New Non-Relativistic String in $AdS_5 \times S^5$

J. Kluso
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Department of Theoretical Physics and Astrophysics, Faculty of Science, Masaryk University, Kotlářská 2, 611 37, Brno, Czech Republic

Abstract

We study non-relativistic limit of $AdS_5 \times S^5$ background and determine corresponding Newton-Cartan fields. We also find canonical form of the new non-relativistic string in this background and discuss its formulation in the uniform light-cone gauge.

¹Email addresses: klu@physics.muni.cz

1 Introduction and Summary

In the past few years it is observed renewed interest in the study of non-relativistic string theories in Newton-Cartan (NC) formulation [1, 2]. Basically, NC gravity provides covariant description of Newton's law. However it is very remarkable that NC description can be extended also into more broader class of theories, as for example field theory and string theory. In fact, non-relativistic string theory was originally introduced in 2000 in two papers [3, 4]. These theories were defined without Newton-Cartan formalism which was firstly introduced in the context of string theory in the remarkable paper [5], for related works, see for example [34, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. Different formulation of non-relativistic string was presented in [6] that was based on T-duality of string theory in the background with null isometry ². In [34] new form of non-relativistic string theory was proposed that has interesting property that non-relativistic string naturally couples to two form field m_{MN} in the similar way as non-relativistic particle couples to mass form m_M .

This new proposal of non-relativistic string was further studied in [40] where canonical formulation of this theory was found. We also analysed the possibility to impose uniform light-cone gauge on this theory and found Hamiltonian on reduced phase space. Uniform light-cone gauge was used in [35, 37, 38, 39] ³ where it was shown that it is very efficient for the study of dynamics of the relativistic string in $AdS_5 \times S^5$. We showed in [40] that such an uniform light cone gauge fixing can be imposed in case of non-relativistic string as well at least at the formal level.

In this paper we continue the analysis of new non-relativistic string when we focus on its explicit formulation as non-relativistic limit of $AdS_5 \times S^5$ background. Our starting point is an important paper [41] where non-relativistic strings in $AdS_5 \times S^5$ was defined by specific limiting procedure. We combine this procedure with the definition of Newton-Cartan fields as was given in [34] and we will be able to find Newton-Cartan background fields for non-relativistic limit of $AdS_5 \times S^5$. Explicitly, we find 2×2 twobein τ_M^A together with field π_M^A that was introduced in [34]. Then we will be able to determine two form m_{MN} and hence corresponding Hamiltonian. As the next step we study gauge fixed form of the theory when we impose uniform light-cone gauge. Solving Hamiltonian constraint we determine Hamiltonian on the reduced phase space. We find that the structure of this Hamiltonian depends on the free parameter that defines generalized uniform light cone gauge [35, 37, 38, 39]. Then we study equations of motion on the reduced phase space. We show that it is possible to have configuration with all free fields to be equal to zero and concentrate on the dynamics of the mode z_1 . However then we find that the equation of motion for z_1 is solved by arbitrary function and hence does not determine dynamics of z_1 at all. We mean that this is a sign that the present form of uniform light cone gauge is not suitable for the specific form of non-relativistic string studied in this paper.

Then in order to study properties of non-relativistic string in more details we

²For related works, see [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 15].

³For review, see for example [36].

focus on its Lagrangian equations of motion. We determine their form for general background. Since these equations of motion are rather complicated in the full generality we restrict ourselves to an analysis of the dynamics of single coordinate z_1 . We find solution that has formally the same form as solution found recently in [42] however there is a crucial difference since we consider extended string along non-compact coordinate and hence we should interpret this solution as the string with infinite number of spikes.

Let us outline our results. The main goal of this paper was to find Hamiltonian for new non-relativistic string in $AdS_5 \times S^5$ background. Performing appropriate limit we determined components of Newton-Cartan fields and then we obtained corresponding Hamiltonian. We studied its gauge fixed form and we argued that the uniform light cone gauge could be too restrictive to obtain interesting dynamics. On the other hand it is possible that different gauge fixing procedure, as for example static gauge, could lead to non-trivial dynamics of free world-sheet fields on AdS_2 background as was shown in [41]. We also studied Lagrangian equations of motion and we found solution corresponding to string extended along one free spatial coordinate and non-trivial dynamics along of z_1 coordinate that agrees with solution found in [42].

This paper is organized as follows. In the next section (2) we review basics facts about new non-relativistic string and its canonical formulation. Then in section (3) we find its form in non-relativistic limit of $AdS_5 \times S^5$. In section (4) we study properties of this string in uniform light-cone gauge. In section (5) we determine Lagrangian equations of motion and study corresponding solution. Finally in appendix (5) we perform non-relativistic limit in coordinates that were used [41] and find corresponding Hamiltonian.

2 Review of New Non-Relativistic String and Its Canonical Formulation

In this section we review basic facts about new non-relativistic string action as was proposed in [34] and that has the form

$$S = -\frac{T}{2} \int d^2 \sigma \sqrt{-\tau} [\tau^{\alpha\beta} h_{MN} + \epsilon^{\alpha\beta} m_{MN}] \partial_\alpha x^M \partial_\beta x^N .$$
 (1)

We firstly describe derivation of this action, following [34]. Let us introduce relativistic vielbein $e_{\overline{M}}^{a}$ so that target space metric has the form

$$g_{MN} = e_M^{\ \underline{a}} e_N^{\ \underline{b}} \eta_{\underline{a}\underline{b}} , \qquad (2)$$

where we use the similar notation as in [34] so that frame indices are $\underline{a}, \underline{b} = 0, \ldots, 9$ and where $\eta_{\underline{a}\underline{b}} = \text{diag}(-1, 1, \ldots, 1)$. Note that space-time indices are $M, N = 0, 1, \ldots, 9$. Following [34] we also introduce parametrization of NSNS two form B_{MN} as

$$B_{MN} = \frac{1}{2} \eta_{\underline{a}\underline{b}} \left(e_{\underline{M}}^{\underline{a}} \pi_{\underline{N}}^{\underline{b}} - e_{\underline{N}}^{\underline{a}} \pi_{\underline{M}}^{\underline{b}} \right) \,. \tag{3}$$

