y}_{\mathrm{d}}}$ denotes the output distribution induced by $P_{\boldsymbol{X}_{\mathrm{d}}}$.

A converse bound on ϵ_{IE} for this scenario can be obtained by a direct application of the metaconverse theorem [1, Th. 27]. Specifically, for every auxiliary distribution $Q_{\boldsymbol{Y}_{\mathrm{d}}}$ on $\mathcal{Y}^{n_{\mathrm{d}}}$, we have that

$$M \leq \sup_{P_{\boldsymbol{X}_{\mathrm{d}}}} \frac{1}{\beta_{1-\epsilon_{\mathrm{IE}}}\left(P_{\boldsymbol{X}_{\mathrm{d}}}P_{\boldsymbol{Y}_{\mathrm{d}}|\boldsymbol{X}_{\mathrm{d}}}, P_{\boldsymbol{X}_{\mathrm{d}}}Q_{\boldsymbol{Y}_{\mathrm{d}}}\right)}. \tag{13}$$

IV. NUMERICAL RESULTS

A. Bounds for the Binary AWGN Channel

In this section, we discuss how to compute the bounds presented in Section III for the memoryless discrete-time binary-input AWGN channel. For this channel, the input-output relation is given by

$$Y_k = X_k + N_k, \quad k = 1, \ldots, n. \tag{14}$$

Here, $X_k \in \mathcal{X} = \{-\sqrt{\rho}, 0, \sqrt{\rho}\}$, where $X_k = 0$ indicates that the transmitter is idle (in other words, $\varnothing = 0$). The additive noise N_k follows a $\mathcal{N}(0, 1)$ distribution; hence, ρ can be thought of as the SNR at the receiver. It then follows that $P_{Y|X=x} = \mathcal{N}(x, 1)$. Throughout this section, we consider as input distribution $P_{\boldsymbol{X}}$ a product distribution $P_{\boldsymbol{X}}(\mathbf{x}) = \prod_{k=1}^n P_X(x_k)$ where $P_X(-\sqrt{\rho}) = p$, $P_X(\sqrt{\rho}) = 1 - p$, and $P_X(0) = 0$, with $p \in [1/2, 1]$. By adjusting p, we can enforce one of the two nonzero symbols to occur more often in the codebook, which, as we shall see, is beneficial from a detection perspective. As auxiliary channel, we choose $Q_{\boldsymbol{Y}}(\mathbf{y}) = \prod_{k=1}^n P_Y(y_k)$, where $P_Y = p\mathcal{N}(-\sqrt{\rho}, 1) + (1 - p)\mathcal{N}(\sqrt{\rho}, 1)$ is the output distribution induced by P_X. To evaluate the achievability bound in Theorem 1, we note that, for this choice of input distribution, we have that $Q_{\boldsymbol{Y}} = P_{\boldsymbol{Y}}$. We then use the Neyman-Pearson lemma to evaluate the α functions in (7)–(9).

Evaluating the converse bound given in Theorem 2 for joint detection and decoding is, however, numerically intractable. Indeed, (10) involves an optimization over all possible input distributions, which cannot be avoided due to the presence of the induced output distribution $P_{\boldsymbol{Y}}$ in (10). One possible approach to sidestep this issue is to upper-bound by one both the indicator function in (10) and the numerator of the first term in (10).

The resulting bound

$$M \leq \sup_{P_{\boldsymbol{X}}} \frac{1}{\beta_{1-\epsilon_{\mathrm{IE}}}\left(P_{\boldsymbol{X}}P_{\boldsymbol{Y}|\boldsymbol{X}}, P_{\boldsymbol{X}}Q_{\boldsymbol{Y}}\right)} \tag{15}$$

which, as already remarked, coincides with the metaconverse bound [1, Th. 27], yields also a converse bound for the genie-aided case. Note that this bound can be evaluated numerically if we set $p = 1/2$ in $Q_{\boldsymbol{Y}}$. Indeed, for this choice one can invoke [1, Lem. 29], and replace the $\beta_{1-\epsilon_{\mathrm{IE}}}\left(P_{\boldsymbol{X}}P_{\boldsymbol{Y}|\boldsymbol{X}}, P_{\boldsymbol{X}}Q_{\boldsymbol{Y}}\right)$ in the denominator of (15) with $\beta_{1-\epsilon_{\mathrm{IE}}}\left(P_{\boldsymbol{Y}|\boldsymbol{X}=\mathbf{x}}, Q_{\boldsymbol{Y}}\right)$, where $\mathbf{x}$ is an arbitrary n-dimensional vector with entries in $\{-\sqrt{\rho}, \sqrt{\rho}\}$. The resulting bound is then independent of $P_{\boldsymbol{X}}$, and the maximization can be omitted.

