Medium effect on anisotropic surface tension of magnetized quark matter

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The thermodynamics of finite size quark matter in the quasiparticle model is self-consistently constructed by an effective bag function, which presents the medium effect to the confinement. We obtained completely analytic surface tension in the strong magnetic field with the multiple reflection expansion. The anisotropic structure is demonstrated by the splitting of the longitudinal and transverse surface tensions. The anisotropy of the surface tension could be enhanced by an increase of the magnetic field. The analytical surface tension is modified by an additional term related to the bag function. For strong enough magnetic fields, the increase of the longitudinal surface tension is proportional to the magnetic field. On the contrary, the transverse component vanishes due to all quarks locating in the lowest landau level.

I. INTRODUCTION

Over the years, many works have been dedicated to the effects of magnetic fields on the quantum chromodynamics (QCD) phase transition [1, 2] and the equation of state in quark (neutron) stars [3–8]. The magnetic field modifies the microscopic properties of quark matter with the corresponding macroscopic implication in compact stars [9–12]. It is well known that in the presence of a magnetic field, the anisotropic effect becomes significant and non-negligible in strong magnetic field, owing to the breaking of the spational rotational symmetry [13–18]. To reflect the anisotropic structure with a rapid longitudinal expansion of QGP created in HICs, the anisotropic Coulomb potential can be produced through an angle-averaged screening mass [19]. Many theoretical works presented the analytic expression for anisotropic pressures under certain approximation [6, 7, 17, 20]. Ferrer et al. showed the anisotropic pressure and estimated the threshold field that separates the isotropic and anisotropic regimes [17]. Later, Isayev and Yang confirmed the splitting of the longitudinal and transverse pressure in their articles [5, 21]. The anisotropic pressure will affect the determination of the compressibility. The compressibility could manifest the anisotropic structure due to the breaking of the rotation symmetry. The discontinuity of longitudinal compressibility with the chemical potential and the temperature captures the signature of a first-order chiral phase transition [22]. With increasing temperature, the appearance of the longitudinal instability prevents the formation of a fully spin-polarized state in neutron matter and only the states with moderate spin polarization are accessible [5]. Recently, Lugones et. al. pushed the investigation of the surface tension in longitudinal and transverse components with respect to the magnetic field in the bag model [23–25]. In this paper, our aim is to investigate the relevant anisotropic surface tension reflecting the breaking of the $O(3)$ rotational symmetry in the deconfinement process.

In principle, the surface tension together with the QCD phase diagram should be investigated in the underlying fundamental theory, lattice QCD (LQCD). However, current LQCD methods are not sufficient to determine the matter structure at larger chemical potentials. The only available methods at relatively low energy are effective models. In literature, the phenomenological models overcome the difficulty of the QCD theory at finite chemical potentials. In order to interpret the chiral phase transition and dynamical symmetry breaking, the Nambu-Jona-Lasinio (NJL) model is widely used in the QCD-like investigation. The NJL model has proved to be very successful in the description of the spontaneous breakdown of chiral symmetry exhibited by the true (nonperturbative) QCD vacuum. It explains very well the spectrum of the low lying mesons which is intimately connected with chiral symmetry as well as many other low energy phenomena of strong interaction. The quark quasiparticle model, as an extended bag model, has been developed in studying the bulk properties of the dense quark matter at finite density and temperature. To describe the strong interaction effects in terms of effective fugacities, the effective fugacity quasiparticle model is proposed by Chandra and Ravisankar [26, 27]. The advantage of the quasiparticle is the successful description of the confinement mechanism by the density- and/or temperature-dependent bag function, via which the first-order deconfining phase transition was constructed and the critical end point was determined [28]. The aim of this work is to investigate the anisotropy of surface tension modified by the medium effect in strong magnetic fields. We also hope that the surface tension is helpful to investigate the deconfinement transition, since the surface tension is relevant for bubble nucleation of quark matter in supernovae [29]. It can play an important role in the hadron-quark phase transition in the presence of a magnetic field when the anisotropic approach is followed as it was done recently [30].

