

# Kaon Decays beyond the Standard Model

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We review a new method in order to determine the parameter  $\bar{\eta}$  of the Cabibbo-Kobayashi-Maskawa matrix from  $K \rightarrow \mu^+ \mu^-$  decays, using interference effects in the time-dependent decay rate. Furthermore, we discuss a new precision relation for the phase-shift of the time-dependent oscillation. The new methodology enables the discovery potential of future time-dependent measurements of  $K \rightarrow \mu^+ \mu^-$  decays for physics beyond the Standard Model.

## 1 Introduction

Even over 75 years after the discovery of the kaon in Manchester<sup>1</sup>, kaon physics is an exciting field with many new developments. On the theory side, there has been a lot of renewed interest in the decay  $K \rightarrow \mu^+ \mu^-$ , see recently Refs.<sup>2,3,4,5,6,7,8</sup>. On the experimental side, recent developments in rare kaon decays have been the  $3.4\sigma$  evidence for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  at NA62<sup>9</sup>, an improved upper limit on  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  from KOTO<sup>10</sup>, as well as new upper limits on  $\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)$ <sup>11</sup> and  $\mathcal{B}(K_{S,L} \rightarrow 2(\mu^+ \mu^-))$ <sup>12</sup> from LHCb. Furthermore, recently, new ideas for the future of kaon physics at CERN have been brought forward<sup>13</sup>.

In this review, I focus on the recent idea to use the time dependence of  $K \rightarrow \mu^+ \mu^-$  decays as a probe for new physics<sup>2,3,7,8</sup>. First experimental studies of this idea have been presented in Ref.<sup>14</sup>. The new idea is that we can in principle very cleanly measure  $\text{Im}(V_{td}^* V_{ts})$ , or equivalently  $\bar{\eta}$ , from  $K \rightarrow \mu^+ \mu^-$ . We can do so by employing time-dependent interference effects. In this way,  $K \rightarrow \mu^+ \mu^-$  is transformed into a third golden mode<sup>15</sup> along  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  which are currently measured at NA62<sup>16</sup> and KOTO<sup>10</sup>, respectively.

Our new method includes  $f_K$  as the main hadronic uncertainty, so it is theoretically clean, however it includes measuring the time-dependent interference effects, which are experimentally challenging.

In Sec. 2 we present the main idea, giving us a new handle on the Wolfenstein parameter  $\bar{\eta}$ . In Sec. 3 we present another precision relation related to the phase shift in the time dependence of  $K \rightarrow \mu^+ \mu^-$  decays. Constraints on new physics are briefly discussed in Sec. 4, before we conclude in Sec. 5.

## 2 Separating Long- and Short-Distance Physics in $K \rightarrow \mu^+ \mu^-$

One of the long-term goals of the physics program of rare kaon decays is to determine the unitarity triangle purely with kaon decays. This gives a crucial intergenerational consistency check of the Standard Model (SM) and new ways to probe for new physics. A key issue for this goal is the identification of observables with a theoretically clean sensitivity to CKM matrix elements. In order to achieve that, we need methods with a theory error on the hadronic physics

at the order of  $\sim 1\%$ . In  $K \rightarrow \mu^+ \mu^-$  we are currently not able to achieve such a theory precision for the long-distance (LD) effects. The question is therefore how to extract the short-distance (SD) physics from  $K \rightarrow \mu^+ \mu^-$  measurements.

In principle, the measurement of the branching ratio  $\mathcal{B}(K_S \rightarrow (\mu\mu)_{l=0})$ , where the index  $l = 0$  indicates the angular momentum of the muons in the final state, provides such a clean probe for SD physics. The reason is that, as a transition from a CP-even to a CP-odd state,  $|A(K_S \rightarrow (\mu\mu)_{l=0})|$  is a CP-violating amplitude. As such it has to a very good approximation no contributions from long-distance (LD) physics. However, in practice final state muons with specific angular momentum  $(\mu\mu)_{l=0}$  and  $(\mu\mu)_{l=1}$  are not available to us because in the decay rate we measure their incoherent sum. The key question is therefore how to access  $\mathcal{B}(K_S \rightarrow (\mu\mu)_{l=0})$ .