A different approach to assess, although in a weaker way, the tightness of the achievability bound in Theorem 1 for our choice of parameters, is to evaluate the converse bound in Theorem 2 for the same input distribution $P_{\boldsymbol{X}}$ chosen to evaluate

the achievability bound. The resulting converse bound should be interpreted as a *random-coding-ensemble converse*. It provides an upper bound on the number of codewords that are compatible with the requirement that the FA, the MD, and the IE probabilities, averaged over all random codebooks whose codewords are drawn independently according to $P_{\boldsymbol{X}}$, do not exceed ϵ_{FA}, ϵ_{MD}, and ϵ_{IE}, respectively.

To evaluate the preamble-based bounds (11), (12), and (13), we set $p = 1/2$ and proceed similarly to the joint-detection-and-decoding case. This time, the input and auxiliary output distributions are defined on vectors of dimension n_{d}.

To evaluate numerically the bounds discussed so far via the Neyman-Pearson lemma, one needs to compute the following three likelihood ratios:

$$\imath(\mathbf{x}; \mathbf{y}) \triangleq \log\left(\frac{\mathrm{d}P_{\boldsymbol{Y}|\boldsymbol{X}=\mathbf{x}}}{\mathrm{d}P_{\boldsymbol{Y}}}\right) \tag{16}$$

$$\jmath(\mathbf{x}; \mathbf{y}) \triangleq \log\left(\frac{\mathrm{d}P_{\boldsymbol{Y}|\boldsymbol{X}=\mathbf{x}}}{\mathrm{d}P_{\boldsymbol{Y}|\boldsymbol{X}=\varnothing}}\right) \tag{17}$$

$$r(\mathbf{y}) \triangleq \log\left(\frac{\mathrm{d}P_{\boldsymbol{Y}}}{\mathrm{d}P_{\boldsymbol{Y}|\boldsymbol{X}=\varnothing}}\right). \tag{18}$$

For our choice of output distribution $Q_{\boldsymbol{Y}}$, these quantities can be evaluated as follows:

$$\imath(\mathbf{x}, \mathbf{y}) = \sum_{k=1}^{n} \log\left(\frac{e^{x_k y_k}}{pe^{-\sqrt{\rho}y_k} + (1-p)e^{\sqrt{\rho}y_k}}\right) \tag{19}$$

$$r(\mathbf{y}) = -\frac{n\rho}{2} + \sum_{k=1}^{n} \log\left(pe^{-\sqrt{\rho}y_k} + (1-p)e^{\sqrt{\rho}y_k}\right) \tag{20}$$

$$\jmath(\mathbf{x}, \mathbf{y}) = -\frac{n\rho}{2} + \sum_{k=1}^{n} x_k y_k. \tag{21}$$

For example, (21) implies that (11) can be expressed in the following parametric form: for every $\gamma \in \mathbb{R}$, we have that

$$\epsilon_{\mathrm{FA}} = P_{\boldsymbol{Y}_{\mathrm{p}}|\boldsymbol{X}=\varnothing}\left[\sum_{i=1}^{n_{\mathrm{p}}} x_i Y_i \geq \gamma + \frac{n_{\mathrm{p}}\rho}{2}\right]$$
$$= Q\left(\frac{\gamma + \frac{n_{\mathrm{p}}\rho}{2}}{\sqrt{n_{\mathrm{p}}\rho}}\right) \tag{22}$$

and

$$\epsilon_{\mathrm{MD}} = P_{\boldsymbol{Y}_{\mathrm{p}}|\boldsymbol{X}_{\mathrm{p}}=\mathbf{x}_{\mathrm{p}}}\left[\sum_{i=1}^{n_{\mathrm{p}}} x_i Y_i < \gamma + \frac{n_{\mathrm{p}}\rho}{2}\right]$$
$$= 1 - Q\left(\frac{\gamma - \frac{n_{\mathrm{p}}\rho}{2}}{\sqrt{n_{\mathrm{p}}\rho}}\right). \tag{23}$$