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This paper is organized as follows. In Section II, we present the self-consistent thermodynamics of the magnetized quark matter in the quasiparticle model. The medium effect is included by introducing the effective bag function. The surface tension is modified by an additional term dependent on the bag function. In Section III, the numerical results for the confinement bag function and surface tension are shown in the strong magnetic field. The detailed discussions are focused on the anisotropy of the surface tension. The last section is a short summary.

II. THERMODYNAMICS OF QUASIPARTICLE MODEL IN STRONG MAGNETIC FIELDS

The main purpose of this paper is to study the properties of the deconfined quark matter in strong magnetic fields. The nonzero quark masses are explored and the exact chiral symmetry are broken. The effective quasi-particle mass should be introduced to include the interaction effect in the quasiparticle approach. The total energy in the ensemble of quasiparticle as a free and degenerate Fermion gas can be written as

$$H_{\mathrm{eff}} = \sum_{i=1}^{d} \sum_{p} \sqrt{p^2 + m_i^{*2}}\, \hat{a}_{i,p}^{\dagger} \hat{a}_{i,p} + B^*(\mu) V \tag{1}$$

where d denotes the degree of the degeneracy including the flavor, color and spin. The chemical potential-dependent bag function B^* denotes the energy difference between the physical vacuum and the perturbative vacuum, which is necessary to ensure thermodynamic consistency.

For the medium dependence of the quark quasiparticle model, the effective quark mass m_i^* is derived at the zero momentum limit of the dispersion relation following from the effective quark propagator by resuming one-loop self energy diagrams in the hard dense loop (HDL) approximation [31]. The in-medium effective mass of quarks can thus be expressed as [31–35]

$$m_i^*(\mu_i) = \frac{m_i}{2} + \sqrt{\frac{m_i^2}{4} + \frac{g^2 \mu_i^2}{6\pi^2}}, \quad (i = u, d, s), \tag{2}$$

where m_i is the current mass of corresponding quarks and the constant g is related to the strong interaction constant α_s by the equation $g = \sqrt{4\pi\alpha_s}$. The quasi-particle idea can be recalled backward to the work by Fowler et.al.[36] that the particle mass may change with the environment parameters. Following the original ideas, the quark mass density dependent model is studied by Chakrabarty et.al.[37]. Accordingly, as a phenomenological method, our quasiparticle model has a similar treatment. They are apparently different in approach but equally satisfactory in result. The in-medium screening mass in Eq.(2) is merely a model assumption on the quasiparticle mass in the present treatment, and can not be justified field-theoretically. The effective mass depends on the leading term of quark self-energy in the hard dense loop approximation. So the expression is only valid for a large chemical potential.

In order to investigate the finite size effect, we apply the multiple reflection expansion (MRE). It is originally proposed by Balian and Bloch in the distribution of eigenvalues of the wave equation inside the volume bounded by s closed surface[38]. The eigenvalue density is smoothed by the asymptotic expansion to eliminate its fluctuation part due to the discrete eigenvalues on boundary condition. Later the MRE is developed in finite size quark matter by Madsen [39],Farhi and Jaffe [40] and Berger and Jaffe [41]. In the MRE framework, the finite- size effects are considered in the modified density of state and the thermodynamic potential density is

$$\begin{aligned} \Omega_i \; = \; & -d_i \frac{2T}{(2\pi)^3} \int \left\{ \ln\left[1 + e^{-(\sqrt{p^2+m_i^2}-\mu_i)/T}\right] \right. \\ & \left. + \ln\left[1 + e^{-(\sqrt{p^2+m_i^2}+\mu_i)/T}\right] \right\} \rho_{\mathrm{MRE}} \mathrm{d}^3 p, \end{aligned} \tag{3}$$

where T is the system temperature and $d_i = 3$ is the color degeneracy factor for i-type quarks. The density of state for a spherical system is modified by the factor in the multi-expansion approach by [42]

$$\rho_{\mathrm{MRE}}(p, m_i, R) = d_i \left[1 + \frac{2\pi^2}{p} \frac{S}{V} f_{\mathrm{S}}(x_i)\right]. \tag{4}$$