To begin with let us write Nambu-Goto form of the action for relativistic string in general background

$$S = -cT_F \int d^2\sigma \sqrt{-\det g_{\alpha\beta}} - cT_F \int d^2\sigma \frac{1}{2} \epsilon^{\alpha\beta} B_{\alpha\beta} , \qquad (4)$$

where $g_{\alpha\beta} = g_{MN}\partial_{\alpha}x^{M}\partial_{\beta}x^{N}$, $B_{\alpha\beta} = B_{MN}\partial_{\alpha}x^{M}\partial_{\beta}x^{N}$ and where $\epsilon^{01} = 1 = -\epsilon_{01}$ and T_{F} is string tension. As in [34] we introduce indices $\underline{a} = (A, a)$ corresponding to directions longitudinal and transverse to string world-sheet where A = 0, 1 are longitudinal and $a = 2, \ldots, 9$ are transverse. Then we have

$$e_{\overline{M}}^{\underline{a}} = (cE_{M}^{A}, e_{M}^{a}) , \quad \pi_{\overline{M}}^{\underline{a}} = (c\Pi_{M}^{A}, \pi_{M}^{a})$$

$$(5)$$

so that

$$g_{\alpha\beta} = c^{2} \eta_{AB} E_{\alpha}^{\ A} E_{\beta}^{\ B} + \delta_{ab} e_{\alpha}^{\ a} e_{\beta}^{\ b} ,$$

$$B_{\alpha\beta} = \frac{1}{2} c^{2} \eta_{AB} (E_{\alpha}^{\ A} \Pi_{\beta}^{\ B} - E_{\beta}^{\ A} \Pi_{\alpha}^{\ B}) + \frac{1}{2} \delta_{ab} (e_{\alpha}^{\ a} \pi_{\beta}^{\ b} - e_{\beta}^{\ a} \pi_{\alpha}^{\ b}) .$$

(6)

We further parametrize longitudinal components in the following way

$$E_{M}^{\ A} = \tau_{M}^{\ A} + \frac{1}{2c^{2}}\pi_{M}^{\ B}\epsilon_{B}^{\ A} , \quad \Pi_{M}^{\ A} = \epsilon_{\ B}^{A}\tau_{M}^{\ B} + \frac{1}{2c^{2}}\pi_{M}^{\ A} ,$$
(7)

where $\epsilon_B^{\ A} = \epsilon_{BC} \eta^{CA}$. Inserting (7) into (2) we finally get

$$g_{\alpha\beta} = c^{2} \tau_{\alpha\beta} + \frac{1}{2} \eta_{AB} (\tau_{\alpha}^{\ A} \pi_{\beta}^{\ C} \epsilon_{C}^{\ B} + \tau_{\beta}^{\ B} \pi_{\alpha}^{\ C} \epsilon_{C}^{\ B}) + h_{\alpha\beta} + \frac{1}{4c^{2}} \eta_{AB} \pi_{\alpha}^{\ C} \epsilon_{C}^{\ A} \pi_{\beta}^{\ D} \epsilon_{D}^{\ B} ,$$

$$(8)$$

where

$$\tau_{\alpha\beta} = \tau_{\alpha}^{\ A} \tau_{\beta}^{\ B} \eta_{AB} , \quad h_{\alpha\beta} = e_{\alpha}^{\ a} e_{\beta}^{\ b} \delta_{ab} .$$
⁽⁹⁾

Then it can be shown that the resulting non-relativistic action has the form

$$S = -\frac{T}{2} \int d^2 \sigma \left[\sqrt{-\tau} \tau^{\alpha\beta} h_{\alpha\beta} + \epsilon^{\alpha\beta} m_{\alpha\beta} \right] \,, \tag{10}$$

where we introduced rescaled tension $cT_F = T$ and we have taken the limit $c \to \infty$. Finally we also introduced matrix $\tau^{\alpha\beta} = \tau^{\alpha}_{\ A} \tau^{\beta}_{\ B} \eta^{AB}$ which is 2×2 matrix inverse to $\tau_{\alpha\beta}$. We also introduced 2×2 twobein $\tau^{\alpha}_{\ A}$ that obeys the condition

$$\tau_{\alpha}{}^{A}\tau_{A}^{\beta} = \delta_{\alpha}^{\beta} , \quad \tau_{\alpha}{}^{A}\tau_{B}^{\alpha} = \delta_{B}^{A} .$$
⁽¹¹⁾

Note that $m_{\alpha\beta} = m_{MN} \partial_{\alpha} x^M \partial_{\beta} x^N$ that is written in (10) is defined as

$$m_{MN} = \frac{1}{2} \eta_{AB} [\tau_M^A \pi_N^B - \tau_N^A \pi_M^B] + \frac{1}{2} \delta_{ab} [e_M^a \pi_N^b - e_N^a \pi_M^b] .$$
(12)

The canonical form of the action (10) was recently analysed in [40] where it was shown that the Hamiltonian is sum of two first class constraints

$$H = \int d\sigma (N^{\tau} \mathcal{H}_{\tau} + N^{\sigma} \mathcal{H}_{\sigma}) , \qquad (13)$$

where

$$\mathcal{H}_{\sigma} = p_M \partial_{\sigma} x^M \approx 0 ,$$

$$\mathcal{H}_{\tau} = -2T \Pi_M \tau^M_{\ A} \eta^{AB} \epsilon_{BD} \tau_{\sigma}^{\ D} + T^2 h_{\sigma\sigma} + \Pi_M h^{MN} \Pi_N \approx 0 ,$$

(14)

where Π_M is defined as

$$\Pi_M = p_M + T m_{MN} \partial_\sigma x^N . \tag{15}$$

After the review of main properties of new non-relativistic string action we proceed to its explicit form when we consider non-relativistic limit of $AdS_5 \times S^5$.

