Significance of classically forbidden regions for short baseline neutrino experiments

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Abstract. Classically forbidden regions (CFRs) are common to both non-relativistic quantum mechanics, and to relativistic quantum field theory. It is known since 2001 that CFR contributes roughly sixteen percent of energy to the ground state of a simple harmonic oscillator (ADUNAS G. Z. et al., Gen. Relativ. Gravit., 33 (2001) 183). Similarly, quantum field theoretic arguments yield a non-zero amplitude for a massive particle to cross the light cone (that is, into the CFR). The signs of these amplitudes are opposite for fermions and antifermions. This has given rise to an erroneous conclusion that amplitude to cross the lightcone is identically zero. This is true as long as a measurement does not reveal the considered object to be a particle or antiparticle. However, neutrino oscillation experiments do measure a neutrino ν , or an antineutrino $\bar{\nu}$. Here we show that in the context of neutrino oscillations these observations have the potential to resolve various short baseline anomalies for a sufficiently light lowest mass eigenstate. In addition, we make a concrete prediction for the upcoming results to be announced later this year by JSNS².

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¹The author declares that during the LSND experiment he was at the Physics Division of the Los Alamos National Laboratory as a Director's Postdoctoral Fellow.

Introduction.

The 1996 claim by LSND collaboration for $\overline{\nu}_{\mu} \rightarrow \overline{\nu}_{e}$ oscillation, and later of $\nu_{\mu} \rightarrow \nu_{e}$ oscillation, remains unconfirmed by the KARMEN neutrino oscillation experiment [1–4]. The physics community is in general agreement with Yellin, that "both experiments have competent personnel" and "both have been working a long enough time to eliminate serious mistakes" [4]. Concurrently, similar anomalies exist in the short baseline experiments with reactors [5,6], and just a few months ago BEST has further re-confirmed the gallium anomaly [7]. Under these circumstances it is natural to suspect that all these short baseline anomalies cannot be entirely understood within the Pontecorvo framework without invoking a family of sterile neutrinos, or to look for a physical origin that has somehow evaded physicists.

It may be relevant to parenthetically note that in the context of gallium anomaly Giunti et al. also conclude that, "one should pursue other possible solutions beyond short-baseline oscillations" and further "that the neutrino oscillation explanation of the Gallium Anomaly is in strong tension with the solar bound on active-sterile neutrino mixing" [8].

Heuristics and their limitations

To initiate a possible first-principle solution beyond short-baseline neutrino oscillations, it is expedient to start with Steven Weinberg's observation in [9]: "... The uncertainty principle tells us that when a particle is at position \boldsymbol{x}_1 at time t_1 , we cannot also define its velocity precisely. In consequence there is certain chance of a particle getting from x_1 to x_2 even if $x_1 - x_2$ is spacelike, that is $|\boldsymbol{x}_1 - \boldsymbol{x}_2| > |x_1^0 - x_2^0|$. To be more precise, the probability of a particle reaching x_2 if it starts at x_1 is nonnegligible as long as

$$(\boldsymbol{x}_1 - \boldsymbol{x}_2)^2 - ({x_1}^0 - {x_2}^0)^2 \le \frac{\hbar^2}{m^2}$$
(1)

where \hbar is Planck's constant (divided by 2π) and m is the particle mass. (Such spacetime intervals are very small even for elementary particle masses; for instance, if m is the mass of a proton then $\hbar/m = 2 \times 10^{-14} \text{ cm} \dots$)"

Tony Zee has also noted that "classically, a particle cannot get outside the lightcone, but a quantum field can "leak" out over a distance of order \hbar/m by the Heisenberg uncertainty principle" [10, p. 24]. 1 Note: Here we have inserted \hbar in order that both quotes are in the same units. Since only the mass squared differences are observable in the Pontecorvo framework the mass of the lowest mass eigenstate remains free (modulo cosmological and astrophysical constraints [11–13]). For the sake of our argument, let its mass be of the order of a nano electron volt (neV),² that is roughy eighteen orders of magnitude smaller than a proton's mass. Then, $\hbar/m \sim 100$ meters. This covers the source-detector distance for all the mentioned short baseline experiments.