Here $x_i \equiv m_i^*/p$ is the ratio of the quark mass m_i over the kinetic moment p. The dimensionless function $f_{\mathrm{S}}(x_i) = -\frac{1}{4\pi^2}\arctan(x_i)$ would play an important role in the modification of the density of state [41, 43]. In conventional form of MRE, the expansion includes the boundary surface and its curvature the density of states for a volume of arbitrary shape. In our work, the curvature term has no influence on our subject and is omitted. For the extremely relativistic particle with $m \ll p$, the density modification is not modified significantly [41]. Generally, the modification of the density of state in the MRE framework constrain the low limit on the infrared cutoff due to the fact that $\rho_{\rm MRE}$ becomes negative at small momenta [44].

In the integrations of the thermodynamic potential, the following replacement should be applied

$$\int \frac{d^3p}{(2\pi)^3} \to \frac{|q_i B_m|}{2\pi} \sum_{\nu} \sum_{s=\pm 1} \int_0^{\infty} \frac{dp_z}{2\pi}, \tag{5}$$

where the magnetic field strengths the degeneracy factor $|qB|$ together with the dimensional reduction. At zero temperature and strong magnetic fields, Eq.(3) is simplified as

$$\Omega_i = \frac{d_i |q_i B_m|}{2\pi^2} \sum_{\nu=0}^{\nu_i^{\max}} (2-\delta_{\nu 0}) \int_{\Lambda_{\rm IR}}^{p_F} (E_i - \mu_i)\, \rho_{\rm MRE} {\rm d}p_z\,, \tag{6}$$

where the single particle energy eigenvalue $E_i = \sqrt{m_i^{*2} + p_z^2 + 2\nu_i |q_i B|}$ sensitively depends on the magnetic fields. The infrared cutoff of the momentum $\Lambda_{\rm IR}$ is required to obtain the non-negative density of state. At zero temperature, the upper limit $\nu_i^{\max}$ of the summation index ν_i can be understood from the positive value requirement on Fermi momentum and is defined by

$$\nu_i^{\max} = \frac{\mu_i^2 - m_i^{*2}}{2|q_i B_m|}. \tag{7}$$

A. The effective bag function

In the framework of the quasiparticle model, the total thermodynamic potential with effective mass $m_i^*(\mu_i)$ should be self-consistently written as

$$\Omega = \sum_i [\Omega_i(\mu_i, m_i^*(\mu_i)) + B_i^*(\mu_i)] + B_0, \tag{8}$$

where the additional term $B_i^*(\mu_i)$ is the medium dependent quantity to be determined. The vacuum energy density B_0 is medium-independent. It has been interpreted as a background field, zero point energy density, or bag pressure [45]. In the standard statistical mechanics, the Hamiltonian or the thermodynamic potential depends on the temperature and the chemical potential related to the conserved charges. If the thermodynamic potential depends on the state variable implicitly via phenomenological parameters $m_i^*(T, \mu_i)$, the corresponding stationarity condition should be required as [46]

$$\left.\frac{\partial \Omega}{\partial m_i^*}\right|_{T,\mu_i} = 0. \tag{9}$$

which has been widely employed in the quasiparticel model at finite temperature and density [33, 47–49]. The condition respects the chiral symmetry restoration in the plasma [50]. At zero temperature, we get the bag function through the integral

$$B_i^*(\mu_i) = -\frac{d_i |q_i B_m|}{2\pi^2} \sum_{\nu=0}^{\nu_i^{\max}} (2-\delta_{\nu 0}) \int_{m_i^*}^{\mu_i} \int_{\Lambda_{\rm IR}}^{p_F} \left(\frac{m_i^*}{E_i}\rho_{\rm MRE} - \frac{3}{2R}\frac{E_i - \mu_i}{E_i^{\,2}}\right) \frac{{\rm d}m_i^*}{{\rm d}\mu_i} {\rm d}p_z {\rm d}\mu_i, \tag{10}$$

where the lower limit means the allowed chemical potential. Specially, the equality $\mu_i = m_i^*$ would lead to the vanishing Fermi momentum and the zero bag function.

