

On the indistinguishability of Chiral QED with parameter-free Faddeevian anomaly and QED under a chiral constraint

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We carry out an investigation imposing a chiral constraint in the phase space of vector and axial-vector Schwinger model. We find that resulting model is identical to gauge non-invariant model which was obtained by the imposition of chiral constraint in the phase-space of in Chiral Schwinger model with the parameter-free Faddeevian anomaly. Three different models having different types of interaction between the matter and gauge field become indistinguishable under a chiral constraint at the quantum mechanical level. The resulting gauge non-invariant model has an equivalent gauge invariant version in the same phase space that can be identified with the vector Schwinger model.

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I. INTRODUCTION

Imposition of chiral constraint has a remarkable feature in the $(1 + 1)$ dimensional field theoretical model. It was first introduced in the seminal work of Harada [1]. Imposition of chiral constraint puts a restriction on the degrees of freedom of a boson with a particular chirality depending on the nature of constraint. In the article [1], it was shown that imposing a chiral constraint in the phase-space of the chiral Schwinger model [2, 4, 5] it could be expressed in terms of a chiral boson, which was the basic ingredient of heterotic String theory [6, 7]. The idea of the imposition of chiral constraint has been used in different situations [8–12]. The fascinating role of imposition of chiral constraint to save the phenomena of s-wave scattering off dilaton black hole from the danger of information loss is observed in [9]. In the article [13], we found that the model generated in [1] by the imposition of the chiral constraint had its origin in the gauge model of chiral boson [14]. Apart from the standard Lorentz co-variant one-parameter class of regularization [2, 3], in the article [15, 16] the authors showed that the chiral Schwinger model was also found to be physically sensible for a Lorentz non-covariant parameter-free regularization which resulted in a Faddeevian type of anomaly [17, 18]. The imposition of chiral constraint in the phasespace of the theory enables us to express this model in terms of chiral boson in [9]. The resulting model in this situation also finds its origin in the gauged version of the chiral boson [14], when the co-variant mass masslike term for the gauge field is replaced by a non-covariant masslike term [13]. The role of the chiral constraint that has been studied so far was restricted on the model where interaction is of chiral nature. A natural question may arise whether the imposition of chiral constraint on the models where the interaction is of vector and axial-vector nature may lead to a physically sensible field-theoretic model as it was found in the case of the model when the interaction was of chiral nature. Keeping it in view, an attempt has been made to impose, the chiral constraint on vector and axial-vector Schwinger model [19, 20] and observe that chiral Schwinger model with Faddeevian anomaly due to Mitra, the vector Schwinger model, and axial-vector Schwinger model all map onto a single gauge non-invariant model which we are going to describe here. This gauge non-invariant model indeed has an equivalent gauge-invariant version in the usual phase-space which can be identified with the vector Schwinger model.

The article is organized as follows. In Sec.II, we have given a brief review of the models in which we are going to impose the chiral constraint. Sec. III is devoted to the imposition of chiral constraint in the vector Schwinger model, axial-vector Schwinger model, and the chiral Schwinger model with parameter-free Faddeevian anomaly. In Sec.IV, we carry out the gauge-invariant reformulation of the model, which has resulted after the imposition of chiral constraint on the said models. Sec. V has devoted to the description of the theoretical spectra of the model resulted after the imposition of the chiral constraint. Sec. VI contains a brief discussion and conclusions.

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II. BRIEF REVIEW OF THE MODELS

A. Chiral Schwinger model

The chiral Schwinger model is described by the fermionic Lagrangian density

$$\mathcal{L}_{CS} = \bar{\psi}\gamma^\mu[i\partial_\mu + gA_\mu(1 - \gamma_5)]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \quad (1)$$

