Analysis of the strong decays of SU(3) partners of the $\Omega(2012)$ baryon

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(Dated: August 25, 2023)

Abstract

We estimate the coupling constants and decay widths of the $SU(3)$ partners of the $\Omega(2012)$ hyperon, as discovered by the BELLE Collaboration, using the light cone sum rules method. Our study includes a comparison of the obtained results for relevant decay widths with those derived within the framework of the flavor $SU(3)$ analysis. We observe a good agreement between the predictions of both approaches. The results we obtain for the branching ratio can provide helpful insights for determining the nature of the $SU(3)$ partners of the $\Omega(2012)$ baryon.

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I. INTRODUCTION

In 2018, the BELLE Collaboration made an exciting announcement regarding the discovery of the $\Omega(2012)$ hyperon. This discovery was based on the $\Omega^{*-} \to \Xi^0 K^-$ and $\Omega^{*-} \to \Xi^- K_s^0$ decay channels, with a measured mass of $m = 2012.4 \pm 0.7\,(\text{stat}) \pm 0.6\,(\text{sys})\,\text{MeV}$, and decay width of $\Gamma_{tot} = 6.4^{+2.5}_{-2.0}\,(\text{stat}) \pm 1.6\,(\text{sys})\,\text{MeV}$ [1]. However, knowing only the mass of the state is not sufficient enough to determine the quantum numbers of a state. For instance, within the QCD sum rule method, the mass of the $\Omega(2012)$ baryon is estimated, assuming it to be either 1P or 2S excitation state [2]. Both assumptions yield the same mass value, although the estimated residues differ. Thus, additional physical quantities, such as the decay width, are necessary to identify the quantum numbers of newly discovered particles.

In a previous study [3], the $\Omega(2012) \to \Xi^0 K^-$ transition was investigated, and its corresponding decay width was estimated by considering two possible scenarios for $\Omega(2012)$: either a $1P$ or $2S$ state. A comparison of the total decay widths obtained in this work led to the conclusion that the $\Omega(2012)$ is itself a $J^P = \frac{3}{2}^-$ state. Moreover, predictions from various theoretical models also converge on the likely quantum numbers $J^P = \frac{3}{2}^-$ for the observed state [4–15].

In this study, considering $\Omega(2012)$ as $J^P = \frac{3}{2}^-$ state, strong couplings of SU(3) partners of this state are investigated within the framework of light cone sum rules (LCSR). It should be noted that this problem was also studied in [16] using the flavor SU(3) symmetry approach.

The structure of this paper is as follows: Section II introduces the LCSR for the strong couplings of the transitions $\frac{3}{2}^- \to \frac{1}{2}^+ +$ pseudoscalar mesons. Section III provides a numerical analysis of the LCSR, focusing on the relevant strong couplings. Within this section, we also present the computed values of the decay widths based on the obtained coupling constants. Additionally, we compare our results with those obtained from the flavor SU(3) symmetry method. Finally, our conclusions are summarized in the last section.

II. LCSR FOR THE STRONG COUPLINGS OF $SU(3)$ PARTNERS OF $\Omega(2012)$

To calculate the strong couplings of $SU(3)$ partners, denoted as $\frac{3}{2}^-$ states in the following discussions, we introduce the vacuum-to-octet baryon correlation function:

$$\Pi_{\mu\nu}(p,q) = i \int d^4x e^{iqx} \left\langle 0 \left| T\Big\{ \eta_\mu(0) J_\nu(x) \Big\} \right| \mathcal{O}(p) \right\rangle \ , \tag{1}$$

where η_μ represents the interpolating current of the decuplet baryons, $J_\nu = \bar{q}_1 \gamma_\nu \gamma_5 q_2$ is the interpolating current of the pseudoscalar mesons, and $|\mathcal{O}(p)\rangle$ represents the octet baryon state.

The interpolating current of the decuplet baryons can be written as:

$$\eta_\mu = \varepsilon^{abc} A \Big\{ \left(q_1^{aT} C \gamma_\mu q_2^b\right) q_3^c + \left(q_2^{aT} C \gamma_\mu q_3^b\right) q_1^c + \left(q_3^{aT} C \gamma_\mu q_1^b\right) q_2^c \Big\} \,, \tag{2}$$

where a, b, c are the color indices, C is the charge conjugation operator, and A is the normalization factor. The quark content of the decuplet baryons and the normalization factor A are presented in Table I.