3 Non-Relativistic $AdS \times S^5$ Background

Following general prescription reviewed in the previous section we would like to find Newton-Cartan fields for non-relativistic limit of $AdS_5 \times S^5$. Let us now consider $AdS_5 \times S^5$ background in Cartesian global coordinates where line element has the form

$$ds^{2} = g_{TT}dT^{2} + g_{Z_{i}Z_{j}}dZ^{i}dZ^{j} + g_{\Phi\Phi}d\Phi^{2} + g_{Y_{i}Y_{j}}dY^{i}dY^{j} , \qquad (16)$$

where

$$g_{TT} = -\left(\frac{1+\frac{Z^2}{4R^2}}{1-\frac{Z^2}{4R^2}}\right)^2 , \quad Z^2 \equiv Z_i Z_i , \quad g_{Z_i Z_j} = \frac{1}{\left(1-\frac{Z^2}{4R^2}\right)^2} \delta_{ij} ,$$
$$g_{\Phi\Phi} = \left(\frac{1-\frac{Y^2}{4R^2}}{1+\frac{Y^2}{4R^2}}\right)^2 , \quad g_{Y_i Y_j} = \left(\frac{1}{1+\frac{Y^2}{4R^2}}\right)^2 \delta_{ij} , \quad Y^2 \equiv Y_i Y^i ,$$
(17)

where i, j = 1, 2, 3, 4 and where R is common radius of AdS_5 and S^5 . It is convenient to write this line element as $ds^2 = e^{\bar{a}}e^{\bar{b}}\eta_{\bar{a}\bar{b}}$. Then, following [41], we define nonrelativistic limit as

$$T \to ct , \quad Z_1 \to cz_1 , \quad Z_m \to z_m, \quad \Phi \to \phi , \quad Y_i \to y_i , \quad R = cR_0 ,$$
 (18)

where m = 2, 3, 4 and where non-relativistic limit corresponds to $c \to \infty$. We start with the vielbein e^{-0} that after rescaling (18) takes the form

$$e^{0} = \frac{1 + \frac{Z^{2}}{4R^{2}}}{1 - \frac{Z^{2}}{4R^{2}}} dX^{0} = c \frac{1 + \frac{z_{1}^{2}}{4R_{0}^{2}}}{1 - \frac{z_{1}^{2}}{4R_{0}^{2}}} dt + \frac{1}{c} \frac{z_{m} z_{m}}{2R_{0}^{2}(1 - \frac{z_{1}^{2}}{4R_{0}^{2}})^{2}} dt .$$
(19)

Then comparing this expression with (5) and (7) we can identify

$$\tau_t^{\ 0} = \frac{1 + \frac{z_1^2}{4R_0^2}}{1 - \frac{z_1^2}{4R_0^2}}, \quad \pi_t^{\ 1} = -\frac{z_m z_m}{R_0^2 (1 - \frac{z_1^2}{4R_0^2})^2}.$$
(20)

In the same way we proceed with e^{-1}

$$e^{-1} = \frac{1}{\left(1 - \frac{Z^2}{4R^2}\right)} dZ^1 = c \frac{1}{1 - \frac{z_1^2}{4R_0^2}} dz_1 + \frac{1}{c} \frac{1}{4R_0^2} \frac{z_m z_m}{\left(1 - \frac{z_1^2}{4R_0^2}\right)^2} dz_1$$
(21)

so that comparing with (7) we get

$$\tau_{z_1}^{\ 1} = \frac{1}{1 - \frac{z_1^2}{4R_0^2}} , \quad \pi_{z_1}^{\ 0} = -\frac{z_m z_m}{2R_0^2 (1 - \frac{z_1^2}{4R_0^2})^2} . \tag{22}$$

In case of the e^{-m} the situation is simpler

$$e^{m} = \frac{1}{1 - \frac{Z^{2}}{4R^{2}}} dZ_{m} = \frac{1}{1 - \frac{z_{1}^{2}}{4R_{0}^{2}}} dz_{m}$$
(23)

and hence

$$e_{z_m}^{\ n} = \frac{1}{1 - \frac{z_1^2}{4R_0^2}} \delta_m^{\ n} \ . \tag{24}$$

In the same way we obtain

$$e_{\phi}^{\ \Phi} = 1 \ , \quad e_{y_i}^{\ j} = \delta_i^{\ j} \ .$$
 (25)

It is important to stress that due to the fact that $R = cR_0 \rightarrow \infty$ the ϕ coordinate is effectively non-compact since original variable Φ was periodic with period $2\pi R$.

Now we are ready to proceed to find corresponding Hamiltonian. We firstly determine components of m_{MN} that, using (20) and (22) have following non-zero elements

$$m_{tz_1} = \frac{\left(1 + \frac{z_1^2}{4R_0^2}\right) z_m z_m}{4R_0^2 \left(1 - \frac{z_1^2}{4R_0^2}\right)^3} .$$
(26)