Here ψ and A_μ are fermion and gauge fields respectively. The field strength tensor is defined by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The indices μ and ν take the value 0 and 1 in (1 + 1) dimension. The nature of the interaction between fermion and gauge field is chiral for this model. The Jackiw-Rajaram version of bosonized Lagrangian had a one-parameter class of the covariant mass-like term for the gauge field that entered into the model in the process of bosonization in order to remove the singularity in the fermionic determinant during the course of eliminating the fermion by integration. The bosonized version of the model (2) with Faddeevian type of anomaly [15, 16, 21, 22] is given by

$$\begin{aligned} \mathcal{L}_{BCS} &= \int dx \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + g(\epsilon_{\mu\nu} + g_{\mu\nu}) \partial^\nu \phi A^\mu + \frac{1}{2} g^2 (A_0^2 - 2A_0 A_1 - 3A_1^2) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right] \\ &= \int dx \left[\frac{1}{2} (\dot{\phi}^2 - \phi'^2) + g(\dot{\phi} + \phi')(A_0 - A_1) + \frac{1}{2} g^2 (A_0^2 - 2A_0 A_1 - 3A_1^2) + \frac{1}{2} (\dot{A}_1 - A_0')^2 \right]. \end{aligned} \quad (2)$$

Here $\epsilon_{\mu\nu}$ is the Levi-Civita symbol in two dimension: $\epsilon^{01} = -\epsilon^{10} = 1$, and Minkowski metric $g_{\mu\nu} = \text{diag}(1, -1)$. Equation (2) was initially found in [15] where Mitra termed it as chiral Schwinger model with Faddeevian regularization since the Gauss law constraint of this theory gave a specific nontrivial contribution. We will discuss it later. Let us now discuss the theoretical spectrum as offered by this model. From the standard definition, the momentum corresponding to the field ϕ , A_1 , and A_0 read

$$\pi_\phi = \dot{\phi} + g(A_0 - A_1), \quad (3)$$

$$\pi_1 = \dot{A}_1 - A_0', \quad (4)$$

$$\pi_0 \approx 0. \quad (5)$$

The following Legendre transformation

$$H_B = \int dx [\pi_\phi \dot{\phi} + \pi_1 \dot{A}_1 - \mathcal{L}_B], \quad (6)$$

leads to the Hamiltonian

$$H_B = \int \mathcal{H}_B dx = \int dx \left[\frac{1}{2} (\pi_1^2 + \phi'^2 + \pi_\phi^2) + \pi_1 A_0' + g(\pi_\phi + \phi')(A_0 - A_1) + 2g^2 A_1^2 \right]. \quad (7)$$

The Gauss law constraint that comes out from the preservation of $\pi_0 \approx 0$ is

$$G = \pi_1' + g(\pi_\phi + \phi')' \approx 0, \quad (8)$$

which has the following nontrivial Poisson bracket

$$[G(x), G(y)] = 2g^2 \delta(x - y)', \quad (9)$$

which we have mentioned already and it is the reason the model is said to carry the Faddeevian anomaly. Note that this Poisson bracket (9) gave the vanishing contribution for the Jackiw-Rajaraman version of the chiral Schwinger model [2]. It was the pioneering observation of Faddeev that anomaly made Poisson bracket between $G(x)$ and $G(y)$ non-vanishing [17, 18]. The constraint became second class itself and gauge invariance got violated. He, however, argued that it would be possible to quantize the theory but in this situation, the system might possess more degrees of freedom. In the article [15], analysis of theoretical spectra with the help of quantization of constrained system due to Dirac [23] was carried out where it was found that in the phase space of the model, along with the constraints (5) and (8), two more secondary constraints were embedded in the phase space, which were the following:

$$\pi_1' + 2e\phi' \approx 0, \quad (10)$$

$$A'_1 + A'_0 \approx 0, \quad (11)$$

and these four constraints altogether formed a second class set. The reduced Hamiltonian of the system obtained after imposition of the four constraints was found out to be

$$H_r = \int dx \mathcal{H}_r = \int dx \left[\frac{1}{2}(\pi_1^2 + \frac{1}{g^2}\pi_1'^2) + \phi'^2 + \pi_1 A'_1 + \frac{1}{g}\pi_1' \phi' + 2g^2 A_1^2 \right]. \quad (12)$$

The Dirac brackets of the field describing the reduced hamiltonian were.