According to the geometric dependence, namely, the power of the radius dependence, the thermodynamic potential density can be considered as

$$\Omega_i = \underbrace{\Omega_{V,i} + B_{V,i}}_{\rm volume} + \underbrace{\Omega_{S,i} + B_{S,i}}_{\rm surface}. \tag{11}$$

The surface terms proportional to the $1/R$ are

$$\Omega_{S,i} = -\frac{d_i|q_iB_m|}{2\pi^2}\sum_{\nu=0}^{\nu_i^{\max}}(2-\delta_{\nu 0})\frac{3}{2R}\int_{\Lambda_{\rm IR}}^{p_F}\frac{E_i-\mu_i}{p}\arctan(x_i)\mathrm{d}p_z, \quad (12)$$

$$B_{S,i} = \frac{d_i|q_iB_m|}{2\pi^2}\sum_{\nu=0}^{\nu_i^{\max}}(2-\delta_{\nu 0})\frac{3}{2R}\int_{m^*}^{\mu_i}\int_{\Lambda_{\rm IR}}^{p_F}\left[\frac{E_i-\mu_i}{E_i^2}+\frac{m_i^*}{E_ip}\arctan(x_i)\right]\frac{dm_i^*}{d\mu_i}\mathrm{d}p_z\mathrm{d}\mu_i. \quad (13)$$

In the bulk limit $R\to\infty$ and for the light quark mass $m=a\mu$ with $a=g/\sqrt{6\pi^2}$, the bag function in Eq. (10) has the analytical expression as

$$B_i^*(\mu_i) = \frac{d_i|q_iB_m|}{4\pi^2}\sum_{\nu=0}^{\nu_i^{\max}}(2-\delta_{\nu 0})a^2\mu^2\left[\frac{1}{2}\ln(\frac{1+k_z}{1-k_z})+\frac{k^2-k_z^2}{a^2}\tanh^{-1}(\frac{1}{k_z})-\frac{k^2-k_z^2}{a^2}\tanh^{-1}(\frac{\sqrt{1-k_z^2}}{ak_z})\right.$$
$$\left.+\frac{k^2-k_z^2}{a^2}\ln(\frac{k\sqrt{1-k_z^2}+ak_z}{k+k_z})-(1-k_z^2)\ln[\frac{\sqrt{1-k_z^2}+ak_z}{\sqrt{1-k_z^2-a^2k_z^2}}]\right], \quad (14)$$

where we define a dimensionless momentum $k=\sqrt{1-a^2}$ and its z-component in the magnetic field $k_z=\sqrt{1-a^2-2\nu|q_iB_m|/\mu_i^2}$. If the magnetic field is so strong that all quarks are lying on the lowest Landau level (LLL), the bag function can be simplified as

$$B_i^*(\mu_i) = -\frac{d_i|q_iB_m|}{4\pi^2}a^2\mu^2(1-a^2)\ln(\frac{1+\sqrt{1-a^2}}{a}). \quad (15)$$