B. Numerical Results for the Binary AWGN Channel

In Fig. 1, we compare the maximum coding rate achievable with joint detection and decoding and with preamble-based detection followed by decoding, with the maximum coding rate achievable with genie-aided detection. In our simulations, we set $\epsilon_{\mathrm{IE}} = 10^{-3}$, $\epsilon_{\mathrm{MD}} = 10^{-4}$, $\epsilon_{\mathrm{FA}} = 10^{-4}$, and consider three different SNR values: $\rho \in \{0\,\mathrm{dB}, 3\,\mathrm{dB}, 6\,\mathrm{dB}\}$.

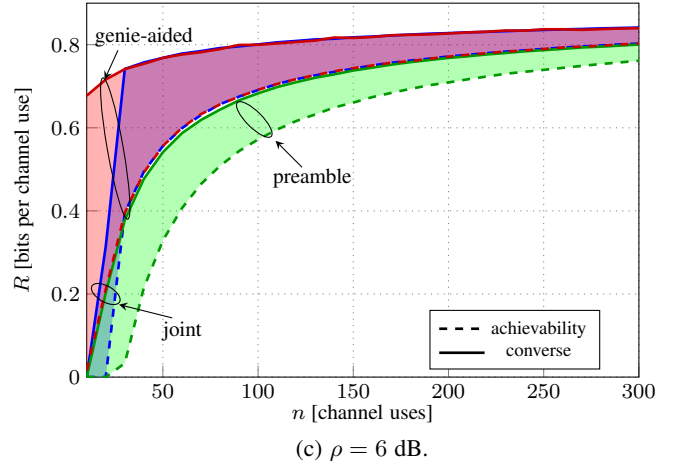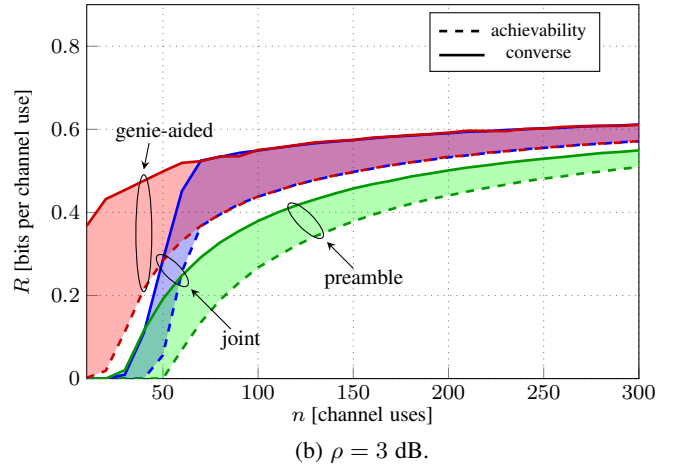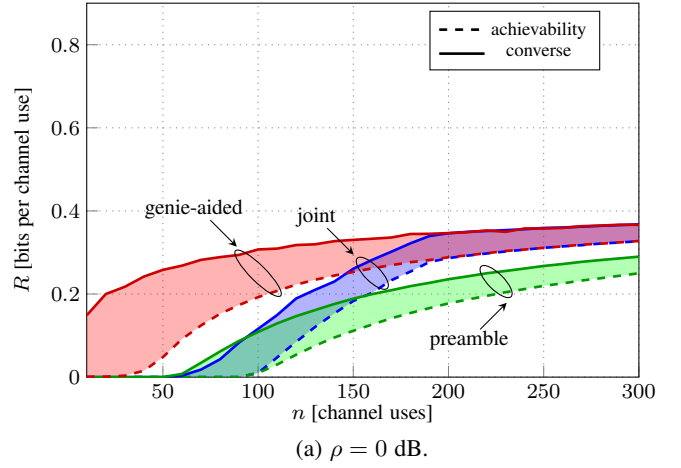The achievability bound for joint detection and decoding given by Theorem 1, and optimized over the choice of p is

(a) $\rho = 0$ dB.