Further, from the relation $\tau_M^A \tau_B^M = \delta_B^A$ we obtain that there are non-zero components of matrix inverse τ_A^M equal to

$$\tau_{0}^{t} = \frac{1}{\tau_{t}^{0}} = \frac{1 - \frac{z_{1}^{2}}{4R_{0}^{2}}}{1 + \frac{z_{1}^{2}}{4R_{0}^{2}}} , \quad \tau_{1}^{z_{1}} = 1 - \frac{z_{1}^{2}}{4R_{0}^{2}} .$$

$$(27)$$

Taking all these results into account we obtain that the Hamiltonian constraint of non-relativistic string is equal to

$$\mathcal{H}_{\tau} = -2T\Pi_{t}\tau_{0}^{t}\tau_{z_{1}}^{1}\partial_{\sigma}z_{1} - 2T\Pi_{z_{1}}\tau_{1}^{z_{1}}\tau_{t}^{0}\partial_{\sigma}t + p_{z_{m}}(1 - \frac{z_{1}^{2}}{4R_{0}^{2}})^{2}p_{z_{m}} + T^{2}\partial_{\sigma}z^{m}\frac{1}{(1 - \frac{z_{1}^{2}}{4R_{0}^{2}})^{2}}\partial_{\sigma}z^{m} + p_{\phi}^{2} + (p_{y_{1}})^{2} + T^{2}(\partial_{\sigma}\phi)^{2} + T^{2}(\partial_{\sigma}y_{i})^{2} ,$$

$$(28)$$

where

$$\Pi_t = p_t + T m_{tz_1} \partial_1 z_1 , \quad \Pi_{z_1} = p_{z_1} + T m_{z_1 t} \partial_1 t .$$
(29)

As the next step we would like to find uniform light-cone gauge fixing form of nonrelativistic string. Discussion of the general case was performed in [40] and here we focus on the non-relativistic limit of $AdS_5 \times S^5$. In order to impose uniform light-cone gauge the background should possesses two abelian isometries where one of them is t. From the form of the non-relativistic background it is clear that the second one can be either ϕ or y_i where now ϕ is non-compact. Without lost of generality we select ϕ as the second coordinate with isometry.

As the next step we introduce light-cone coordinates and momenta [35, 36, 37]

$$x^{-} = \phi - t , \quad x^{+} = \frac{1}{2}(\phi + t) + \alpha x^{-} ,$$

$$p_{+} = p_{\phi} + p_{t} , \quad p_{-} = \frac{1}{2}(p_{\phi} - p_{t}) - \alpha p_{+} ,$$
(30)

with inverse relations

$$\phi = x^{+} + x^{-}(\frac{1}{2} - \alpha) , \quad t = x^{+} - x^{-}(\frac{1}{2} + \alpha) ,$$

$$p_{t} = p_{+}(\frac{1}{2} - \alpha) - p_{-} , \quad p_{\phi} = p_{-} + p_{+}(\frac{1}{2} + \alpha) ,$$
(31)

where α is free parameter. Let us insert these relations to the Hamiltonian constraint given above and we get

$$\begin{aligned} \mathcal{H}_{\tau} &= -2T((p_{+}(\frac{1}{2}-\alpha)-p_{-})+Tm_{tz_{1}}\partial_{\sigma}z_{1})\tau_{0}^{t}\tau_{z_{1}}^{-1}\partial_{\sigma}z_{1} - \\ &-2T(p_{z_{1}}+Tm_{z_{1}t}(\partial_{\sigma}x^{+}-\partial_{\sigma}x^{-}(\frac{1}{2}+\alpha)))\tau_{1}^{z_{1}}\tau_{t}^{-0}\partial_{\sigma}(x^{+}-x^{-}(\frac{1}{2}+\alpha)) + \\ &+p_{z_{m}}(1-\frac{z_{1}^{2}}{4R_{0}^{2}})^{2}p_{z_{m}}+T^{2}\partial_{\sigma}z^{m}\frac{1}{(1-\frac{z_{1}^{2}}{4R_{0}^{2}})^{2}}\partial_{\sigma}z^{m} + \\ &+(p_{y_{1}})^{2}+T^{2}(\partial_{\sigma}y_{i})^{2}+(p_{-}+p_{+}(\frac{1}{2}+\alpha))^{2}+T^{2}(\partial_{\sigma}x^{+}+\partial_{\sigma}x^{-}(\frac{1}{2}-\alpha))^{2} . \end{aligned}$$

$$(32)$$

Now we are ready to impose uniform light-cone gauge by introducing following gauge fixing functions [35]

$$\mathcal{G}^+ \equiv x^+ - \tau \approx 0$$
, $\mathcal{G}^- = p_- - T \approx 0$, $a = \frac{1}{2} + \alpha$. (33)

Clearly $\mathcal{G}^+, \mathcal{G}^-$ have non-zero Poisson brackets with $\mathcal{H}_{\tau}, \mathcal{H}_{\sigma}$ and hence together form set of second class constraints that vanish strongly. As a result constraints $\mathcal{H}_{\tau} = 0, \mathcal{H}_{\sigma} = 0$ can be explicitly solved. We firstly solve $\mathcal{H}_{\sigma} = 0$ for $\partial_{\sigma} x^-$ and we get

$$T\partial_{\sigma}x^{-} = -p_{y_{i}}\partial_{\sigma}y^{i} - p_{z_{m}}\partial_{\sigma}z_{m} - p_{z_{1}}\partial_{\sigma}z_{1} \equiv -\tilde{\mathcal{H}}_{\sigma} .$$

$$(34)$$

Further, Hamiltonian constraint $\mathcal{H}_{\tau} = 0$ can be solved for p_+ which is related to the Hamiltonian on the reduced phase space as $\mathcal{H}_{red} = -p_+$. To do this we insert $\partial_{\sigma} x^-$ given in (34) into (32) and using also (33) we get quadratic equation for p_+