B. The anisotropic surface tension

In literature the surface tension was investigated with the two main approaches. One is the multiple reflection expansion approximation [51]. The other method is the geometric approach [52, 53]. The surface tension is characteristic of the two phase. For example, the amount of the surface tension between the liquid drop and the gas phase is dominant to the raindrop formation. The surface tension between true vacuum and perturbative phase was usually neglected in comparison to the confinement bag constant. The early work is typically traced back to the paper [41] by Berger et al. They suggested that the surface tension parameter can be calculated from the surface modification of the fermion density of states with larger values of the surface tension could survive the early Universe. The surface tension also depends strongly on the dynamical effects and the skin thickness [54]. In contrast, there is a suggestion that the surface tension for the interface separating the quark and the hadron phase should be smaller to make the mixed phase occur [55, 56]. Recently, in order to employ the excess energy associated with inhomogeneous configurations, the definition of a differential surface tension from the bubble formation in the discrete case to systems with continuous symmetry [57]. The special value of the surface tension is poorly known. At least, it is possible that there is a critical surface tension, above which the structure of the mixed phase will become unstable [58].

Once the thermodynamic potential is known, the longitudinal pressure is obtained as

$$P^{\parallel} = -\Omega, \quad (16)$$

As is mentioned that in the presence of the strong magnetic field, the rotational symmetry breaking would be demonstrated not only by the anisotropic pressure structure but also by the surface tension for the finite volume matter. Therefore, the longitudinal surface tension can be derived by the longitudinal pressure [59].

$$\sigma_i^{\parallel} = \frac{R}{3}(\Omega_{S,i}+B_{S,i}), \quad (17)$$

The electromagnetic contribution of Maxwell term $B_m^2/2$ is omitted to the pressure due to that it has no influence on the surface tension. The transverse pressure is usually defined as $P^{\perp}=P^{\parallel}-MB_m$, where the magnetization susceptibility M can be derived by the relation $M=-\frac{\partial\Omega}{\partial B}$. In the present work, we have the transverse pressure from the i-flavor quarks

$$P_i^{\perp} = \frac{d_i|q_iB_m|^2}{2\pi^2}\sum_{\nu=1}^{\nu_i^{\max}}\nu\left\{\int_{\Lambda_{\rm IR}}^{p_F}\frac{\rho_{\rm MRE}{\rm d}p_z}{E_i} - \int_{m^*}^{\mu_i}\int_{\Lambda_{\rm IR}}^{p_F}\left[\frac{m}{E^3}+\frac{3}{R}\Big(\frac{1}{p^2E_i}-2\frac{\mu_i}{E_i^4}+\frac{m}{p^3E_i^3}(E_i^2+p^2)\arctan(\frac{m_i}{p})\Big)\right]\frac{dm_i^*}{d\mu_i}{\rm d}p_z{\rm d}\mu_i\right\}. \quad (18)$$

Similar to the expression of the longitudinal surface tension $\sigma^{\parallel}$, the transverse surface tension contribution per flavor can be divided into two parts as

$$\sigma_i^{\perp} = \frac{R}{3}(\Omega'_{S,i}+B'_{S,i}), \quad (19)$$

where the notation O' stands for a transformation of the quantity O, namely, $O' = O + B_m\frac{\partial O}{\partial B_m}$. Applying the operation on both the free term Ω and the bag function B^*, one can find the following expressions

$$\Omega'_{S,i} = \frac{d_i|q_iB_m|^2}{2\pi^2}\sum_{\nu=1}^{\nu_i^{\max}}\nu\int_{\Lambda_{\rm IR}}^{p_F}\frac{3}{Rp}\arctan(x_i)\frac{{\rm d}p_z}{E_i}. \quad (20)$$

$$B'_{S,i} = \frac{d_i|q_iB_m|^2}{2\pi^2}\sum_{\nu=0}^{\nu_i^{\max}}\nu\frac{3}{R}\int_{m^*}^{\mu_i}\int_{\Lambda_{\rm IR}}^{p_F}\left[\frac{1}{p^2E_i}-2\frac{\mu_i}{E_i^4}+\frac{m}{p^3E_i^3}(E_i^2+p^2)\arctan(\frac{m_i}{p})\right]\frac{dm_i^*}{d\mu_i}{\rm d}p_z{\rm d}\mu_i. \quad (21)$$

It can be well understood that the anisotropy of dense quark matter may stem from the magnetization along the field direction resulted by two reasons, namely, the arrangement of free particles and the medium effect.