	A	q_1	q_2	q_3
Δ^+	$\sqrt{\frac{1}{3}}$	u	u	d
Σ^{*+}	$\sqrt{\frac{1}{3}}$	u	u	s
Σ^{*0}	$\sqrt{\frac{2}{3}}$	u	d	s
Σ^{*-}	$\sqrt{\frac{1}{3}}$	d	d	s
Ξ^{*0}	$\sqrt{\frac{1}{3}}$	s	s	u
Ξ^{*-}	$\sqrt{\frac{1}{3}}$	s	s	d

TABLE I: The quark content of the decuplet baryons and the normalization factor A.

To derive the Light Cone Sum Rules (LCSR) for the strong coupling constants, the approach involves computing the correlation function in two ways: in terms of hadrons and in terms of quark-gluon fields within the deep Euclidean domain. By applying the quark-hadron duality ansatz, the relevant sum rules can be derived.

The calculation of the strong coupling constants in the framework of the LCSR is based on the fact that they appear in double dispersion relation for the same correlation function given in Eq. (1). In other words, calculating the strong coupling constant requires the use of double dispersion relation for the correlation function by making use of the axial vector current.

Before delving into the details of the calculations, it is important to highlight the following aspect: the interpolating current for the decuplet baryons interacts not only with the ground positive parity states $J^P = \frac{3}{2}^+$ but also with the negative parity states $J^P = \frac{3}{2}^-$ and even with states of $J^P = \frac{1}{2}^-$.

To eliminate the contributions from unwanted states $J^P = \frac{3}{2}^+$ and $J^P = \frac{1}{2}^-$, a technique involving linear contributions of different Lorentz structures is employed (for more details about this approach refer to [17]).

Following the standard procedure, we insert the total set of set of baryons with $J^P = \frac{3}{2}^+$ into the correlation function as well as the corresponding pseudoscalar mesons. Then, we get,

$$\Pi_{\mu\nu}(p,q)=\sum_{i=\pm}\frac{\left\langle 0\left|\eta_{\mu}\right|\frac{3}{2}^{i}(p')\right\rangle}{m_i^2-p'^2}\frac{\left\langle\frac{3}{2}^{i}(p')\mathcal{P}(q)|\mathcal{O}(p)\right\rangle}{m_{\mathcal{P}}^2-q^2}\left\langle 0\left|J_\nu(x)\right|\mathcal{P}(q)\right\rangle \ , \tag{3}$$

where summation is over positive and negative states, and $m_{\mathcal{P}}$ is the mass of the corresponding pseudoscalar meson. The matrix elements in the above equation are defined as,

$$\begin{aligned}
\left\langle 0\middle|\eta_\mu\middle|\frac{3}{2}^{+}(p')\right\rangle&=\lambda_{+}u_\mu(p') \ ,\\
\left\langle 0\middle|\eta_\mu\middle|\frac{3}{2}^{-}(p')\right\rangle&=\lambda_{-}\gamma_5 u_\mu(p') \ ,\\
\left\langle\frac{3}{2}^{+}(p')\mathcal{P}(q)\middle|\mathcal{O}(p)\right\rangle&=g_{+}\bar{u}_\alpha(p')\gamma_5 u(p)q^\alpha \ ,\\
\left\langle\frac{3}{2}^{-}(p')\mathcal{P}(q)\middle|\mathcal{O}(p)\right\rangle&=g_{-}\bar{u}_\alpha(p')u(p)q^\alpha \ ,\\
\left\langle 0\left|J_\nu\right|\mathcal{P}(q)\right\rangle&=if_{\mathcal{P}}q_\nu \ ,
\end{aligned} \tag{4}$$

where $\lambda_{\pm}$ are the residues of the related $\frac{3}{2}^{\pm}$ baryons, $g_{\pm}$ stands for the coupling constants of the $J^P=\frac{3}{2}^{\pm}$ baryons with the octet baryons and the pseudoscalar mesons, $f_{\mathcal{P}}$ is the decay constant of the pseudoscalar meson and q denotes its 4-momentum, and $u_\mu(p')$ and $u(p)$ are the Rarita-Schwinger and Dirac spinors respectively. Performing summation over the spins of the Rarita-Schwinger spinors with the help of the following formula,