However, these heuristics can be misleading as in the $m \to 0$ limit the light cone completely opens up! As such one must seek a fully quantum field theoretic expression for "leaking off" into the CFR that does not suffer from this problem.

Beyond the heuristics.

To go beyond the heuristics, in quantum field theoretic framework if x and x' are separated by a space-like interval then the amplitude for a fermionic *particle* to reach x' starting from x is [14, eq. 2.13]

$$A(x \to x') = +\frac{1}{\pi^2} \frac{m^2}{\sqrt{r^2 - t^2}} K_1 \left(m \sqrt{r^2 - t^2} \right), \tag{2}$$

where $r = |\mathbf{x}' - \mathbf{x}|, t = |x'^0 - x^0|$, and $K_{\nu}(\ldots)$ is the modified Bessel function of the second kind of order ν . The total amplitude contributing at r from *all* spacelike separations – that is, from the entire CFR – is calculated by integrating $A(x \to x')$ from t = 0 to t = r. To the leading order (that is, approximating $A(x \to x')$ to $\mathcal{O}(t^2)$ in the series expansion, and implementing the indicated integration)

$$A(r, \text{CFR}) = +\frac{m^2}{6\pi^2 r} \left(mr^2 K_2(mr) + 6r K_1(mr)\right).$$
(3)

For antiparticles the plus sign in equations (2) and (3) after the equal sign should be changed to a minus sign. There is also a multiplicative factor η on the right hand sides of (2) and (3) that is determined from the requirement that the total probability of a particle being in the CAR plus the CFR must be

 $^{^{2}}$ This may appear a bit audacious. But to the best of our knowledge such small mass scale is not prohibited by any existing data. However, once such an assumption is made some very concrete predictions follow.



Figure 1: Unnormalised $\mathcal{P}(x \to x')$ for a fixed r = 1 (in units 6.53 neV)⁻¹.

unity

$$\eta \stackrel{\text{def}}{=} \sqrt{\eta^* \eta} = \left(\frac{1}{A^* A|_{\mathsf{CAR}} + A^* A|_{\mathsf{CFR}}}\right)^{1/2}.$$
(4)

Since the probability for CAR (CAR, is abbreviation for classically allowed region) is \gg probability for CFR, the η upto a possible phase factor may be taken as

$$\eta \approx \frac{1}{\sqrt{A^* A}|_{\text{CAR}}} \tag{5}$$

As such η is practically constant in the CFR and is not explicitly noted in our considerations. To illustrate the m dependence of the probability $\mathcal{P}(x \to x')$ that derives from (3) we plot the un-normalised \mathcal{P} as a function of m. This is depicted in Figure 1. The result deviates somewhat from what would be expected from the heuristics: for m = 0 the probability to go off the light-cone is zero. For a given r, in CFR it increases to a maximum and then falls off and vanishes again for $m \to \infty$.

An intuitive way to understand the inevitability of our results is to realise that almost all quantum systems are endowed with CFRs. Quantum tunneling is just one such example in which one may access CFRs . Explicit calculations for a simple harmonic oscillator in its ground state are given in reference [15]; where it was found that CFR's contribution to the ground state energy equals $[1 - \text{erf}(1)] \times \frac{1}{2}\hbar\omega$.³ Since $[1 - \text{erf}(1)] \approx 0.16$, roughly 16% of the ground state energy is contributed by the CFR. If one considers a quantum field as a system of infinitely many simple harmonic oscillators, then that a massive particle may have a finite probability to access CFR becomes transparent.