III. NUMERICAL RESULT AND CONCLUSION

In the present paper we take the current mass m_0=5.6 MeV and $\mu_u = \mu_d = \mu$ for isospin-symmetric quarks and the bag constant $B_0 = (145\text{MeV})^4$ for bulk case. The main advantage of the quasiparticle model is the combination of the medium effect into both the effective quark mass and the bag function B^*. For the intensity of the field $eB_m < \mu^2$, the validity of the spherical geometry could be approximately available to some extent.

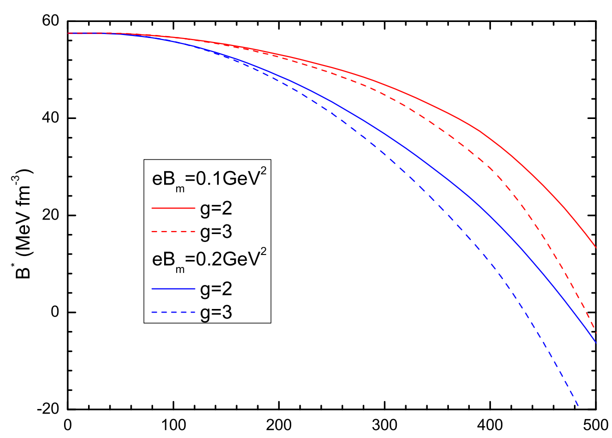

FIG. 1: The effective bag function $B^*(\mu)$ at the coupling constant is shown as a function of the chemical potential.

The chemical potential dependent bag function plays an important role in the description of the deconfinement transition. The bag function would play as a function of the chemical potential and the finite size. The magnetic field effect and the coupling constant are considered separately. In Fig. 1, the bag function decreases with the increase of the chemical potential, which indicates a signal of the deconfinement transition. As the coupling constant becomes larger and/or the magnetic field becomes stronger, the decreasing behavior of the bag function would happen at a smaller chemical potential, which indicates a critical chemical potential is similar to the temperature behavior characterized by the inverse magnetic catalysis effect [60]. In Fig. 2, the effect of the finite size volume is shown on the bag function. The two horizontal dotted lines are the corresponding bag functions for bulk strange quark matter. The bag function increases as the spherical size decreases. As the spherical radius R approach the infinite value, the bag function is gradually close to the constant B_0 for the bulk matter, which indicates the disappearance of the finite size effect.

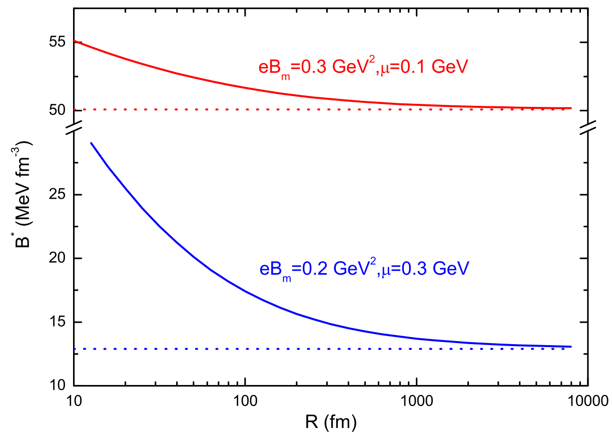

FIG. 2: The effective bag function $B^*(\mu)$ at the coupling constant is shown as a function of the radius of the spherical system.