$$\sum_{s'}u_\mu(p',s')\bar{u}_\alpha(p',s')=-(\not{p}'+m)\left[g_{\mu\alpha}-\frac{1}{3}\gamma_\mu\gamma_\nu-\frac{2p'_\mu p'_\alpha}{3m^2}+\frac{p'_\mu\gamma_\alpha-p'_\alpha\gamma_\mu}{3m}\right] \ , \tag{5}$$

and using Eqs. (3) and (4) one can obtain the expression of the correlation function from the hadronic part. It should be reminded here that the interpolating current interacts not only with spin $\frac{3}{2}$ states, but also with spin $\frac{1}{2}$ states.

Using the condition $\gamma^\mu\eta_\mu=0$, it can easily be shown that

$$\left\langle 0\middle|\eta_\mu\middle|\frac{1}{2}(p')\right\rangle\sim\left[\alpha\gamma_\mu-\beta p'_\mu\right]u(p') \ . \tag{6}$$

It follows from this equation that any structure containing γ_μ or p'_μ is "contaminated" by the contributions of spin $\frac{1}{2}$-states. Hence, to remove the contributions of spin $\frac{1}{2}$-states, these structures are all discarded. Another problem is all Dirac structures not being independent of each other. To overcome this issue, Dirac structures need to be arranged in a specific order. In the present work we choose the ordering $\gamma_\mu \not{p}' \not{q} \gamma_\nu \gamma_5$.

Keeping these notes in mind, and using Eqs. (3), (4) and (5), we obtain the correlation function from the phenomenological part as follows:

$$\Pi_{\mu\nu} = \frac{\lambda_+ g_+ (-\not{q} + m_+ - m_{\mathcal{O}}) \gamma_5 q_\mu q_\nu f_{\mathcal{P}}}{(m_+^2 - p'^2)(m_{\mathcal{P}}^2 - q^2)} u(p) + \frac{\lambda_- g_- (\not{q} + m_- + m_{\mathcal{O}}) \gamma_5 q_\mu q_\nu f_{\mathcal{P}}}{(m_-^2 - p'^2)(m_{\mathcal{P}}^2 - q^2)} u(p) \ , \tag{7}$$

where $m_{\mathcal{O}}$ is the mass of the relevant octet baryon, $m_+ (m_-)$ is the mass of the spin-$\frac{3}{2}$ positive (negative) parity baryon, respectively.

As a last step, we need to eliminate the contributions of $J^P = \frac{3}{2}^+$ states. For this purpose, we use the linear combinations of the invariant functions corresponding to different Lorentz structures.

We now turn our attention to the calculation of the correlation function by using the operator product expansion (OPE) in the deep Euclidean region for the variables $p'^2 = (p-q)^2$, and $q^2 \ll 0$. The new element of the calculation is the appearance of the double spectral density of the invariant functions. For the calculation of the double spectral densities it is enough to find the double spectral representations of the master integrals of the form,

$$I_{n,k} = \int du \frac{u^k}{[m^2 - (pu-q)^2]^n} \ ; \quad n = 1, 2, 3.$$

We now present the details of the calculations for the spectral density for $n = 1$ case. The cases $n = 2$ and $n = 3$ are calculated in the similar manner. First of all we will show how the doubly spectral density can be obtained from the invariant amplitudes. The invariant amplitudes can be written in terms of the double spectral representation as follows

$$\Pi[(p-q)^2, q^2] = \int ds_1 \int ds_2 \frac{\rho(s_1, s_2)}{[s_1 - (p-q)^2](s_2 - q^2)} + \cdots \tag{8}$$

The spectral density can be obtained from $\Pi[(p-q)^2, q^2]$ by applying two subsequent double Borel transformations. After first double Borel transformation over the variables $-(p-q)^2$ and $-q^2$ we get,

$$\Pi^{\mathcal{B}_1}(M_1^2, M_2^2) = \int ds_1 \int ds_2 e^{-s_1/M_1^2 - s_2/M_2^2} \rho(s_1, s_2) \ . \tag{9}$$