At this point it is instructive to take a step back and look at the anatomy of long- and short-distance physics in  $K \rightarrow \mu^+ \mu^-$  in general:<sup>3</sup>

- CP-conserving amplitudes: both SD and LD contributions.  
 CP-odd  $\rightarrow$  CP-odd:  $|A(K_L \rightarrow (\mu\mu)_{l=0})|$ ,  
 CP-even  $\rightarrow$  CP-even:  $|A(K_S \rightarrow (\mu\mu)_{l=1})|$ .
- CP-violating amplitudes: only SD contributions.  
 CP-even  $\rightarrow$  CP-odd:  $|A(K_S \rightarrow (\mu\mu)_{l=0})|$ ,  
 CP-odd  $\rightarrow$  CP-even:  $|A(K_L \rightarrow (\mu\mu)_{l=1})|$ .
- Relative phases: both SD and LD contributions.  
 $\varphi_0 \equiv \arg(\mathcal{A}^*(K_S \rightarrow (\mu\mu)_{l=0})\mathcal{A}(K_L \rightarrow (\mu\mu)_{l=0}))$ ,  
 $\varphi_1 \equiv \arg(\mathcal{A}^*(K_S \rightarrow (\mu\mu)_{l=1})\mathcal{A}(K_L \rightarrow (\mu\mu)_{l=1}))$ .

In our discussion we neglect the small CP violation from mixing, *i.e.*, we take the limit  $\varepsilon_K = 0$ , which can however also be incorporated into the analysis as shown in Ref.<sup>7</sup>. In the SM, the SD operator does not generate a  $(\mu\mu)_{l=1}$  state due to CPT, see, *e.g.*, the appendix of Ref.<sup>3</sup>, and therefore  $|A(K_L \rightarrow (\mu\mu)_{l=1})| = 0$  and  $\varphi_1 = 0$ . Because of that, we are left in total with four theory parameters, one of which, namely  $|A(K_S \rightarrow (\mu\mu)_{l=0})|$ , is purely due to SD physics. As said above, we can cleanly calculate it in the SM<sup>7,17,18,19</sup>

$$\mathcal{B}(K_S \rightarrow (\mu\mu)_{l=0}) = 1.7 \cdot 10^{-13} \times \left( \frac{A^2 \lambda^5 \bar{\eta}}{1.3 \cdot 10^{-4}} \right). \quad (1)$$

The hadronic uncertainties from  $f_K$ <sup>20</sup> as well as from higher-order QCD/EW corrections<sup>7</sup> in the prefactor in Eq. (1) are at the level of  $\sim 1\%$ . The observable  $\mathcal{B}(K_S \rightarrow (\mu\mu)_{l=0})$  therefore opens the way to a theoretically clean extraction of  $\bar{\eta}$ , and importantly we can also calculate it cleanly in models beyond the Standard Model (BSM).

Now, the solution to the problem how to access  $\mathcal{B}(K_S \rightarrow (\mu\mu)_{l=0})$  experimentally is as follows. It consists in measuring the time dependence of  $K \rightarrow \mu^+ \mu^-$ , which can be written as<sup>21</sup>

$$\frac{d\Gamma}{dt} \propto C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2C_{\text{Int.}} \cos(\Delta m_K t - \varphi_0) e^{-\frac{\Gamma_L + \Gamma_S}{2} t}, \quad (2)$$

where  $C_{L,S}$  are related to the  $K_{L,S}$  decay rates, respectively,  $C_{\text{Int.}}$  is due to the interference between  $K_L$  and  $K_S$  decays and  $\varphi_0$  is the phase shift of the oscillation. Furthermore,  $\Gamma \equiv (\Gamma_S + \Gamma_L)/2$  and  $\Delta m$  is the kaon mass difference. For the example of a pure  $K^0$  beam, the four experimental observables in Eq. (2) can be expressed as follows by the four theory parameters:

$$\begin{aligned} C_L &= |A(K_L)_{l=0}|^2, & C_S &= |A(K_S)_{l=0}|^2 + \beta_\mu^2 |A(K_S)_{l=1}|^2, & (3) \\ C_{\text{Int.}} &= |A(K_S)_{l=0}| |A(K_L)_{l=0}|, & \varphi_0 &= \arg(A(K_S)_{l=0}^* A(K_L)_{l=0}), & (4) \end{aligned}$$

where  $\beta_\mu = \sqrt{1 - 4m_\mu^2/m_{K^0}^2}$ . Consequently, we can completely solve the system and obtain<sup>3</sup>

$$\mathcal{B}(K_S \rightarrow (\mu^+ \mu^-)_{l=0}) = \mathcal{B}(K_L \rightarrow \mu^+ \mu^-) \cdot \frac{\tau_S}{\tau_L} \cdot \frac{C_{\text{Int.}}^2}{C_L^2}. \quad (5)$$

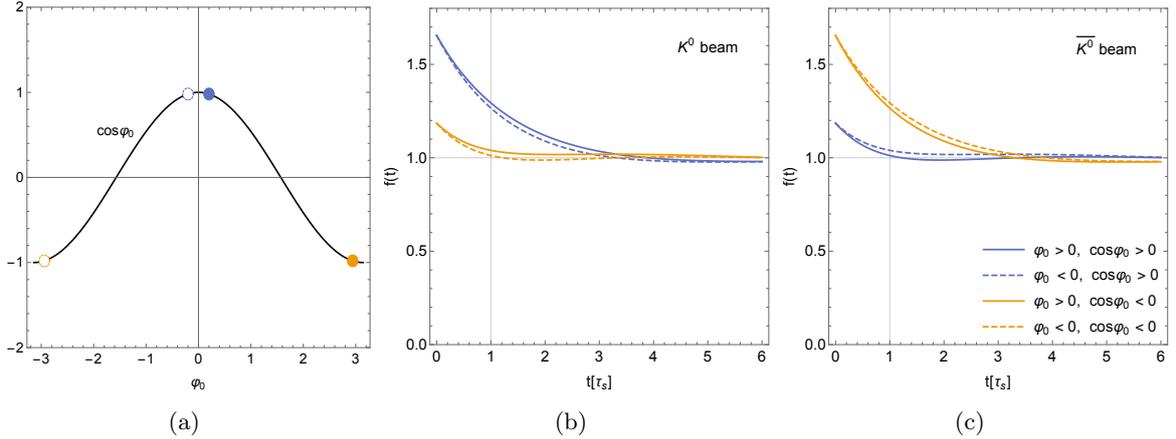


Figure 1 – The four solutions of Eq. (7) for  $\varphi_0$  (a) and their implications for the time dependence of a pure  $K^0$  (b) or  $\bar{K}^0$  beam (c). Figure reproduced from Ref. <sup>8</sup>.

The comparison of Eq. (5) as extracted from future data with the SM result Eq. (1) enables the extraction of  $\bar{\eta}$ . This implies that it is crucial to obtain the interference terms in the time dependence Eq. (2). We note that Eq. (5) is only valid in the limit of a pure  $K^0$  beam. In a realistic scenario of a mixed  $K^0/\bar{K}^0$  beam, a dilution factor reduces the sensitivity by a respective amount, see Ref. <sup>3</sup> for details.