$$-2T(1-a)p_{+}\tau_{0}^{t}\tau_{z_{1}}^{1}\partial_{\sigma}z_{1} + 2T^{2}(1-m_{tz_{1}}\partial_{\sigma}z_{1})\tau_{0}^{t}\tau_{z_{1}}^{1}\partial_{\sigma}z_{1} - -2(p_{z_{1}}+m_{z_{1}t}\tilde{\mathcal{H}}_{\sigma}a)\tau_{1}^{z_{1}}\tau_{t}^{0}(a\tilde{\mathcal{H}}_{\sigma}) + +p_{z_{m}}(1-\frac{z_{1}^{2}}{4R_{0}^{2}})^{2}p_{z_{m}} + T^{2}\partial_{\sigma}z^{m}\frac{1}{(1-\frac{z_{1}^{2}}{4R_{0}^{2}})^{2}}\partial_{\sigma}z^{m} + +(p_{y_{1}})^{2} + T^{2}(\partial_{\sigma}y_{i})^{2} + T^{2} + 2Tp_{+}a + a^{2}p_{+}^{2} + \tilde{\mathcal{H}}_{\sigma}^{2}(1-a)^{2} = 0$$

$$(35)$$

that can be solved for p_+ as

$$p_{+} = \frac{T(1-a)\tau_{0}^{t}\tau_{z_{1}}^{-1}\partial_{\sigma}z_{1} - Ta}{a^{2}} - \frac{1}{2a^{2}}\sqrt{\mathcal{K}} , \qquad (36)$$

where we defined \mathcal{K} as

$$\mathcal{K} = [2T(1-a)\tau_0^t \tau_{z_1}^{-1} \partial_\sigma z_1 - 2Ta]^2 - 4a^2 \left(2T^2(1-m_{tz_1}\partial_\sigma z_1)\tau_0^t \tau_{z_1}^{-1} \partial_\sigma z_1 - -2(p_{z_1}+m_{z_1t}\tilde{\mathcal{H}}_{\sigma}a)\tau_1^{z_1} \tau_t^{-0}a\tilde{\mathcal{H}}_{\sigma} + p_{z_m}(1-\frac{z_1^2}{4R_0^2})^2 p_{z_m} + T^2\partial_\sigma z^m \frac{1}{(1-\frac{z_1^2}{4R_0^2})^2} \partial_\sigma z^m + (p_{y_1})^2 + T^2(\partial_\sigma y_i)^2 + T^2 + (1-a)^2\tilde{\mathcal{H}}_{\sigma}^2\right).$$
(37)

Previous form of the Hamiltonian density on the reduced phase space is valid for $a \neq 0$. Explicitly, it is not valid for $\alpha = -\frac{1}{2}$, that, according to [35, 36, 37], defines temporal gauge

$$\phi = x^+ + x^-$$
, $t = x^+$, $p_t = p_+ - p_-$, $p_\phi = p_-$. (38)

In this case we should start again with the Hamiltonian constraint (32) that for a = 0 has the form

$$\mathcal{H}_{\tau} = -2Tp_{+}\tau_{0}^{t}\tau_{z_{1}}^{1}\partial_{\sigma}z_{1} + 2T^{2}(1 - m_{tz_{1}}\partial_{\sigma}z_{1})\tau_{0}^{t}\tau_{z_{1}}^{1}\partial_{\sigma}z_{1} + p_{z_{m}}(1 - \frac{z_{1}^{2}}{4R_{0}^{2}})^{2}p_{z_{m}} + T^{2}\partial_{\sigma}z^{m}\frac{1}{(1 - \frac{z_{1}^{2}}{4R_{0}^{2}})^{2}}\partial_{\sigma}z^{m} + (p_{y_{i}})^{2} + (\partial_{\sigma}y_{i})^{2} + T^{2} + (\tilde{\mathcal{H}}_{\sigma})^{2} = 0$$
(39)

that can be solved for p_+ as

$$p_{+} = \frac{1}{2T\tau_{0}^{t}\tau_{z_{1}}^{1}\partial_{\sigma}z_{1}} [2T^{2}(1 - m_{tz_{1}}\partial_{\sigma}z_{1})\tau_{0}^{t}\tau_{z_{1}}^{1}\partial_{\sigma}z_{1} + p_{z_{m}}(1 - \frac{z_{1}^{2}}{4R_{0}^{2}})^{2}p_{z_{m}} + T^{2}\partial_{\sigma}z^{m}\frac{1}{(1 - \frac{z_{1}^{2}}{4R_{0}^{2}})^{2}}\partial_{\sigma}z^{m} + (p_{y_{i}})^{2} + T^{2}(\partial_{\sigma}y)^{2} + T^{2} + \tilde{\mathcal{H}}_{\sigma}^{2}].$$

$$(40)$$

Let us return to (32) and determine its explicit form for some special cases. For $\alpha = 0$ we get uniform light-cone gauge when

$$\phi = x^{+} + \frac{1}{2}x^{-}$$
, $t = x^{+} - \frac{1}{2}x^{-}$, $p_{t} = \frac{1}{2}p_{+} - p_{-}$, $p_{\phi} = p_{-} + \frac{1}{2}p_{+}$, (41)

where p_+ is equal to

$$p_{+} = 2[T\tau_{0}^{t}\tau_{z_{1}}^{1}\partial_{\sigma}z_{1} - T] - 2\sqrt{\mathcal{K}} ,$$

$$\mathcal{K} = [T\tau_{0}^{t}\tau_{z_{1}}^{1}\partial_{\sigma}z_{1} - T]^{2} - [2T^{2}(1 - m_{tz_{1}}\partial_{\sigma}z_{1})\tau_{0}^{t}\tau_{z_{1}}^{1}\partial_{\sigma}z_{1} - (p_{z_{1}} + \frac{1}{2}m_{z_{1}t}\tilde{\mathcal{H}}_{\sigma})\tau_{1}^{z_{1}}\tau_{t}^{0}\tilde{\mathcal{H}}_{\sigma} + p_{z_{m}}(1 - \frac{z_{1}^{2}}{4R_{0}^{2}})^{2}p_{z_{m}} + T^{2}\partial_{\sigma}z^{m}\frac{1}{(1 - \frac{z_{1}^{2}}{4R_{0}^{2}})^{2}}\partial_{\sigma}z^{m} + (p_{y_{1}})^{2} + T^{2}(\partial_{\sigma}y_{i})^{2} + T^{2} + \frac{1}{4}\tilde{\mathcal{H}}_{\sigma}^{2}] .$$