(b) $\rho = 3$ dB.

(c) $\rho = 6$ dB.

Fig. 1: Rate bounds for the binary AWGN with $\epsilon_{\mathrm{IE}} = 10^{-3}$, $\epsilon_{\mathrm{MD}} = 10^{-4}$, and $\epsilon_{\mathrm{FA}} = 10^{-4}$.

depicted with blue dashed lines, whereas we use green dashed lines to indicate the achievability bound for the case of preamble-based detection, obtained by evaluating (22), (23) and (12). This bound is optimized over the choice of n_{p}. Furthermore, we use red dashed lines to indicate an achievability bound for the genie-aided-detection case. Specifically, we consider the DT bound [1, Th. 17], evaluated for the input distribution described
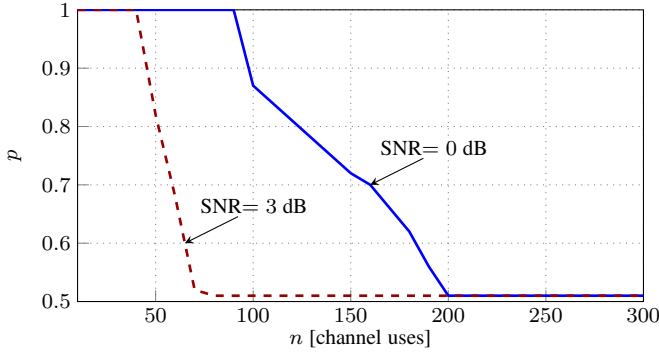
Fig. 2: Optimal value of the parameter p in the achievability bound provided in Theorem 1 for joint detection and decoding. Here, $\epsilon_{\mathrm{IE}} = 10^{-3}$, $\epsilon_{\mathrm{MD}} = 10^{-4}$, and $\epsilon_{\mathrm{FA}} = 10^{-4}$.

in Section IV-A and $p = 1/2$.

On the converse side, we use blue solid lines to indicate the ensemble-converse bound discussed in Section IV-A, optimized over the choice of p. The converse bound for the preamble-based detection, obtained by evaluating (22), (23), and (13) is indicated by green solid lines. Finally, we use red solid lines to indicate the metaconverse bound (15) for the genie-aided case.

We observe that the genie-aided-detection bounds approach the joint detection and decoding bounds as the blocklength and the SNR increase. In our numerical examples, this occurs when $n \geq 190$ for an SNR value of $0\,\mathrm{dB}$, when $n \geq 70$ for an SNR value of $3\,\mathrm{dB}$, and for $n \geq 20$ for an SNR value of $6\,\mathrm{dB}$. On the contrary, the preamble-based strategy is suboptimal over the entire range of blocklength values considered in the figure, although the performance gap decreases as the SNR increases. This is expected, since detection is simplified when the power available for the transmission of the preamble increases (see (22) and (23)), in which case a shorter preamble sequence suffices. Thus, the penalty due to the transmission of the preamble decreases as the SNR and the blocklength grow.

In Fig. 2, we depict the value of p that maximizes the achievability bound given in Theorem 1 for the case of joint detection and decoding, as a function of the blocklength, for the case $\rho = 0\,\mathrm{dB}$ and $\rho = 3\,\mathrm{dB}$. The other parameters are as in Fig. 1. As illustrated in the figure, skewing the input distribution is beneficial for blocklength values where detection is the performance bottleneck. This corresponds to the range of blocklength values for which the joint detection and decoding scheme performs strictly better than the preamble-based scheme, but strictly worse than the genie-aided scheme. Note that by setting $p = 0.5$, we obtain the capacity achieving input distribution, which maximizes the coding rate in the large blocklength limit. However, uniform inputs are not a good choice for detection when packets are short. For example, in the scenario depicted in Fig. 1a, by using uniform inputs, the achievable rate with joint detection and decoding would be equal to zero until $n = 190$, to then rapidly approach the genie-aided bound for slightly larger blocklength values. As shown in the figure, higher rates can be obtained for blocklengths smaller than 190 by optimizing p.

V. CONCLUSION

We have presented nonasymptotic bounds on the largest coding rate achievable in a frame-synchronous communication system where the receiver has to decide on the presence of a packet prior to decoding. The bounds, which rely on the $\beta\beta$ framework introduced in [14], apply to the scenario where detection is performed jointly with decoding over the entire data packet, and to the scenario in which a dedicated preamble is used for detection.