### 3 A Precision Prediction for the Phase Shift

Apart from Eq. (5), there is one more precision relation for an observable of the time dependence of  $K \rightarrow \mu^+\mu^-$  that can be used for a SM test. The phase shift  $\varphi_0$  in Eq. (2) is related to a known ratio of branching ratios as <sup>8</sup>

$$\cos^2 \varphi_0 = C_{\text{QED}}^2 \cdot \frac{\mathcal{B}(K_L \rightarrow \gamma\gamma)}{\mathcal{B}(K_L \rightarrow \mu^+\mu^-)}, \quad (6)$$

where  $C_{\text{QED}}^2$  is a known QED factor. Eq. (6) implies the model-independent prediction <sup>8</sup>

$$\cos^2 \varphi_0 = 0.96 \pm 0.02_{\text{exp}} \pm 0.02_{\text{th}}, \quad (7)$$

that can be tested with future measurements of the time dependence of  $K \rightarrow \mu^+\mu^-$ . In Eq. (7), the experimental error comes from the one of the involved branching ratios. The theory error comes from higher order QED corrections and from additional intermediate on-shell contributions that give a small correction to the dominant two-photon contribution <sup>22</sup>. Eq. (7) has actually four model-independent solutions for  $\varphi_0$ , each of which implies a different time evolution. We show these solutions in Fig. 1.

### 4 Constraints on New Physics

Although no measurement of the time dependence of  $K \rightarrow \mu^+\mu^-$  is available, the LHCb constraint on the branching ratio <sup>11</sup>

$$\mathcal{B}(K_S \rightarrow \mu^+\mu^-) < 2.1 \cdot 10^{-10} \quad (8)$$

constrains relevant parameter space of new physics models already now, as is worked out in detail in Ref. <sup>5</sup>. Therein, in order to be conservative, the bound Eq. (8) is interpreted as a bound on  $\mathcal{B}(K_S \rightarrow \mu^+\mu^-)_{t=0}$ , resulting in a lot of room for BSM physics to be tested <sup>5</sup>

$$\frac{\mathcal{B}(K_S \rightarrow \mu^+\mu^-)_{t=0}^{\text{SM}}}{\mathcal{B}(K_S \rightarrow \mu^+\mu^-)_{t=0}} \leq 1280. \quad (9)$$

However, scalar leptoquark or two-Higgs doublet models that saturate the bound Eq. (9) can be constructed, at the same time being consistent with existing constraints, see Ref. <sup>5</sup> for details. The decays  $K \rightarrow \mu^+\mu^-$  and  $K_L \rightarrow \pi^0\nu\bar{\nu}$ , although sensitive to the same CKM matrix element combination in the SM, are sensitive to different new physics operators in BSM models<sup>5</sup>. Future updated bounds of the constraint Eq. (8) are important to probe the parameter space of BSM models further.

## 5 Conclusion

The time dependence of  $K \rightarrow \mu^+\mu^-$  gives two independent SM tests. The coefficient of the interference term of the time-dependent decay rate is sensitive to  $\mathcal{B}(K_S \rightarrow \mu^+\mu^-)_{l=0}$ , which in the SM is proportional to the Wolfenstein parameter  $\bar{\eta}$ . The second SM test is given by a precision relation of the oscillation phase shift, which is predicted model-independently up to a four-fold ambiguity. The leptonic kaon decay mode  $K \rightarrow \mu^+\mu^-$  turns out to be theoretically clean and experimentally challenging, similar in that respect to the related decay modes  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  and  $K_L \rightarrow \pi^0\nu\bar{\nu}$ .