$$(42)$$

Finally we can consider special case when $\alpha = \frac{1}{2}$ corresponding to a = 1. In this case we find

$$p_{+} = -T - \frac{1}{2}\sqrt{\mathcal{K}} ,$$

$$\mathcal{K} = 4T^{2} - 4(2T^{2}(1 - m_{tz_{1}}\partial_{\sigma}z_{1})\tau_{0}^{t}\tau_{z_{1}}^{-1}\partial_{\sigma}z_{1} - 2(p_{z_{1}} + m_{z_{1}t}\tilde{\mathcal{H}}_{\sigma})\tau_{1}^{z_{1}}\tau_{t}^{-0}a\tilde{\mathcal{H}}_{\sigma} + p_{z_{m}}(1 - \frac{z_{1}^{2}}{4R_{0}^{2}})^{2}p_{z_{m}} + T^{2}\partial_{\sigma}z^{m}\frac{1}{(1 - \frac{z_{1}^{2}}{4R_{0}^{2}})^{2}}\partial_{\sigma}z^{m} + (p_{y_{1}})^{2} + T^{2}(\partial_{\sigma}y_{i})^{2} + T^{2}) .$$

$$(43)$$

We see that the case $\alpha = -\frac{1}{2}$ is exceptional since in this case the Hamiltonian density on the reduced phase space is quadratic in momenta while generally the Hamiltonian has square root structure. In the next section we will analyse some classical solutions of the equations of motion on the reduced phase space.

4 Properties of Non-Relativistic String in Uniform Light-Cone gauge

In this section we will discuss properties of non-relativistic string theory on reduced phase space. First of all we consider equations of motion for y_i, p_{y_i}, z_m, p_m that due to the fact that the background fields do not depend on them have the form

$$\partial_{\tau} y_{i} = \{y_{i}, H_{red}\} = \frac{1}{2a^{2}\sqrt{\mathcal{K}}} (T^{2} p_{y_{i}} + (\dots)\partial_{\sigma} y_{i}) ,$$

$$\partial_{\tau} z_{m} = \{z_{m}, H_{red}\} = \frac{1}{2a^{2}\sqrt{\mathcal{K}}} ((1 - \frac{z_{1}^{2}}{4R_{0}^{2}})^{2} p_{z_{m}} + (\dots)\partial_{\sigma} z_{m}) ,$$

(44)

where $H_{red} = \int d\sigma \mathcal{H}_{red}$ and where (\dots) mean terms which are not important for us. The equations above can be solved by the ansatz $z_m = p_{z_m} = y_i = p_{y_i} = 0$. In fact, this ansatz also solves equations of motion for p_{z_m} and p_{y_i} . As a result we can consider Hamiltonian density for z_1 only that has the form

$$\mathcal{H}_{red}^{z_1} = -\frac{T(1-a)\tau_0^t \tau_{z_1}^1 \partial_\sigma z_1 - Ta}{a^2} + \frac{1}{2a^2}\sqrt{\mathcal{K}} ,$$

$$\mathcal{K} = [2T(1-a)\tau_0^t \tau_{z_1}^1 \partial_\sigma z_1 - 2Ta]^2 - -4a^2 (2T^2 \tau_0^t \tau_{z_1}^1 \partial_\sigma z_1 - 2p_{z_1} \tau_1^{z_1} \tau_t^0 a p_{z_1} \partial_\sigma z_1 + T^2 + (1-a)^2 (p_{z_1} \partial_\sigma z_1)^2) .$$
(45)

In order to find equation of motion for z_1 it is convenient to find Lagrangian from (45). To do this we firstly determine canonical equation of motion using (45)

$$\partial_{\tau} z_1 = \frac{1}{4a^2 \sqrt{\mathcal{K}}} (8a^2 \tau_1^{z_1} \tau_t^{\ 0} a \partial_{\sigma} z_1 - 2(1-a)^2 (\partial_{\sigma} z_1)^2) p_{z_1} .$$
(46)

Then $\mathcal{L}_{red}^{z_1}$ is given by standard formula

$$\mathcal{L}_{red}^{z_1} = p_{z_1} \partial_\tau z_1 - \mathcal{H}_{red}^{z_1} = \frac{T(1-a)\tau_0^t \tau_{z_1}^{-1} \partial_\sigma z_1 - Ta}{a^2} - \frac{T}{a} \sqrt{(1-a)^2 (\tau_0^t \tau_{z_1}^{-1} \partial_\sigma z_1)^2 - 2a\tau_0^t \tau_{z_1}^{-1} \partial_\sigma z_1 + a^4 (\tau_{z_1}^{-1} \tau_0^t)^2 (\partial_\tau z_1)^2} = \frac{T(1-a)g \partial_\sigma z_1 - Ta}{a^2} - \frac{T}{a} \sqrt{\mathbf{B}} ,$$
(47)

where we introduced g and \mathbf{B} defined as

$$g \equiv \tau_0^t \tau_{z_1}^{-1} , \quad \mathbf{B} = (1-a)^2 g^2 (\partial_\sigma z_1)^2 - 2ag \partial_\sigma z_1 + a^2 g^2 (\partial_\tau z_1)^2 .$$
(48)