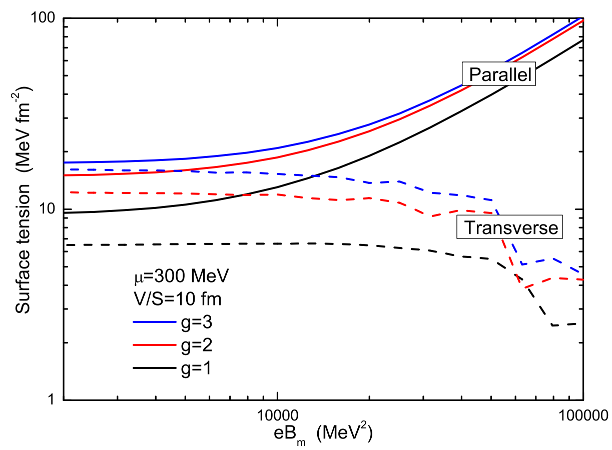

FIG. 3: The parallel and transverse surface tensions with $V/S = 10$ fm and $\mu = 300$ MeV at vanishing temperature are shown as function of the magnetic field strength.

At the larger chemical potential, a direct transition from the vacuum to quark matter happens possibly depending on the surface tension of bubble quark matter [61]. The surface tension is plotted as a function of the magnetic field in Fig. 3. The chemical potential and the finite size are adopted as $\mu = 300$ MeV and $V/S = 10$ fm. The different coupling constants $g = 1$, 2, and 3 are marked by the black, the red, and the blue curves from bottom to top. The longitudinal and transverse surface tensions are denoted by the solid and dashed curves respectively. It can be found that the longitudinal component increases with the magnetic field eB_m and coupling constant g. In particular, the longitudinal surface tension is proportional to the magnetic field in the strong field limit. It can be simply understood that the magnetic field only has contribution to the coefficient in front of the integral of Eqs. (12) (13). On the contrary, the transverse surface tension would decrease as the increasing magnetic field. Furthermore, the transverse component feels the Landau level effect more sensitively, which results in a oscillation behavior. At weak magnetic field, the value of the surface tension is in agreement with the result estimated as the order as $(70\ \text{MeV})^3$ [40]. As the magnetic field becomes much stronger, the anisotropy structure would be enhanced greatly.

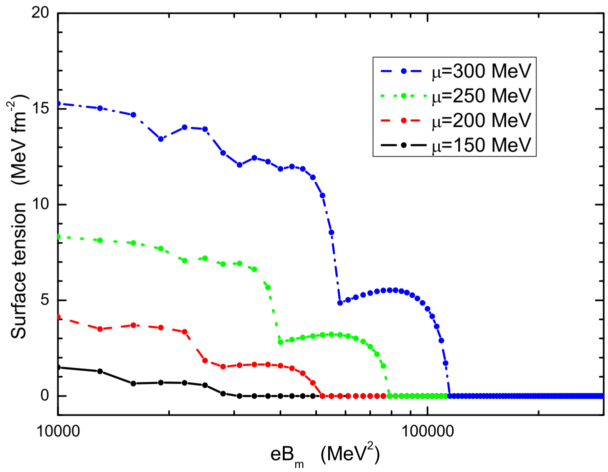

FIG. 4: The behavior of the transverse surface tension $\sigma^{\perp}$ is shown as a function of the magnetic field at four different chemical potentials with $V/S = 10$ fm.

In Fig. 4, the transverse surface tension is plotted as function of the magnetic field at four different chemical potentials $\mu = 150, 200, 250, 300$ MeV and the fixed size $V/S = 10$ fm. The oscillation behavior is shown clearly in the weaker magnetic field. The more quarks located in high Landau levels have a finite contribution to the motion perpendicular to the magnetic field. At the much stronger magnetic field, the transverse surface tension would drop down and vanish in the end, which can be understood that all quarks located in the LLL have no contribution to the transverse motion. It is suggested that the vanishing of the transverse surface tension is a signal of all charged particle occupied in the LLL. Moreover, the threshold value of the magnetic field for the vanishing of $\sigma^{\perp}$ would increase with the increase of the chemical potential.