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## References

1. G. D. Rochester and C. C. Butler. Evidence for the Existence of New Unstable Elementary Particles. *Nature*, 160:855–857, 1947.
2. Giancarlo D’Ambrosio and Teppei Kitahara. Direct  $CP$  Violation in  $K \rightarrow \mu^+\mu^-$ . *Phys. Rev. Lett.*, 119(20):201802, 2017.
3. Avital Dery, Mitrajyoti Ghosh, Yuval Grossman, and Stefan Schacht.  $K \rightarrow \mu^+\mu^-$  as a clean probe of short-distance physics. *JHEP*, 07:103, 2021.
4. Andrzej J. Buras and Elena Venturini. Searching for New Physics in Rare  $K$  and  $B$  Decays without  $|V_{cb}|$  and  $|V_{ub}|$  Uncertainties. *Acta Phys. Polon. B*, 53(6):A1, 9 2021.
5. Avital Dery and Mitrajyoti Ghosh.  $K \rightarrow \mu^+\mu^-$  beyond the standard model. *JHEP*, 03:048, 2022.
6. G. D’Ambrosio, A. M. Iyer, F. Mahmoudi, and S. Neshatpour. Anatomy of kaon decays and prospects for lepton flavour universality violation. *JHEP*, 09:148, 2022.
7. Joachim Brod and Emmanuel Stamou. Impact of indirect  $CP$  violation on  $\text{Br}(K_S \rightarrow \mu^+\mu^-)_{\ell=0}$ . 9 2022.
8. Avital Dery, Mitrajyoti Ghosh, Yuval Grossman, Teppei Kitahara, and Stefan Schacht. A Precision Relation between  $\Gamma(K \rightarrow \mu^+\mu^-)(t)$  and  $\mathcal{B}(K_L \rightarrow \mu^+\mu^-)/\mathcal{B}(K_L \rightarrow \gamma\gamma)$ . *JHEP*, 03:014, 2023.
9. Eduardo Cortina Gil et al. Measurement of the very rare  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  decay. *JHEP*, 06:093, 2021.
10. J. K. Ahn et al. Study of the  $K_L \rightarrow \pi^0\nu\bar{\nu}$  Decay at the J-PARC KOTO Experiment. *Phys. Rev. Lett.*, 126(12):121801, 2021.
11. Roel Aaij et al. Constraints on the  $K_S^0 \rightarrow \mu^+\mu^-$  Branching Fraction. *Phys. Rev. Lett.*, 125(23):231801, 2020.
12. Search for  $K_{S(L)}^0 \rightarrow \mu^+\mu^-\mu^+\mu^-$  decays at LHCb. 12 2022.
13. E. Cortina Gil et al. HIKE, High Intensity Kaon Experiments at the CERN SPS. 11 2022.
14. Radoslav Marchevski. First thought on a high-intensity  $K_S$  experiment. In *International Conference on Kaon Physics 2022*, 1 2023.
15. Avital Dery.  $K \rightarrow \mu+\mu-$  as a third kaon golden mode. *J. Phys. Conf. Ser.*, 2446(1):012034, 2023.
16. Eduardo Cortina Gil et al. An investigation of the very rare  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  decay. *JHEP*, 11:042, 2020.
17. Gino Isidori and Rene Unterdorfer. On the short distance constraints from  $K_{L,S} \rightarrow \mu^+\mu^-$ . *JHEP*, 01:009, 2004.
18. D. Gomez Dumm and A. Pich. Long distance contributions to the  $K_L \rightarrow \mu^+\mu^-$  decay width. *Phys. Rev. Lett.*, 80:4633–4636, 1998.
19. T. Inami and C. S. Lim. Effects of Superheavy Quarks and Leptons in Low-Energy Weak Processes  $k(L) \rightarrow \mu$  anti- $\mu$ ,  $K \rightarrow \pi$  Neutrino anti-neutrino and  $K^0 \leftrightarrow$  anti- $K^0$ . *Prog. Theor. Phys.*, 65:297, 1981. [Erratum: *Prog.Theor.Phys.* 65, 1772 (1981)].
20. Y. Aoki et al. FLAG Review 2021. *Eur. Phys. J. C*, 82(10):869, 2022.
21. R. L. Workman et al. Review of Particle Physics. *PTEP*, 2022:083C01, 2022.
22. B. R. Martin, E. De Rafael, and J. Smith. Neutral kaon decays into lepton pairs. *Phys. Rev. D*, 2:179–200, 1970.