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Gravitational $p \to \Delta^+$ transition form factors in chiral perturbation theory

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Abstract: The gravitational form factors of the transition from the proton to the Δ^+ resonance are calculated to leading one-loop order using a manifestly Lorentz-invariant formulation of chiral perturbation theory. We take into account the leading electromagnetic and strong isospin-violating effects. The loop contributions to the transition form factors are found to be free of power-counting violating pieces, which is consistent with the absence of tree-level diagrams at the considered order. In this sense, our results can be regarded as predictions of chiral perturbation theory.

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Contents

1 Introduction

The linear response of the effective action to the change of the space-time metric specifies mechanical properties of particles. In particular, static characteristics, like the mass, spin and the D-term correspond to the hadron gravitational form factors (GFFs) at zero momentum transfer [1, 2]. In recent years, GFFs have attracted increasing attention for characterizing properties of hadrons with different spins due to their connection to generalized parton distributions (GPDs). Parameterizations of the energy-momentum tensor (EMT) matrix elements in terms of the GFFs have been considered for spin-0 [2], spin-1 [3–5], and for arbitrary-spin hadrons [6]. Mechanical properties, energy and spin densities as well as spatial distributions of the pressure and shear forces have been introduced for spin-0 and spin-1/2 hadrons in Ref. [7], and generalized to higher-spin systems in Refs. [5, 8, 9].

The nucleon GFFs can be extracted from experimental measurements of exclusive processes like deeply virtual Compton scattering (DVCS) [10, 11] and hard exclusive meson production [12]. The connection to GFFs can be seen in the QCD description of these processes, where the *symmetric* EMT appears naturally in the operator product expansion [10]. The first results of measurements of the D-term in hard QCD processes for the nucleon and the pion can be found in Refs. [13–16]. Recently, the mechanical radius of the proton has been determined from experimental data on DVCS cross sections and polarized electron beam spin asymmetries [17]. The GFFs have also been studied in lattice QCD, see, e.g., Refs. [18–23] and references therein.

While the electromagnetic $p \to \Delta^+$ transition has been extensively studied over the past two decades on both the theoretical and experimental sides, see, e.g., Refs. [24–28], the gravitational $p \to \Delta^+$ transition form factors (GTFFs) gained attention only since a few years [29]. The GTFFs can be accessed experimentally through their connection to the transition GPDs [30, 31], obtained by expanding the non-local QCD operators with various quantum numbers. Non-perturbative properties of the nucleon-Δ transition GPDs have been studied, e.g., by applying the approach of large N_c limit of QCD, as discussed in Sec. 2.7 of Ref. [32]. In Ref. [33], the transition GPDs have been connected with the DVCS amplitude within the process $e^- N \to e^- \gamma \pi N$, while in Ref. [34] these quantities have been studied using exclusive electroproduction of $\pi^- \Delta^{++}$.

In Ref. [29], the matrix element of the symmetric EMT corresponding to the $p \to \Delta^+$ transition has been studied for the first time, where a parametrization for the transitions $\frac{1}{2}^\pm \to \frac{3}{2}^\pm$ and $\frac{1}{2}^\pm \to \frac{3}{2}^\mp$ has been suggested in terms of five conserved and four non-conserved GTFFs. The first calculations of the GTFFs of the $N \to \Delta$ transition were done in Ref. [35] using the QCD light-cone sum rules. The interpretation and understanding of the GTFFs have generated much interest recently. In particular, the concept of QCD angular momentum (AM) [36–38] has been extended to $N \to \Delta$ transitions in Ref. [39]. These quantities were calculated in the $1/N_c$ expansion, and their connection to the transition GPDs of the hard exclusive electroproduction processes was discussed. Properties of the AM of various transitions were further explored in Ref. [40], where their decomposition into the orbital AM and the intrinsic spin components was studied.

For systematic studies of low-energy hadronic processes involving the Δ resonances and induced by gravity one may rely on the effective chiral Lagrangian for the nucleons, pions, photons and delta resonances in curved spacetime. Effective Lagrangian of pions in curved spacetime has been derived in Ref. [41], and the GFFs of the pion are considered in Ref. [42]. The leading and subleading effective chiral Lagrangians for nucleons, delta resonances and pions in curved spacetime, along with the calculation of the leading one-loop contributions to the GFFs of the nucleons and the Δ resonances can be found in Refs. [43, 44].

In this work we calculate the GTFFs of the $p \to \Delta^+$ transition in the framework of manifestly Lorentz-invariant chiral perturbation theory (ChPT) up-to-and-including the third order in the small-scale expansion [45]. As gravity conserves isospin, such kind of processes are possible only if the isospin symmetry is broken, i.e. if $m_u \neq m_d$ and/or if the electromagnetic interaction is taken into account. We include both effects at the corresponding leading orders to calculate the one-loop contributions to the GTFFs.