$$[\phi(x), \phi(y)]^* = -\frac{1}{4}\epsilon(x-y), \quad (13)$$

$$[A(x), A(y)]^* = \delta'(x-y), \quad (14)$$

$$[A(x), \phi(y)]^* = \delta(x-y). \quad (15)$$

Here ” * ” indicates the dirac brackets. The theoretical spectrum contained a massive and a mass less boson chiral boson described by the following equations

$$[\square + 4g^2]A_1 = 0, \quad (16)$$

$$(\partial_0 + \partial_1)\mathcal{H} = 0, \mathcal{H} = \phi + \frac{1}{g}(\dot{A}_1 + A'_0). \quad (17)$$

Let us now turn to the bosonized vector Schwinger model

B. Vector Schwinger model

The vector Schwinger model in the fermionic version is defined by the Lagrangian density

$$\mathcal{L}_{VS} = \bar{\psi}\gamma^\mu [i\partial_\mu + eA_\mu]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \quad (18)$$

The bosonized version of the model reads

$$\begin{aligned} \mathcal{L}_{BVS} &= \int dx \left[\frac{1}{2}\partial_\mu \phi \partial^\mu \phi + e\epsilon_{\mu\nu} \partial^\nu \phi A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right], \\ &= \int dx \left[\frac{1}{2}(\dot{\phi}^2 - \phi'^2) + e(A_1 \dot{\phi} - A_0 \phi') + \frac{1}{2}(\dot{A}_1 - A'_0)^2 \right]. \end{aligned} \quad (19)$$

The momenta corresponding to the fields are

$$\pi_\phi = \dot{\phi} + eA_1, \quad (20)$$

$$\pi_0 \approx 0, \quad (21)$$

$$\pi_1 = \dot{A}_1 - A'_0. \quad (22)$$

The canonical Hamiltonian is found out to be

$$H_{CV} = \int dx \left[\frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2) + \pi_1 A'_0 + eA_0 \phi' - eA_1 \pi_\phi \right] \quad (23)$$

The two first class constraints which are present in the phase space of the system are

$$\pi_0 \approx 0, \quad (24)$$

$$\pi_1 - e\phi' \approx 0 \quad (25)$$

Two gauge fixing conditions chosen here are in order to quantize the theory such that the real physical degrees of freedom be identified. The conditions are

$$A_0 \approx 0, \quad \pi_\phi \approx 0. \quad (26)$$

The reduced Hamiltonian obtained after imposition of the constraints and the gauge fixing conditions reads

$$H_R = \int dx \left[\frac{1}{2} \pi_1^2 + \frac{1}{2e^2} \pi_1'^2 + \frac{1}{2} e^2 A_1^2 \right] \quad (27)$$

Here Dirac brackets remain canonical. This reduced Hamiltonian with the use of canonical Dirac brackets renders the following two first-order equations of motion:

$$\dot{\pi}_1 = -e^2 A_1, \quad (28)$$

$$\dot{A}_1 = \pi_1 - \frac{1}{2e^2} \pi_1''. \quad (29)$$

The above two equations ultimately reduce to the following two second order differential equations of motion:

$$[\square + e^2] \pi_1 = 0, \quad [\square + e^2] A_1 = 0. \quad (30)$$

The above two equations represent a massive boson with mass e and its momentum conjugate.

C. Axia-vector Schwinger model

The same model with axial vector interaction is described by the Lagrangian density

$$\mathcal{L}_{AVS} = \bar{\psi} \gamma^\mu [i\partial_\mu + \tilde{e}\gamma_5 A_\mu] \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \quad (31)$$

Here Vector interaction between matter and gauge field is replaced by axial vector. The bosonized version of the model reads

$$\begin{aligned} \mathcal{L}_{BAVS} &= \int dx \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \tilde{e} \partial^\mu \phi A_\mu + \frac{1}{2} \tilde{e}^2 A_\mu A^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right] \\ &= \int dx \left[\frac{1}{2} (\dot{\phi}^2 - \phi'^2) + \tilde{e} (A_0 \dot{\phi} - A_1 \phi') + \frac{1}{2} \tilde{e}^2 (A_0^2 - A_1^2) + \frac{1}{2} (\dot{A}_1 - A_1')^2 \right], \end{aligned} \quad (32)$$