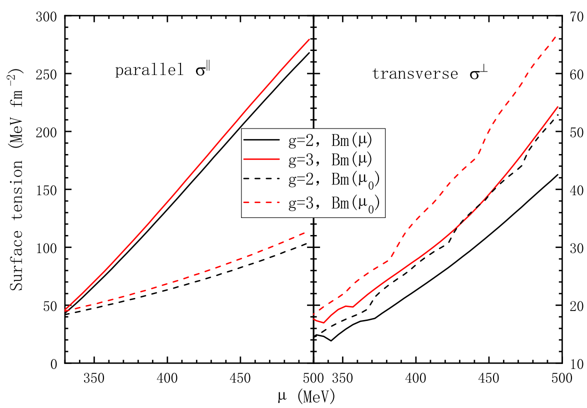

FIG. 5: The parallel and transverse surface tensions are shown as functions of the chemical potential in a uniform magnetic field $B_{\rm m}(\mu_0)$ and a nonuniform $B_{\rm m}(\mu)$ [62]

To mimic a realistic magnetic field in compact stars, the magnetic field is recently suggested to increase polynomial instead of exponential as the chemical potential [62]. In Fig. 5, the anisotropic surface tension is investigated as a function of the chemical potential. The nonuniform magnetic field $B_{\rm m}(\mu)$ and the fixed strength $B_{\rm m}(\mu_0)$ are marked by the solid lines and the dashed lines, respectively. The initial point μ_0 is associated to the surface chemical potential μ_0 of compact stars [62, 63]. The parallel surface tension $\sigma^{\parallel}$ on left panel and transverse one $\sigma^{\perp}$ on right panel are calculated at different couplings $g = 2$ and 3. The transverse surface tension is apparently smaller than the parallel one in the whole range of the chemical potential, which reflects the anisotropic structure. It should be emphasized that the transverse surface tension would decrease as the increasing the magnetic field at high densities, which is indicated in Fig. 4. However, the surface tension is enlarged by the increasing chemical potential. Therefore, the transverse surface tension is shown as an increasing function of the chemical potential. The tiny oscillation behavior occurs at low chemical potential due to the Landau level transition. The transverse surface tension under the magnetic field $B_{\rm m}(\mu)$ is smaller than that of the case $B_{\rm m}(\mu_0)$. On the contrary, the parallel tension is larger and grows more rapidly under the the magnetic field profile $B_{\rm m}(\mu)$.

IV. SUMMARY

In this paper, the thermodynamics of magnetized quark matter in the finite size has been obtained in the quasiparticle model. The dense medium effect is included through the effective bag function, which depends on the chemical potential and the magnetic field. The bag function plays as an appropriately chosen vacuum energy constant ensuring thermodynamic consistency. On the other hand, it provides a measure for nonperturbative physics which cannot be described by the effective masses. Its variation would be helpful to investigate the deconfinement transition. As expected, it has been numerically shown that the bag function is a decreasing function of the chemical potential. The drop down behavior would be strengthened by the increase of both the magnetic field and the coupling interaction constant. It was verified that the bag function B^* would gradually approach the bulk limit B_0 as the size becomes infinitely large. We have employed the extended quasiparticle model to the surface tension. It is found that the medium effect represented by the bag function would lead to an additional term in the surface tension. The anisotropy of the longitudinal and transverse surface tensions is enhanced by an increase of the magnetic field. The longitudinal surface tension is an increasing function of the magnetic field. But the transverse component would decrease and drop down to zero at extreme strong magnetic fields. The vanishing of transverse surface tension coincides with the condensation of all quarks in LLL. Finally, the new expression of the surface tension modified by the medium effect would be useful to generalize the current investigation on the bubble formation in the QCD transition. Last but no least, the infrared cutoff for avoiding the negative density of state is relevant with the confinement phenomenon. The more reasonable confinement mechanism should be produced by the self-consistent combination of the infrared cutoff and the bag constant in future work.

Acknowledgments

The authors would like to thank support from the National Natural Science Foundation of China under the Grants No. 11875181, No. 11705163, and No. 12147215. This work was also sponsored by the Fund for Shanxi "1331 Project" Key Subjects Construction.

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