Before implementing second double Borel transformation, we introduce new variables $\sigma_1 = \frac{1}{M_i^2}$. The second double Borel transformation can be performed over the new Borel parameter τ_i by using the relation,

$$\mathcal{B}_\tau e^{-s\sigma} = \delta\left(\frac{1}{\tau} - s\right) \ . \tag{10}$$

As a result we get

$$\mathcal{B}_{\tau_1}\mathcal{B}_{\tau_2}\Pi^{\mathcal{B}_1}(M_1^2, M_2^2) = \rho\left(\frac{1}{\tau_1}, \frac{1}{\tau_2}\right) \ . \tag{11}$$

Hence, double spectral density can be obtained as follows,

$$\rho(s_1, s_2) = \mathcal{B}_{\frac{1}{s_1}}(\sigma_1)\mathcal{B}_{\frac{1}{s_2}}(\sigma_2)\Pi^{\mathcal{B}}\left(\frac{1}{\sigma_1}, \frac{1}{\sigma_2}\right) \ .$$

Let us now pay our attention to the double spectral density for the $n = 1$ case. Using

$$-(pu - q)^2 = -u(p - q)^2 - \bar{u}q^2 + u\bar{u}m_{\mathcal{O}}^2 \ ,$$

where $\bar{u} = 1 - u$. $I_{1,k}$ can be written as,

$$\begin{aligned} I_{1,k} =& \int du \frac{u^k}{[m^2 - u(p-q)^2 - \bar{u}q^2 + \bar{u}um_{\mathcal{O}}^2]} \\ =& \int du \frac{u^k}{\mathcal{D}} \ , \end{aligned}$$

where m is the corresponding quark mass. Using the Schwinger representation for the denominator and carrying out the first double Borel transformation over the variables $-(p-q)^2$ and $-q^2$, we get

$$\begin{aligned} I_{1,k} =& \frac{\sigma_2^k}{(\sigma_1+\sigma_2)^{k+1}} \, exp\left[-m_{\mathcal{O}}^2 \frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2} - m^2(\sigma_1+\sigma_2)\right] \ , \\ =& \frac{\sigma_2^k}{(\sigma_1+\sigma_2)^{k+1}} \, exp\left[m_{\mathcal{O}}^2 \frac{\sigma_1^2+\sigma_2^2}{2(\sigma_1+\sigma_2)} - \left(m^2 + \frac{m_{\mathcal{O}}^2}{2}\right)(\sigma_1+\sigma_2)\right] \ , \end{aligned}$$

where $\sigma_i = \frac{1}{M_i^2}$. In order to perform the second double Borel transformation we use the relation,

$$\sqrt{\frac{\sigma_1+\sigma_2}{2\pi}} \int_{-\infty}^{+\infty} dx_i \, exp\left[-\frac{\sigma_1+\sigma_2}{2}x_i^2 - \sigma_i m_{\mathcal{O}} x_i\right] = exp\left[\frac{m_{\mathcal{O}}^2\sigma_i^2}{2(\sigma_1+\sigma_2)}\right] \ .$$

Then we get,

$$I_{1,k}^{\mathcal{B}} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dx_1 \int_{-\infty}^{+\infty} dx_2 \, \frac{\sigma_2^k}{(\sigma_1+\sigma_2)^k} \, exp\Bigg[- \sigma_1\left(m^2 + \frac{(m_{\mathcal{O}}+x_1)^2 + x_2^2}{2}\right)$$

$$-\sigma_2\left(m^2+\frac{(m_{\mathcal{O}}+x_2)^2+x_1^2}{2}\right)\Bigg] \, ,$$

$$=\frac{1}{2\pi}\frac{1}{\Gamma(k)}\int_{-\infty}^{+\infty}dx_1\int_{-\infty}^{+\infty}dx_2\int_0^{\infty}dt\, t^{k-1}\sigma_2^k\, exp\Bigg[-\sigma_1\left(m^2+\frac{(m_{\mathcal{O}}+x_1)^2+x_2^2}{2}+t\right)$$
$$-\sigma_2\left(m^2+\frac{(m_{\mathcal{O}}+x_2)^2+x_1^2}{2}+t\right)\Bigg] \, ,$$
$$=\frac{1}{2\pi}\frac{1}{\Gamma(k)}\int_{-\infty}^{+\infty}dx_1\int_{-\infty}^{+\infty}dx_2\int_0^{\infty}dt\, t^{k-1}\, exp\left[-\sigma_1\left(m^2+\frac{(m_{\mathcal{O}}+x_1)^2+x_2^2}{2}+t\right)\right]$$
$$\times\left(-\frac{\partial}{\partial t}\right)^k exp\left[-\sigma_2\left(m^2+\frac{(m_{\mathcal{O}}+x_2)^2+x_1^2}{2}+t\right)\right] \, .$$