We now proceed to study the phasespace structure of the axial-vector Schwinger model. To this end, we consider the bosonized Lagrangian of the axial-vector Schwinger model (32). The momenta corresponding to the fields ϕ , A_0 , and A_1 are

$$\pi_\phi = \dot{\phi} + \tilde{e} A_0, \quad (33)$$

$$\pi_0 \approx 0, \quad (34)$$

$$\pi_1 = \dot{A}_1 - A_1' \quad (35)$$

A straightforward calculation leads to following canonical Hamiltonian.

$$H_{CAV} = \int dx \left[\frac{1}{2} (\pi_1^2 + \pi_\phi^2 + \phi'^2) + \pi_1 A_1' + \tilde{e} A_1 \phi' - \tilde{e} A_0 \pi_\phi \right]. \quad (36)$$

Here equation (34) is the primary constraint of the theory and the preservation of this constraint leads to

$$\pi_1 + e\pi_\phi \approx 0, \quad (37)$$

which is the secondary constraint. The constraints (34) and (37) form a first-class set. So like the vector Schwinger model, two gauge fixing conditions are needed to find out the physical degrees of freedom. The gauge fixing conditions are chosen as

$$A_0 \approx 0, \quad \phi' \approx 0. \quad (38)$$

The reduced Hamiltonian which comes out after imposition of the constraint and the gauge fixing conditions reads

$$H_R = \int dx \left[\frac{1}{2} \pi_1^2 + \frac{1}{2\tilde{e}^2} \pi_1'^2 + \frac{1}{2} \tilde{e}^2 A_1^2 \right]. \quad (39)$$

It has been found that the Dirac Brackets are canonical for this system too. The canonical dirac brackets along with the Hamiltonian (39) lead to the following equations of motion

$$\dot{\pi}_1 = -\tilde{e}^2 A_1, \quad (40)$$

$$\dot{A}_1 = \pi_1 - \frac{1}{2\tilde{e}^2} \pi_1''. \quad (41)$$

The above two equations ultimately reduce to

$$[\square + \tilde{e}^2] \pi_1 = 0, \quad [\square + \tilde{e}^2] A_1 = 0. \quad (42)$$

Like the vector Schwinger model, these two equations also indicate that the spectrum contains a massive boson with mass e and its momentum conjugate.

In all the cases the bosonized version of the model is equivalent to the fermionic version. However in the bosonized version the models contain a quantum correction because the process of bosonization involves the integrating out of the fermions from the actions of the models that lead to fermionic determinant which carries singularity, and to remove the singularity regularization is needed. So, the different counter terms may result depending on the choice of regularization. However, the model with different counter terms are equivalent to the original fermionic version of the models

III. IMPOSITION OF CHIRAL CONSTRAINTS ON THE MODELS

We have already mentioned that the imposition of chiral constraint rendered remarkable services in different situations. The mathematical form chiral constraint, which was introduced in [1] was

$$\Omega(x) = \pi_\phi(x) - \phi'(x) \approx 0. \quad (43)$$

. It may be of the form $\Omega(x) = \pi_\phi(x) + \phi'(x) \approx 0$. The constraint with this form will restrict the degrees of freedom of the boson with the chirality opposite to the chirality restricted by the constraint (43). These are a second-class constraint since the Poisson bracket of the constraints with themselves are non-vanishing and the inverse of these exist.

$$[\Omega(x), \Omega(y)] = -2\delta'(x - y). \quad (44)$$

We saw its remarkable role in the article [1] where the author was able to describe the Chiral Schwinger model in terms of Chiral boson. We are now intended to impose this chiral constraint in the phase space of the three models defined with three different types of interaction. The models are discussed in Sec.II. Let us first consider the chiral Schwinger model with parameter-free Faddeevian anomaly.