Note also that (47) is valid for $a \neq 0$. Then performing variation of (47) we get following equation of motion for z_1

$$-\frac{1}{\sqrt{\mathbf{B}}}(1-a)^{2}gg'(\partial_{\sigma}z_{1})^{2} + \partial_{\sigma}\left[\frac{1}{\sqrt{\mathbf{B}}}g^{2}\partial_{\sigma}z_{1}\right] - \frac{ag}{2}\frac{1}{\mathbf{B}^{3/2}}\partial_{\sigma}\mathbf{B} - a^{4}\frac{gg'}{\sqrt{\mathbf{B}}}(\partial_{\tau}z_{1})^{2} + a^{4}\partial_{\tau}\left[\frac{g^{2}}{\sqrt{\mathbf{B}}}\partial_{\tau}z_{1}\right] = 0.$$
(49)

This is rather complicated equation and it is difficult to solve it in the full generality. On the other hand we can certainly gain in sign into its form when we consider some simpler ansatz as for example $z_z = z_z(\tau)$. However it turns out that this is too restrictive since it is easy to see that the equation above is solved for any $z_1(\tau)$. It is possible that more general ansatz could be desirable but we are not going proceed along this way.

we rather proceed to the more interesting situation when a = 0. It is easy to see that as in general case $a \neq 0$ we can consistently set $p_m = z_m = y_i = p_{y_i} = 0$ so that the reduced Hamiltonian density has the form

$$\mathcal{H}_{a=0}^{z_1} = -\frac{1}{2Tg\partial_{\sigma}z_1} [2T^2g\partial_{\sigma}z_1 + T^2 + p_{z_1}^2(\partial_{\sigma}z_1)^2] .$$
(50)

Then the first canonical equation has the form

$$\partial_{\tau} z_1 = \{ z_1, H_{a=0}^{z_1} \} = -\frac{1}{Tg} p_{z_1} \partial_{\sigma} z_1 \tag{51}$$

so that Lagrangian density has the form

$$\mathcal{L}_{a=0}^{z_1} = p_{z_1} \partial_\tau z_1 - \mathcal{H}_{a=0}^{z_1} = -\frac{Tg}{2\partial_\sigma z_1} (\partial_\tau z_1)^2 + T\partial_\sigma z_1 .$$
(52)

It is easy to derive corresponding equation of motion for z_1

$$-\frac{T}{2}\frac{dg}{dz_1}\frac{(\partial_\tau z_1)^2}{\partial_\sigma z_1} - T\partial_\sigma \left[\frac{g}{2}\frac{(\partial_\tau z_1)^2}{(\partial_\sigma z_1)^2}\right] + T\partial_\tau \left[\frac{g}{\partial_\sigma z_1}\partial_\tau z_1\right] = 0.$$

Clearly this equation has solution $z_1 = z_1(\sigma)$ for any function z_1 . On the other hand let us consider an ansatz $z_1 = f(\sigma - v\tau)$ so that $\partial_{\tau} z_1 = -vf'$, $\partial_{\sigma} z_1 = f'$ and hence the equation of motion has the form

$$-\frac{T}{2}\frac{dg}{dz_1}v^2f' - \frac{T}{2}v^2\partial_{\sigma}g - Tv\partial_{\tau}g = -T\frac{dg}{dz_1}v^2f' + Tv^2\frac{dg}{dz_1}f' = 0.$$
(53)

In other words this equation is obeyed by any function $f(\sigma - v\tau)$. This is again interesting property of non-relativistic string in uniform light-cone gauge.

5 Lagrangian Equations of Motion

In this section we find equations of motion of new-non-relativistic string with application to the $AdS_5 \times S^5$ case. Recall that the Lagrangian has the form

$$S = -\frac{T}{2} \int d^2 \sigma \left[\sqrt{-\tau} \tau^{\alpha\beta} h_{\alpha\beta} + \epsilon^{\alpha\beta} m_{\alpha\beta} \right] \,. \tag{54}$$

From this action we derive following equations of motion for x^M

$$\frac{1}{2}\sqrt{-\tau}\partial_{M}\tau_{KL}\partial_{\alpha}x^{K}\partial_{\beta}x^{L}\tau^{\beta\alpha}\tau^{\gamma\delta}h_{\gamma\delta} - \partial_{\alpha}[\sqrt{-\tau}\tau^{\alpha\beta}\tau_{MN}\partial_{\beta}x^{N}\tau^{\gamma\delta}h_{\gamma\delta}] + \\
+\sqrt{-\tau}\tau^{\alpha\beta}\partial_{M}h_{KL}\partial_{\alpha}x^{K}\partial_{\beta}x^{L} + \epsilon^{\gamma\delta}\partial_{M}m_{KL}\partial_{\gamma}x^{K}\partial_{\delta}x^{L} - \\
-\sqrt{-\tau}\tau^{\alpha\delta}\partial_{M}\tau_{KL}\partial_{\delta}x^{K}\partial_{\gamma}x^{L}\tau^{\delta\beta}h_{\beta\alpha} + 2\partial_{\alpha}[\sqrt{-\tau}\tau^{\gamma\alpha}\tau_{MN}\partial_{\beta}x^{N}\tau^{\beta\delta}h_{\delta\gamma}] - \\
-2\partial_{\alpha}[\sqrt{-\tau}\tau^{\alpha\beta}h_{MN}\partial_{\beta}x^{N}] - 2\partial_{\alpha}[\epsilon^{\alpha\beta}m_{MN}\partial_{\beta}x^{N}] = 0.$$
(55)

These equations of motion are very complicated in the full generality and we rather proceed in different way when we try to analyse non-relativistic string in $AdS_5 \times S^5$ background. As in previous section we consider an ansatz $z_m = y_i = 0$ so that we are interested in dynamics of t and z_1 only. We further presume that world-sheet time τ coincides with t and also that ϕ coincides with σ . Explicitly, we presume following ansatz

$$t = \kappa \tau$$
, $\phi = \sigma$, $z_1 = f(\sigma - v\tau)$. (56)