After performing the second Borel transformation, we obtain the the spectral density corresponding to $I_{1,k}$ as is given below,

$$\rho_{1,k}(s_1,s_2)=\frac{1}{2\pi}\frac{1}{\Gamma(k)}\left(-\frac{\partial}{\partial s_2}\right)^k\int_{-\infty}^{+\infty}dx_1\int_{-\infty}^{+\infty}dx_2\int_0^{\infty}dt\, t^{k-1}\delta\left[s_1-\left(m^2+\frac{(m_{\mathcal{O}}+x_1)^2+x_2^2}{2}+t\right)\right]$$
$$\times\delta\left[s_2-\left(m^2+\frac{(m_{\mathcal{O}}+x_2)^2+x_1^2}{2}+t\right)\right]$$
$$=\frac{1}{2\pi}\frac{1}{\Gamma(k)}\left(-\frac{\partial}{\partial s_2}\right)^k\int_{-\infty}^{+\infty}dx_1\int_{-\infty}^{+\infty}dx_2\int_0^{\infty}dt\, t^{k-1}\delta\left[s_1-\left(m^2+\frac{(m_{\mathcal{O}}+x_1)^2+x_2^2}{2}+t\right)\right]$$
$$\times\delta\left[s_2-\left(m^2+\frac{(m_{\mathcal{O}}+x_2)^2+x_1^2}{2}+t\right)\right] \, .$$

Using two Dirac delta functions, one can easily perform integrals over t and x_2 whose result is given below,

$$\rho_{1,k}(s_1,s_2)=\frac{1}{2\pi\Gamma(k)m_{\mathcal{O}}}\left(-\frac{\partial}{\partial s_2}\right)^k\int_{-\infty}^{+\infty}dx_1\left[s_1-\left(m^2+\frac{(m_{\mathcal{O}}+x_1)^2+x_2^2}{2}\right)\right]^{k-1}$$
$$\times\Theta\left[s_1-\left(m^2+\frac{(m_{\mathcal{O}}+x_1)^2+x_2^2}{2}\right)\right] \, ,$$

where

$$x_2=\frac{s_2-s_1}{m_{\mathcal{O}}}+x_1 \, ,$$

and $\Theta(x)$ is the Heaviside step function which restricts the integral over x_1 between the limits

$y_\pm(s_1,s_2)$ where

$$y_\pm(s_1,s_2) = \frac{-m_\mathcal{O}^2 + s_1 - s_2 \pm \sqrt{\Delta}}{2m_\mathcal{O}} ~,$$

and

$$\Delta = -m_\mathcal{O}^4 - (s_1-s_2)^2 + 2m_\mathcal{O}^2(-2m^2+s_1+s_2) ~.$$

Thus as a result of above summarized calculations, the spectral density can takes the following form,

$$\rho_{1,k}(s_1,s_2)=\frac{1}{2\pi}\frac{1}{\Gamma(k)}\frac{1}{m_\mathcal{O}}\left(-\frac{\partial}{\partial s_2}\right)^k \int_{y_-}^{y_+} dx \left[(y_+ - x)(x-y_-)\right]^k \Theta(\Delta) ~.$$

In order to evaluate the x integral, we introduce a new variable through the relation,

$$x = (y_+ - y_-)y + y_- ~,$$

so that the spectral density can be written as,

$$\rho_{1,k}(s_1,s_2)=\frac{1}{2\pi}\frac{\Gamma(k)}{\Gamma(2k)}\frac{1}{m_\mathcal{O}^{2k}}\left(-\frac{\partial}{\partial s_2}\right)^k \left[\Delta^{k-\frac{1}{2}}\Theta(\Delta)\right] ~. \qquad (12)$$

Double spectral densities for $I_{2,k}$ and $I_{3,k}$ can be calculated with the help of the following relations,

$$I_{2,k}=\left(-\frac{\partial}{\partial m^2}\right)I_{1,k} ~,~ \text{and,}$$
$$I_{3,k}=\frac{1}{2}\left(-\frac{\partial}{\partial m^2}\right)^2 I_{1,k} ~,$$

(see also [17] for the calculation of the spectral densities $I_{2,k}$ and $I_{3,k}$).