A. Imposition of chiral constraints on the chiral Schwinger models with parameter-free Faddeevian anomaly

The bosonized version of Lagrangian of this model is described in Eqn. (2). If we imposing the constraint $\Omega(x) \approx 0$, into the phasespace of the model the generating functional can be written down as

$$Z_{CH} = \int d\phi d\pi_\phi dA_1 d\pi_1 \delta(\pi_\phi - \phi') \sqrt{\det[\Omega(x), \Omega(y)]} e^{i \int d^2x [\pi_\phi \dot{\phi} + \pi_1 \dot{A}_1 - \mathcal{H}_B]} \quad (45)$$

After a few steps of algebra we land onto to the following:

$$Z_{CH} = N \int d\phi dA_1 e^{i \int d^2x \mathcal{L}_{CH}}, \quad (46)$$

Where N stands for the normalization constant, and \mathcal{L}_{CH} has the expression

$$\mathcal{L}_{CH} = \dot{\phi}\phi' - \phi'^2 + 2g(A_0 - A_1)\phi' + 2g^2A_1^2 + \frac{1}{2}(\dot{A}_1 - A'_0)^2. \quad (47)$$

This can be identified with the gauged Lagrangian for chiral boson obtained from the bosonized Lagrangian with parameter Faddeevian regularization [15] just by imposing the chiral constraint in its phase space. Harada in [1], obtained the same type of result for the usual chiral Schwinger model with one parameter class of regularization proposed by Jackiw and Rajaraman [2]. The Lagrangian (47) can be thought of as the gauged version of chiral boson described by Floreanini and Jackiw [24].

B. Imposition of chiral constraints on the vector Schwinger models

Let us now impose the chiral constraint in the pasespace of the vector Schwinger model. The bosonized version of the Lagrangian of this model is given in Eqn. (19). The chiral constraint which was imposed in the chiral Schwinger model with the Faddeevian anomaly was

$$\Omega(x) = \pi_\phi(x) - \phi'(x) \approx 0. \quad (48)$$

The Poisson bracket of this constraint with itself is non-vanishing as usual

$$[\Omega(x), \Omega(y)] = -2\delta'(x - y). \quad (49)$$

. So it is a second class constraint. The way Harada imposed the chiral constraint [1] in the chiral Schwinger model can be extended to the Schwinger model without any hindrance since no physical principle will be violated here too like the Chiral Schwinger model: All though the nature of the interaction is different in the models, in the kinetic term boson's of both the chirality are present and the constrain in Eqn. (43) puts restriction on the kinematics of the boson. If we imposing the constraint $\Omega(x) \approx 0$, the generating functional corresponding this model can be written down as follows:

$$Z_{CHV} = \int d\phi d\pi_\phi dA_1 d\pi_1 \delta(\pi_\phi - \phi') \sqrt{\det[\Omega(x), \Omega(y)]} e^{i \int d^2x [\pi_\phi \dot{\phi} + \pi_1 \dot{A}_1 - \mathcal{H}_{BVS}]} \quad (50)$$

A straightforward calculation leads to

$$Z_{CHV} = \tilde{N} \int d\phi dA_1 e^{i \int d^2x \mathcal{L}_{CHV}}, \quad (51)$$

where \tilde{N} is the normalization constant. The Lagrangian density \mathcal{L}_{CHV} reads

$$\mathcal{L}_{CHV} = \dot{\phi}\phi' - \phi'^2 + 2e(A_0 - A_1)\phi' + 2e^2A_1^2 + \frac{1}{2}(\dot{A}_1 - A'_0)^2. \quad (52)$$

It is gauge non-invariant Lagrangian here Lorentz co-variance is not manifested.