Numerical examples for the case of binary-input AWGN channels indicate that in the short-packet regime, joint detection and decoding yields significant gains in terms of maximum coding rate over preamble-based detection followed by decoding. Furthermore, there exists a range of blocklength values for which departing from a uniform input distribution is beneficial, since it facilitates detection. The tightness of the proposed bounds in the error-exponent regime considered in [7], [9], [10] remains to be investigated.

APPENDIX A
PROOF OF THEOREM 1

Fix a codebook $\{\mathbf{c}_m\}_{m=1}^M$ and let $Z^{(1)}(\mathbf{y})$ and $Z^{(2)}(\mathbf{x}, \mathbf{y})$ denote the tests achieving $\alpha_{\delta^{(1)}}(P_{\mathbf{Y}}, P_{\mathbf{Y}|\mathbf{X}=\varnothing})$ and $\alpha_{\delta^{(2)}}(P_{\mathbf{X},\mathbf{Y}}, P_{\mathbf{X}}Q_{\mathbf{Y}})$, respectively. For a given received signal $\mathbf{y}$, the decoder evaluates $Z^{(1)}(\mathbf{y})$. If $Z^{(1)}(\mathbf{y}) = 0$, the decoder declares the transmitter to be idle. If $Z^{(1)}(\mathbf{y}) = 1$, the decoder declares the transmitter to be active and determines the transmitted message as follows: it computes $Z^{(2)}(\mathbf{c}_m, \mathbf{y})$ for all $m = 1, \ldots, M$ and returns the smallest index $\tilde{m}$ for which $Z^{(2)}(\mathbf{c}_{\tilde{m}}, \mathbf{y}) = 1$. If no such index is found, it declares a decoding error. For this decoder, it follows that

$$\epsilon_{\mathrm{FA}}\big(\{\mathbf{c}_m\}_{m=1}^M\big) = P_{\mathbf{Y}|\mathbf{X}=\varnothing}[Z^{(1)}(\mathbf{Y}) = 1] \tag{24}$$

$$\epsilon_{\mathrm{MD}}\big(\{\mathbf{c}_m\}_{m=1}^M\big) = \frac{1}{M}\sum_{m=1}^M P_{\mathbf{Y}|\mathbf{X}=\mathbf{c}_m}[Z^{(1)}(\mathbf{Y}) = 0] \tag{25}$$

$$\epsilon_{\mathrm{IE}}\big(\{\mathbf{c}_m\}_{m=1}^M\big) = \frac{1}{M}\sum_{m=1}^M P_{\mathbf{Y}|\mathbf{X}=\mathbf{c}_m}\bigg[Z^{(1)}(\mathbf{Y}) = 0 \text{ or }$$
$$Z^{(2)}(\mathbf{c}_m, \mathbf{Y}) = 0 \text{ or } Z^{(2)}(\mathbf{c}_{m'}, \mathbf{Y}) = 1, m' < m\bigg]. \tag{26}$$

We obtain (7) and (8) by averaging (24) and (25) over all random codebooks $\{\mathbf{C}_m\}_{m=1}^M$ whose codewords are generated independently from $P_{\mathbf{X}}$ and by using the definition of the test $Z^{(1)}(\mathbf{y})$. To obtain (9) by averaging (26) over $\{\mathbf{C}_m\}_{m=1}^M$, we proceed as follows. We first apply the union bound on the probability of the union of the three events on the right-hand side of (26) to obtain three probability terms. The first term can be evaluated as

$$\mathbb{E}_{\{\mathbf{C}_m\}_{m=1}^M}\bigg[\frac{1}{M}\sum_{m=1}^M P_{\mathbf{Y}|\mathbf{X}=\mathbf{C}_m}[Z^{(1)}(\mathbf{Y}) = 0]\bigg]$$
$$= \alpha_{\delta^{(1)}}(P_{\mathbf{Y}}, P_{\mathbf{Y}|\mathbf{X}=\varnothing}). \tag{27}$$