Matching the OPE results with the double dispersion relations for the relevant Lorentz structures for the hadrons, applying the quark-hadron duality ansatz, and performing double Borel transformation with respect to the variables $-(p-q)^2$ and $-q^2$, we obtain the LCSR for the relevant coupling constants whose explicit form can be written as,

$$g_-=\frac{e^{m_-^2/M_1^2}\, e^{m_\mathcal{P}^2/M_2^2}}{f_\mathcal{P}\lambda_-(m_+ + m_-)}\frac{1}{\pi^2}\int_0^{s_0} ds_1$$

$$\times \int_{t_1(s_1)}^{t_2(s_1)} ds_2\, e^{-s_1/M_1^2} e^{-s_2/M_2^2}\, \mathrm{Im}_{s_1} \mathrm{Im}_{s_2} \Big\{ \Pi_1 (m_+ - m_{\mathcal{O}}) + \Pi_2 \Big\} \,, \tag{13}$$

where Π_1 and Π_2 are the invariant functions of the Lorentz structures $\not{q}\gamma_5 q_\mu q_\nu$ and $\gamma_5 q_\mu q_\nu$, respectively, and

$$t_{1,2} = s_1 + m_{\mathcal{O}}^2 \mp 2 m_{\mathcal{O}} \sqrt{s_1 - m^2} \,.$$

III. NUMERICAL ANALYSIS

The present section is devoted to the numerical analysis of the coupling constants derived in the previous section within LCSR. The main nonperturbative input of the considered LCSR is the distribution amplitudes (DAs) of the octet baryons, namely N, Σ and Ξ. The explicit expressions of the relevant DAs as well as the values of the parameters (f, λ_1, and λ_2) determined from the analysis of mass sum rules [18–21] are presented in Table II for completeness. The masses of the $SU(3)$ partners of $\Omega(2012)$ are obtained in [16] and presented below.

	f (GeV^2)	λ_1 (GeV^2)	λ_2 (GeV^2)
N	$(5.3 \pm 0.5) \times 10^{-3}$	$-(2.7 \pm 0.9) \times 10^{-2}$	$(5.1 \pm 1.9) \times 10^{-2}$
Σ	$(9.4 \pm 0.4) \times 10^{-3}$	$-(2.5 \pm 0.1) \times 10^{-2}$	$(4.4 \pm 0.1) \times 10^{-2}$
Ξ	$(9.9 \pm 0.4) \times 10^{-3}$	$-(2.8 \pm 0.1) \times 10^{-2}$	$(5.2 \pm 0.2) \times 10^{-2}$

TABLE II: Numerical values of the coupling constants used in the calculations are presented for completeness (see [18–21] for more details).

$$m_- = \begin{cases} 1700 \pm 90 \text{ MeV} & \text{for } \Delta, \\ 1805 \pm 100 \text{ MeV} & \text{for } \Sigma, \\ 1910 \pm 110 \text{ MeV} & \text{for } \Xi. \end{cases}$$

These mass values are used in our numerical analysis. Moreover, for the masses of the ground state baryons, we adapted values from PDG [22]. In addition, the value of the quark condensate is taken as $\langle \bar{q}q \rangle = -(246^{+28}_{-19} \text{ MeV})^3$ [17].

The residues of the negative parity $J^P = \frac{3}{2}^-$ baryons are related with the residues of the radial excitations of the decuplet baryons as follows,

$$\lambda_- = \lambda_{rad} \sqrt{\frac{m_- - m_+}{m_- + m_+}} \,.$$

The residues of radial excitations of the decuplet baryons are calculated in [2]. Using these results one can easily determine the residues of the $J^P = \frac{3}{2}^-$ baryons.

The working regions of the Borel mass parameters used in the numerical analysis are presented in Table III. Determination of these regions is based on the criteria that both power corrections and continuum contributions should be suppressed. The continuum threshold s_0 is obtained from the condition that the mass of the considered states reproduce the experimental values about 10% accuracy..