C. Imposition of chiral constraints on the axial vector Schwinger models

The bosonized version of the Lagrangian of the axial-vector Schwinger model is given in Eqn. (32). Let us now impose the same chiral constraint $\Omega(x) = \pi_\phi(x) - \phi'(x) \approx 0$ in the phasespace of the axial-vector Schwinger models. If we do so the generating functional of the axial-vector Schwinger will be written down as

$$Z_{CHA} = \int d\phi d\pi_\phi dA_1 d\pi_1 \delta(\pi_\phi - \phi') \sqrt{\det[\Omega(x), \Omega(y)]} e^{i \int d^2x [\pi_\phi \dot{\phi} + \pi_1 \dot{A}_1 - \mathcal{H}_{BACS}]}. \quad (53)$$

After a little algebra it reduces to

$$Z_{CHA} = \bar{N} \int d\phi dA_1 e^{i \int d^2x \mathcal{L}_{CHA}}, \quad (54)$$

Here \mathcal{L}_{CHA} is given by

$$\mathcal{L}_{CHA} = \dot{\phi}\phi' - \phi'^2 + 2\tilde{e}(A_0 - A_1)\phi' + 2\tilde{e}^2 A_1^2 + \frac{1}{2}(\dot{A}_1 - A_0')^2, \quad (55)$$

and \bar{N} represents the normalization constant. It is also a gauge non-invariant Lagrangian where Lorentz co-variance is not manifested. It is important to note that the Lagrangians obtained after imposition of chiral constraint in the phasespaces of the Chiral Schwinger model with parameter-free Faddeevian anomaly, Vector Schwinger model, and the axial-vector Schwinger model are not different. It is surprising indeed that three models with different interactions become indistinguishable after the imposition of chiral constraint. It is undoubtedly a novel aspect of imposition of chiral constraint

IV. DESCRIPTION OF THEORETICAL SPECTRA OF THE RESULTING MODEL OBTAINED AFTER IMPOSITION OF THE CHIRAL CONSTRAINT

The constraint structure and the theoretical spectra of this model is known [15]. The primary constraint of the theory are

$$\Omega_1 = \pi_0 \approx 0 \quad (56)$$

$$\Omega_2 = \pi_\phi - \phi' \approx 0 \quad (57)$$

The canonical Hamiltonian is

$$H_{CB} = \int \mathcal{H}_{CB} dx = \int dx \left[\frac{1}{2} \pi_1^2 + \phi'^2 + \pi_1 A_0' - 2g\phi'(A_0 - A_1) + 2g^2 A_1^2 \right]. \quad (58)$$

The secondary constraints are

$$\Omega_3 = \pi_1' + 2g\phi' \approx 0, \quad (59)$$

$$\Omega_4 = (A_1 + A_0)' \approx 0. \quad (60)$$

The reduced Hamiltonian reads

$$H_{CR} = \int dx \mathcal{H}_{CR} = \int dx \left[\frac{1}{2} \pi_1^2 + \frac{1}{4g^2} \pi_1'^2 + \pi_1 A_1' + 2g^2 A_1^2 \right]. \quad (61)$$

The non-canonical bracket reads

$$[A_1(x), A_1(y)] = \frac{1}{2g^2} \delta'(x - y) = 0 \quad (62)$$

The theoretical spectra contains only a massive boson with mass $2g$

$$[\square + 4g^2]A_1 = 0 \quad (63)$$

V. GAUGE INVARIANT REFORMULATION USING MITRA-RAJARAMAN'S FORMALISM

Mitra and Rajaraman developed an ingenious formalism in their seminal work [25, 26] for obtaining a gauge-invariant theory by reducing the number of constraints from a second class set of constraints belonging to a theory retaining only the first-class subset. The remarkable feature of this formalism is that the gauge invariance is received in the usual phasespace of the theory. Unlike Stueckelberg formalism [27–29], the extension of phasespace is not required here. For a gauge theory fixing of the gauge is required in order to single out the physical degrees of freedom and it

is done by imposing a suitable number of gauge fixing conditions. The first-class set of constraints that the theory is endowed with form a second class set together with the gauge fixing condition. As a result, the theory turns into an equivalent second-class system. In the article [25, 26], an inverse to the above procedure is invoked that enables to have a first-class gauge-invariant system corresponding to a second class theory gauge variant theory.