Significance for short-baseline neutrino experiments

For comparing LSND with KARMEN we take units such that r = 1 for LSND.⁴ Then r = 17.6/30 for KARMEN. The ratio of the number of events expected for LSND versus KARMEN for the $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ channel is then

$$\alpha \frac{\mathcal{P}(x \to x')\big|_{r=1}}{\mathcal{P}(x \to x')\big|_{r=17.6/30}} \tag{6}$$

with α introduced to account for various detector/beam-related parameters

$$\alpha \stackrel{\text{def}}{=} \left(\frac{20000 \text{ Coulombs}}{2897 \text{ Coulombs}}\right) \left(\frac{167 \text{ tons}}{56 \text{ tons}}\right) \left(\frac{17.6 \text{ meters}}{30.0 \text{ meters}}\right)^2 \tag{7}$$

The first term in α represents data for KARMEN (three months after the 1996 upgrade): 2897 coulombs of integrated proton beam; for LSND with over 20000 coulombs collected between 1993 and 1997. The second term accounts for the difference in the detector volumes. The third term accounts for source-detector distance: 30 meters for LSND versus 17.6 meters for KAR-MEN; it accounts for diminution in the flux. Other issues, such as different duty factors, can apparently be incorporated (though this author is not competent enough to do that). Our source for these details is reference [4].

Figure 2 depicts the ratio of the number of events expected for LSND versus KARMEN as a function of m. In the stated context, there is an indication in favour of LSND.

In making these estimates we have assumed that the dominant contribution comes from the lowest lying mass eigenstate. The mixing matrix element

³The notation is standard, and erf(...) denotes the error function.

⁴The mass m is then measured in units of 6.53 neV.



Figure 2: The ratio of the number of events expected for LSND versus KAR-MEN as a function of m in the $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ mode. For the $\nu_{\mu} \rightarrow \nu_{e}$ mode, LSND gains an additional advantage by a factor 34 due to target differences and drift spaces – see [4] for details.

that enters in the projection to $\bar{\nu}_e$ from the lowest mass eigenstate cancels in taking the ratio (6). For reactor experiments one must be careful: while for the short baseline experiments, "leaking" off the lightcone may be important; the Pontecorvo mechanism may takeover for larger baselines.

The bottom line is thus: repeat a LSND-type experiment with, if possible, KARMEN-like feature that allows for better space-time resolution of events. Such an effort is already underway in Korea's JSNS² experiment [16]. In its first phase the source-detector distance is 24 meters. In its second phase an additional detector will be placed at a distance of 48 meters. It is designed to be a direct test of the $\overline{\nu}_e$ excess events observed by LSND. With m measured in units of 6.53 neV, the ratio of the number of events per unit volume of the far versus the near detector as a function of m reads

$$\frac{\left(15K_1\left(\frac{8m}{5}\right) + 4mK_2\left(\frac{8m}{5}\right)\right)^2}{4\left(15K_1\left(\frac{4m}{5}\right) + 2mK_2\left(\frac{4m}{5}\right)\right)^2}\tag{8}$$

where we have included the effect of flux reduction for the far detector relative to that of the near detector. This variation with m is displayed in Figure 3.

While here our focus has been on accelerator based experiments, we take note that in the context of reactor experiments, data taken by Neutrino-4 needs to be re-interpreted along the lines outlined here [17]. The concerns about its energy resolution pointed out by Giunti et al. in [6] then completely evaporate. Instead, because of the segmented detector design its spatio-temporal resolution becomes an advantage. Additionally, its ability to change the detector-source distance makes the Neutrino-4 an important tool for studying the framework described here.

Conclusion.

Here we have argued that LSND and KAMEN anomaly, and various other short baseline anomalies, may be resolved by first principle arguments. While Pontecorvo framework for neutrino oscillations allows us to understand, not only what was once called a 'solar neutrino anomaly,' but also the atmospheric neutrino data, it fails to account for various short baseline observations. In the process it seems that a fundamental mechanism has been missed. That mechanism is for a massive particle, with neV range mass, to cross the light-cone and trigger a detector at space-like separation. A positive result shall not only resolve the short baseline anomalies but it shall



Figure 3: The ratio given by equation (8) as a function of m.

also provide a mass of the lowest mass eigenstate without invoking a sterile neutrino.

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