	Borel mass parameters		Continuum threshold
	M_1^2 (GeV^2)	M_2^2 (GeV^2)	s_0 (GeV^2)
$\Delta \to N\pi$	$3 \div 4$	0.775 ± 0.025	5.1 ± 0.1
$\Sigma \to NK$	$3 \div 4$	0.750 ± 0.025	5.1 ± 0.1
$\Sigma \to \Lambda\pi$	$3 \div 4$	0.750 ± 0.025	5.1 ± 0.1
$\Sigma \to \Sigma\pi$	$3 \div 4$	0.750 ± 0.025	5.1 ± 0.1
$\Xi \to \Lambda K$	$3 \div 4$	0.750 ± 0.050	6.1 ± 0.1
$\Xi \to \Sigma K$	$3 \div 4$	0.750 ± 0.050	6.1 ± 0.1
$\Xi \to \Xi\pi$	$3 \div 4$	0.750 ± 0.025	6.1 ± 0.1

TABLE III: Working regions of the Borel mass parameters and continuum threshold s_0.

Having the values of all input parameters at hand, we can proceed to perform the numerical analysis of the relevant coupling constants. As an example, in Fig. 1, we present the dependency of the coupling constant on M_2^2 at the fixed values of the continuum threshold s_0 and M_1^2 for the $\Delta^+ \to N\pi^+$ transition. From this figure we observe that there exists good stability of the coupling constant when M^2 varies in its working region (see Table III). The obtained coupling constants are presented in Table IV. The errors in the results for the coupling constants can be attributed to the uncertainties in the input parameters as well as to the Borel mass parameters M_1^2, M_2^2, and continuum threshold s_0.

After the determination of coupling constants, we can calculate the corresponding decay channels. Using the matrix elements for the considered $\frac{3}{2}^- \to \frac{1}{2}^+ +$ pseudoscalar meson transitions, the decay width can be written as,

$$\Gamma = \frac{g_-^2}{24\pi m_-^2}\Big[(m_- - m_{\mathcal{O}})^2 - m_{\mathcal{P}}^2\Big]|\vec{p}|^3 ~, \tag{14}$$

where

$$|\vec{p}| = \frac{1}{2m_-}\sqrt{m_-^4 + m_{\mathcal{O}}^4 + m_{\mathcal{P}}^4 - 2m_-^2 m_{\mathcal{O}}^2 - 2m_-^2 m_{\mathcal{P}}^2 - 2m_{\mathcal{O}}^2 m_{\mathcal{P}}^2} \ ,$$

is the momentum of octet baryon, $m_{\mathcal{O}}$ and $m_{\mathcal{P}}$ are the mass of the octet baryon and pseudoscalar meson, respectively, Using the values of the coupling constants obtained within this work, we estimated the decay widths of the relevant transitions that are summarized in Table IV. For comparison we also present the results of the decay widths obtained within frame of the Flavor $SU(3)$ analysis [16]. We would like to make the following remark at this point. From the expression of the decay width, we see that it is quite sensitive to the mass splitting among the $SU(3)$ partners of the $\Omega(2012)$ and ground state baryons. Thus, to calculate the coupling constants and decay widths of the transitions under consideration, we used the same masses as in [16].

Decay channels	g_- (GeV^{-1})	Γ (MeV) (This work)	Γ (MeV) [16]
$\Delta \to N\pi$	-11.0 ± 1.0	$48.8 \times (1.0 \pm 0.2)$	39 - 58
$\Sigma \to NK$	-5.6 ± 0.7	$9.7 \times (1.0 \pm 0.3)$	7 - 12
$\Sigma \to \Lambda\pi$	-7.0 ± 0.8	$14.0 \times (1.0 \pm 0.3)$	11 - 18
$\Sigma \to \Sigma\pi$	5.5 ± 0.7	$5.6 \times (1.0 \pm 0.3)$	4 - 7
$\Xi \to \Lambda K$	-6.9 ± 1.4	$8.0 \times (1.0 \pm 0.3)$	5 - 10
$\Xi \to \Sigma K$	6.8 ± 1.5	$4.0 \times (1.0 \pm 0.4)$	2 - 5
$\Xi \to \Xi\pi$	7.0 ± 1.1	$7.4 \times (1.0 \pm 0.3)$	5 - 9

TABLE IV: Decay widths of the $J^P = \frac{3}{2}^-$ baryons.