The reduction of the number of constraints is done in such a way that the constraints which are eliminated may be thought of as the gauge fixings of the first-class set of constraints that retains in. This formulation crucially depends on the constraints that embed in the phasespace of the theory and different gauge-invariant version may result which solely depend on which set of first-class constraints are retained. Since no extension of phase space is done here the physical contents of the resulting gauge-invariant actions remain unaltered. What follows next is the use of this formalism to have a gauge-invariant version of the chiral Schwinger model with parameter-free Faddeevian regularization when it is described in terms of chiral boson [15]

This model described in (47) contain two primary constraints

$$\Omega_1 = \pi_0 \approx 0, \quad (64)$$

$$\Omega_2 = \pi_\phi - \phi' \approx 0. \quad (65)$$

. The effective Hamiltonian of this system is therefore given by

$$H_{eff} = H_{CB} + v_1\pi_0 + v_2(\pi_\phi - \phi'), \quad (66)$$

where the canonical Hamiltonian H_{CB} is given by

$$H_{CB} = \int dx \left[\frac{1}{2}\pi_1^2 + \pi_1 A'_0 + \phi'^2 - 2g(A_0 - A_1) + 2g^2 A_1^2 \right] \quad (67)$$

There are two secondary constraints in the phase space of the theory which are the following

$$\Omega_3 = \pi'_1 + 2g\phi' \approx 0, \quad (68)$$

$$\Omega_4 = A'_1 + A'_0 \approx 0. \quad (69)$$

The Poisson bracket between $\Omega_1 \approx 0$ and $\Omega_3 \approx 0$ vanishes. Therefore, these two constraints form a first-class subset from the full set of four constraints. Following the formalism developed in [25, 26], if we are intended to retain only these two constraints (64), and (65) the Hamiltonian needs the following modification

$$\begin{aligned} H_{MOD} = & \int dx \left[\frac{1}{2}\pi_1^2 + \pi_1 A'_0 - 2e(A_0 - A_1)\phi' + 2g^2 A_1^2 + \pi_\phi \phi' - e\pi_\phi(A_0 - A_1) \right. \\ & \left. + \frac{1}{2}(\pi_\phi - \phi')^2 + g(\pi_\phi - \phi')(A_0 + A_1) + w\pi_0 \right]. \end{aligned} \quad (70)$$

Note that this modified Hamiltonian (70) retains only two first-class constraints $\Omega_1 \approx 0$ and $\Omega_3 \approx 0$, because the preservation of the constraint $\Omega_1 \approx 0$, and $\Omega_3 \approx 0$ with respect to the modified Hamiltonian does not lead to any new constraint and it does not alter the physical contents of the theory since the modified Hamiltonian contains only those fields with which it was defined. The equations of motion that follow from the modified first-class Hamiltonian(70) are

$$\dot{\phi} = [\phi, H_{MOD}] = \pi_\phi + 2gA_1, \quad (71)$$

$$\dot{A}_0 = [A_0, H_{MOD}] = -u, \quad (72)$$

$$\dot{A}_1 = [A_1, H_{MOD}] = \pi_1. \quad (73)$$

Through a Legendre transformation we obtain the Lagrangian corresponding to the modified Hamiltonian (70)

$$\begin{aligned} L = & \int dx \left[\pi_\phi \dot{\phi} + \pi_1 \dot{A}_1 + \pi_0 \dot{A}_0 - \left(\frac{1}{2}\pi_1^2 + \pi_1 A'_0 + 2gA_1 \pi_\phi \right. \right. \\ & \left. \left. + \pi_\phi \phi' - 2gA_0 \phi' + \frac{1}{2}(\pi_\phi - \phi')^2 + 2g^2 A_1^2 + w\pi_0 \right) \right]. \end{aligned} \quad (74)$$

By the use of equations (71), (72), and (73) Lagrangian (74) is expressed in a simplified form after a little algebra:

$$\begin{aligned} L &= \int dx \left[\frac{1}{2}(\dot{\phi}^2 - \phi'^2) - 2g(A_1\dot{\phi} - A_0\phi') + \frac{1}{2}(\dot{A}_1 - \dot{A}_0)^2 \right] \\ &= \int dx \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + 2g\epsilon_{\mu\nu}\partial^\nu\phi A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right] \end{aligned} \quad (75)$$