Similarly, the second term is given by

$$\mathbb{E}_{\{C_m\}_{m=1}^M}\left[\frac{1}{M}\sum_{m=1}^M P_{Y|X=C_m}[Z^{(2)}(C_m,Y)=0]\right]$$
$$=\alpha_{\delta^{(2)}}(P_{X,Y},P_XQ_Y). \quad (28)$$

To evaluate the third term, it is convenient to define a random variable W uniformly distributed over the message set $[1,\ldots,M]$, and to consider the randomized test $Z^{(3)}(\mathbf{y})$, which returns 1 if $Z^{(2)}(\mathbf{c}_m,\mathbf{y})=1$ for some $m<W$. Then

$$\mathbb{E}_{\{C_m\}_{m=1}^M}\left[\frac{1}{M}\sum_{m=1}^M P_{Y|X=C_m}[Z^{(2)}(C_{m'},Y)=1,\right.$$
$$\left. m'<m\right]=P_Y[Z^{(3)}=1]. \quad (29)$$

Now note that

$$P_Y[Z^{(3)}(Y)=1]=1-P_Y[Z^{(3)}(Y)=0]$$
$$\leq 1-\alpha_{Q_Y[Z^{(3)}(Y)=1]}(P_Y,Q_Y). \quad (30)$$

Furthermore,

$$Q_Y[Z^{(3)}(Y)=1]\leq\frac{1}{M}\sum_{j=1}^M(j-1)P_XQ_Y[Z^{(2)}(X,Y)=1]$$
$$=\frac{M-1}{2}\delta^{(2)}. \quad (31)$$

Since the function α_β is nonincreasing in β, we conclude that

$$P_Y[Z^{(3)}(Y)=1]\leq 1-\alpha_{\frac{M-1}{2}\delta^{(2)}}(P_Y,Q_Y). \quad (32)$$

The desired result follows by substituting (32) into (29), and then by combining (27), (28), and (29). To conclude the proof, we proceed similarly to [17, Th. 19], and note that, by Caratheodory's theorem (see e.g. [18, Th. 15.3.5]), there exists a randomized code that achieves simultaneously (7)–(9) and involves time-sharing between four deterministic codes.

APPENDIX B
PROOF OF THEOREM 2

Fix a coding scheme and let P_X be the distribution on $\mathcal{X}^n$ induced by the encoder when the messages are uniform. Let P_Y be the corresponding output distribution. Finally, denote by $\mathcal{R}_0,\mathcal{R}_1,\ldots,\mathcal{R}_M$ the decoding regions. Note that, by assumption, we have that

$$P_Y[Y\notin\mathcal{R}_0]\geq 1-\epsilon_{\mathrm{MD}} \quad (33)$$
$$P_{Y|X=\varnothing}[Y\in\mathcal{R}_0]\geq 1-\epsilon_{\mathrm{FA}}. \quad (34)$$

Let now $Z(\mathbf{x},\mathbf{y})=\mathbb{1}\{\widehat{W}=W,\mathbf{y}\notin\mathcal{R}_0\}$ where $\widehat{W}$ denotes the estimated message by the decoder when the transmitted message is W. Note that

$$P_{X,Y}[Z(X,Y)=1]=P_{X,Y}[\widehat{W}=W]\geq 1-\epsilon_{\mathrm{IE}}. \quad (35)$$

Furthermore, we also have that

$$P_XQ_Y[Z(X,Y)=1]\leq\frac{Q_Y[Y\notin\mathcal{R}_0]}{M}. \quad (36)$$

It then follows that

$$\beta_{1-\epsilon_{\mathrm{IE}}}(P_{X,Y},P_XQ_Y)\leq\frac{Q_Y[Y\notin\mathcal{R}_0]}{M}. \quad (37)$$

Note finally that

$$\beta_{1-\epsilon_{\mathrm{MD}}}(P_Y,Q_Y)\leq Q_Y[Y\notin\mathcal{R}_0]$$
$$=1-Q_Y[Y\in\mathcal{R}_0]$$
$$\leq 1-\beta_{1-\epsilon_{\mathrm{FA}}}(P_{Y|X=\varnothing},Q_Y). \quad (38)$$

We obtain the desired bound by using (38) in (37) to remove the dependence on $\mathcal{R}_0$, and by maximizing over P_X, to obtain a bound that is valid for every code.

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