As a final remark, we compare our results with the values obtained within the framework of the flavor $SU(3)$ method [16]. In this analysis, the coupling constant for $\Omega \to \Xi K$ is taken as the input parameter, and all the remaining couplings are expressed in terms of this coupling with the help of $SU(3)$ symmetry relations. Using the experimental value of the decay width $\Omega \to \Xi K$, one can determine the coupling constant of this transition with the help of Eq. (14), and hence all the other coupling constants can be determined. When we compare our results for the coupling constants and decay widths of the considered decays with those obtained within flavor $SU(3)$ analysis, we see they are compatible. Small deviations in the results can be attributed to the $SU(3)$ violation effects.

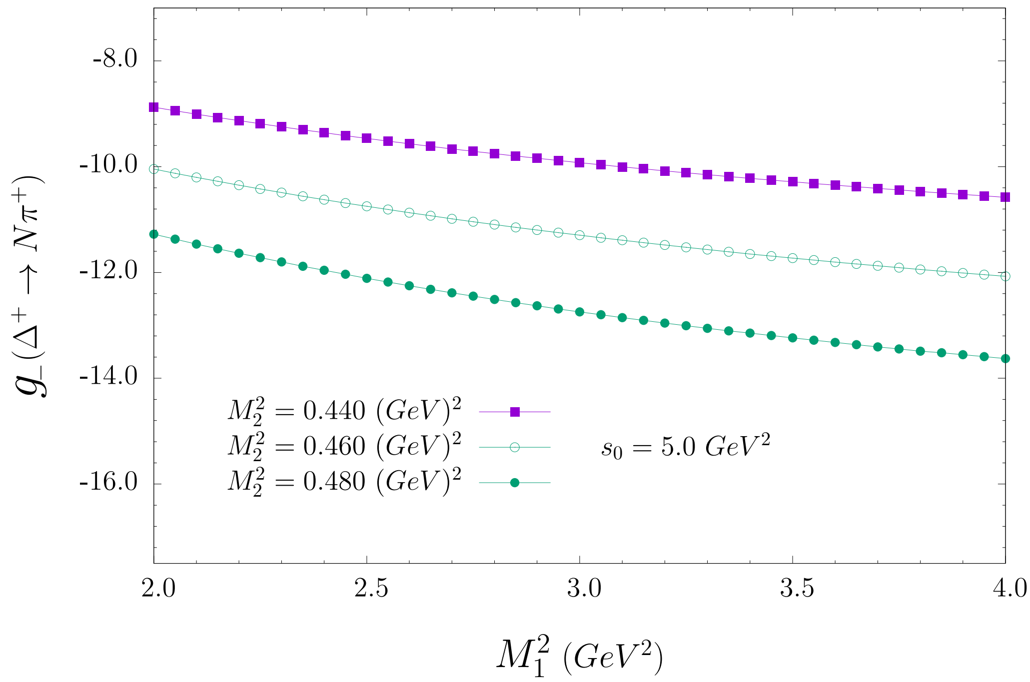

FIG. 1: The dependency of the coupling constant of the $\Delta^+ \to N\pi^+$ transition on the Borel mass parameter M_1^2, at several fixed values of the Borel parameter M_2^2, and the continuum threshold $s_0 = 5.0\ GeV^2$.

IV. CONCLUSION

In conclusion, we employed the LCSR method to compute the strong coupling constants and decay widths for the $SU(3)$ partners of the $\Omega(2012)$ baryon in $\frac{3}{2}^- \to \frac{1}{2}^+ +$ pseudoscalar meson transitions. The "contamination" caused by the $J^P = \frac{3}{2}^+$ baryons are eliminated by considering the linear combinations of the sum rules obtained from different Lorentz structures. By comparing our decay width results with the findings of [16], we ascertain the compatibility of our decay width predictions with the outcomes of the flavor $SU(3)$ symmetry analysis. Small discrepancy between the two methods' predictions may be attributed to the $SU(3)$ violation effects. Our results on the branching ratios can give useful hints about the nature of the $SU(3)$ partners of $\Omega(2012)$ baryon.

ACKNOWLEDGMENTS

We appreciate Y. M. Wang for discussion about numerical analysis. We also thank A. Ozpineci for useful remarks.

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