The Lagrangian(75), corresponds to the Hamiltonian(70), and it has the identical set of equations of motion (71), (72), and (73). It is straightforward to see that the Lagrangian (75) has two primary constraints (64) and (65) only. This Lagrangian can easily be identified with the bosonized version of the well-celebrated vector Schwinger model [19, 20]. Here coupling strength is $2g$. We can set $g = \frac{1}{2}e$ to make it identical to with equation (19).

It is known from the seminal work of Dirac [23] that the gauge transformation generator is constructed from the first-class constraints of a theory. The gauge transformation generator in this situation can be written down as

$$\mathcal{G} = \int dx(\Lambda_1\Omega_1 + \Lambda_2\Omega_3). \quad (76)$$

Here Λ_1 and Λ_2 are two arbitrary parameters that will be fixed later. The gauge transformation for the fields ϕ , A_1 , and A_0 that stems out from the generator read

$$\delta\phi = 0, \delta A_1 = -\Lambda'_1, \delta A_0 = -\Lambda_2. \quad (77)$$

It is straightforward to see that the under the transformation (77), the Lagrangian (75) remains invariant if the the parameters Λ_1 and Λ_2 are constrained to

$$\Lambda_2 = \dot{\Lambda}_1. \quad (78)$$

This transformation is equivalent to the familiar form of the gauge transformation $A_\mu \rightarrow A_\mu + \frac{1}{2g}\partial_\mu\Lambda$. There is something interesting that we must mention here. Note that the gauge-invariant version of the chiral Schwinger model with parameter-free Faddeevian regularization land on to the celebrated Schwinger model and the Schwinger model under a chiral constraint lands onto the chiral Schwinger model with the parameter-free Faddeevian regularization. It is surprising that the nature of interaction of the two models to start with was different. However the imposition of the chiral constraint in the phase-space makes the nature of interaction indistinguishable and that ultimately lead identical theoretical spectra.

VI. DISCUSSION AND CONCLUSIONS

In this article we consider three different models having different types of interaction between the matter and the gauge field. The models are with parameter-free Faddeevian anomaly, the vector Schwinger model, and the axial-vector Schwinger model. We should mention here that the Schwinger itself can be described with two different types of interaction namely vector and axial-vector interaction. We have used the bosonized version of all these models, which contain quantum corrections that enter through the regularization needed in order to remove the singularity in the fermionic determinant that appears during the process of integrating out the fermions. So different counter-terms result because of the choice of regularization. For the vector and axial-vector Schwinger model we consider the usual regularization. However, for the Chiral Schwinger model, we use the parameter-free Faddeevian regularization developed in [15, 16, 21, 22]. We impose chiral constraint following the article [1] in the phase space of all these three models and found that all the three models map onto a gauge non-invariant model and it has no manifestly Lorentz covariant structure, but it is known from the article [16], that the model exhibits Lorentz invariant spectrum. It is also observed that the gauge-invariant version of the resulting model maps onto the vector bosonized version of the vector Schwinger model.

It is surprising that three different models having different types of interaction between the matter and gauge field become indistinguishable under a chiral constraint at the quantum mechanical level. To unveil the precise reason for obtaining this the remarkable results need further involved investigations. However, some comments can be made. The models considered here are described in (1+1) dimensional spacetime manifold. The constraint structure and the constrained subspace of the models are such that under this specific chiral constraint the physical phasespace become indistinguishable and that, in turn, result in the identical theoretical spectra.

Besides, the imposition of chiral constraint in the vector Schwinger model, axial-vector Schwinger model, and chiral Schwinger model with parameter-free Faddeevian anomaly leads to single gauge non-invariant model which has a

gauge invariant structure in the same phase space that can be identified with the gauge invariant vector Schwinger model. It also favors the indistinguishable nature of the models.

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