Inserting this ansatz into action (54) we obtain

$$S = \frac{T}{\kappa} \int d^2 \sigma \left[g \frac{1}{f'} + \frac{v^2}{g} f' \right] , \quad g = \sqrt{-\frac{\tau_{tt}}{\tau_{z_1 z_1}}} , \quad f' \equiv \frac{df}{dx} , \quad x = \sigma - v\tau .$$
 (57)

Performing variation of (57) with respect to f we get following equation

$$\frac{dg}{df}\frac{1}{f'} + \frac{d}{dx}\left(\frac{g}{f'^2}\right) - \frac{v^2}{g^2}\frac{dg}{df}f' - v^2\frac{d}{dx}\left(\frac{1}{g}\right) = 0$$
(58)

that can be simplified into the form

$$\frac{1}{f'}\frac{d}{dx}\left(\frac{g}{f'}\right) = 0 \ . \tag{59}$$

The first integral of this equation is equal to

$$\frac{g}{f'} = K , \qquad (60)$$

where K is constant. Then using the fact that $g(f) = 1 + \frac{f^2}{4R_0^2}$ we obtain final result

$$f = 4R_0 |\tan \frac{K(\sigma - v\tau)}{4R_0}|$$
 (61)

Note that this is similar solution as was discussed recently in [42] and that has physical interpretation as an infinite array of spikes on the world-sheet of non-relativistic string that is extended along ϕ direction. Certainly it could be possible to consider more general ansatz and analyse corresponding equations of motion in the similar way as in [42].

Appendix: Non-relativistic string in $AdS_5 \times S^5$ in an alternative set of coordinates

In this appendix we present formulation of non-relativistic string in $AdS_5 \times S^5$ background using coordinates that were introduced in [41]. In this formulation the vector indices for AdS_5 are labelled with m = 0, 1, 2, 3, 4 while for S^5 we have m' = 1', 2', 3', 4', 5' with the flat metric $\eta_{mn} = \text{diag}(-1, 1, 1, 1, 1)$ and $\delta_{m'n'} =$ diag(1, 1, 1, 1, 1). Then vielbein has following form

$$e^{0} = dT \cosh \rho , \quad \rho = \frac{\sqrt{X^{a} \eta_{ab} X^{b}}}{R} , \quad e^{1} = dX^{1} \cosh \rho \cos \frac{T}{R} ,$$

$$e^{a} = dX^{a} + dX^{b} (\eta_{b}^{a} - \frac{X_{b} X^{a}}{\rho^{2} R^{2}}) (\frac{\sinh \rho}{\rho} - 1) ,$$

$$e^{m'} = dX^{m'} + dX^{n'} (\eta_{n'}^{m'} - \frac{X_{n'} X^{m'}}{r^{2} R^{2}}) (\frac{\sin r}{r} - 1) , \quad r = \frac{\sqrt{X^{m'} \delta_{m'n'} X^{n'}}}{R} ,$$
(62)

where a = 2, 3, 4. Let us now take non-relativistic limit in the form

$$T = \omega t , \quad X^1 = \omega x^1 , \quad R = \omega R_0 , \qquad (63)$$

so that we obtain

$$e^{0} = cdt(1 + \frac{\hat{\rho}^{2}}{2c^{2}})$$
, (64)

from which we can deduce following components of $\tau_M^{\ A}$ and $\pi_M^{\ A}$

$$\tau_t^{\ 0} = 1 , \quad \pi_t^{\ 1} = -\hat{\rho}^2 , \quad \hat{\rho} = \sqrt{x^a x_a} / R_0 .$$
 (65)

We further have

$$e^{1} = cdx^{1}\left(1 + \frac{\hat{\rho}^{2}}{2c^{2}}\right)\cos\frac{t}{R_{0}}$$
(66)

that again implies

$$\tau_1^{\ 1} = \cos\frac{t}{R_0} , \quad \pi_1^{\ 0} = -\hat{\rho}^2 \cos\frac{t}{R_0} .$$
(67)

Finally we have

$$e^{a} = dx^{a} + dx^{b}(\eta_{b}^{a} - \frac{x_{b}x^{a}}{\hat{\rho}^{2}R_{0}^{2}})(1 + O(c^{-3}) - 1) = dx^{a}.$$
(68)

Now we are ready to proceed to find corresponding Hamiltonian. As the first step we determine following non-zero components of m_{MN}

$$n_{t1} = \hat{\rho}^2 \cos \frac{t}{R_0} \,. \tag{69}$$

Further, from the relation $\tau_M^{\ A} \tau_B^M = \delta_B^A$ we obtain that there are non-zero components

1

$$\tau_0^t = \frac{1}{\tau_t^0} = 1 , \quad \tau_1^1 = \frac{1}{\cos \frac{t}{R_0}}$$
(70)

so that Hamiltonian constraint is equal to

$$\mathcal{H}_{\tau} = -2T\Pi_t \cos\frac{t}{R_0} \partial_{\sigma} x^1 - 2T\Pi_1 \frac{1}{\cos\frac{t}{R_0}} \partial_{\sigma} t + p_{x^a} \delta^{ab} p_{x^b} + T^2 \partial_{\sigma} x^a \delta_{ab} \partial_{\sigma} x^b + p_{m'} \delta^{m'n'} p_{n'} + T^2 \partial_{\sigma} x^{m'} \delta_{m'n'} \partial_{\sigma} x^{n'} ,$$

$$\tag{71}$$

where

$$\Pi_t = p_t + T m_{t1} \partial_\sigma x^1 , \quad \Pi_1 = p_1 + T m_{1t} \partial_\sigma t .$$
(72)

Form of the Hamiltonian constraint implies that it is not possible to impose uniform light-cone gauge due to its explicit dependence on t. Certainly it is possible to study non-relativistic string in the background defined by (65) and (67) but the Lagrangian formulation is the same as in [41] which is well known and hence we will not study it in this paper.

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