Our paper is organized as follows: In section 2, we specify the relevant terms of the effective Lagrangian of the nucleons, pions, photons and delta resonances in curved spacetime. We calculate the GTFFs of the $p \to \Delta^+$ transition in section 3. The results of our calculations are summarized in section 4. In the appendices, we list the isospin symmetry breaking terms in the action and the expression for the parts of the EMT, which are relevant for our study.

2 Effective Lagrangian in curved spacetime and the energy-momentum tensor

The action corresponding to the leading-order effective Lagrangian for nucleons, pions, photons and delta resonances, interacting with an external gravitational field, can be easily obtained from the corresponding expressions in flat spacetime [41, 46–49]. It has the following form:

$$S_{\gamma}^{(2)} = \int d^4x\sqrt{-g}\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{m_\gamma^2}{2}A_\mu A^\mu\right\}, \tag{2.1}$$

$$S_{\pi}^{(2)} = \int d^4x\sqrt{-g}\left\{\frac{F^2}{4}\,\mathrm{Tr}(D_\mu U(D^\mu U)^\dagger)+\frac{F^2}{4}\,\mathrm{Tr}(\chi U^\dagger+U\chi^\dagger)\right\}, \tag{2.2}$$

$$S_{\mathrm{N}\pi}^{(1)} = \int d^4x\sqrt{-g}\left\{\bar\Psi\, i\gamma^\mu\overset{\leftrightarrow}{\nabla}_\mu\Psi - m\bar\Psi\Psi+\frac{g_A}{2}\,\bar\Psi\gamma^\mu\gamma_5 u_\mu\Psi\right\}, \tag{2.3}$$

$$\begin{aligned}S_{\Delta\pi}^{(1)} = -\int d^4x\sqrt{-g}\Big\{&\bar\Psi^{i\mu}\, i\gamma^\alpha\overset{\leftrightarrow}{\nabla}_\alpha\Psi^i_\mu - m_\Delta\,\bar\Psi^i_\mu\Psi^{i\mu} - g^{\lambda\sigma}\left(\bar\Psi^i_\mu i\gamma^\mu\overset{\leftrightarrow}{\nabla}_\lambda\Psi^i_\sigma+\bar\Psi^i_\lambda i\gamma^\mu\overset{\leftrightarrow}{\nabla}_\sigma\Psi^i_\mu\right)\\ &+i\bar\Psi^i_\mu\gamma^\mu\gamma^\alpha\gamma^\nu\overset{\leftrightarrow}{\nabla}_\alpha\Psi^i_\nu+m_\Delta\bar\Psi^i_\mu\gamma^\mu\gamma^\nu\Psi^i_\nu+\frac{g_1}{2}\,g^{\mu\nu}\bar\Psi^i_\mu u_\alpha\gamma^\alpha\gamma_5\Psi^i_\nu\\ &+\frac{g_2}{2}\bar\Psi^i_\mu\left(u^\mu\gamma^\nu+u^\nu\gamma^\mu\right)\gamma_5\Psi^i_\nu+\frac{g_3}{2}\bar\Psi^i_\mu u_\alpha\gamma^\mu\gamma^\alpha\gamma_5\gamma^\nu\Psi^i_\nu\Big\},\end{aligned} \tag{2.4}$$

$$\begin{aligned}S_{\Delta\mathrm{N}\pi}^{(1,2)} = \int d^4x\sqrt{-g}\,\Big\{&-g_{\pi N\Delta}\bar\Psi\left(g^{\mu\nu}-\gamma^\mu\gamma^\nu\right)u_{\mu,i}\Psi_{\nu,i}\\ &+d_3^{(2)}i\bar\Psi f_+^{i\mu\nu}\gamma_5\gamma_\mu\left(g_{\nu\lambda}-\left[z_n+\frac{1}{2}\right]\gamma_\nu\gamma_\lambda\right)\Psi^{i\lambda}+\mathrm{H.c.}\Big\}.\end{aligned} \tag{2.5}$$

The Δ resonances are represented by the Rarita-Schwinger fields Ψ_i^μ, which contain the isospin-3/2 projectors $\xi_{ij}^{\frac{3}{2}} = \delta_{ij}-\tau_i\tau_j/3$, i.e. they satisfy the condition $\Psi_i^\mu = \xi_{ij}^{\frac{3}{2}}\Psi_j^\mu$. Further, $g^{\mu\nu}$ is the metric tensor field and $\gamma_\mu \equiv e_\mu^a\gamma_a$, where e_μ^a denote the vielbein gravitational fields. In the photon Lagrangian we included the mass term $m_\gamma^2\,A_\mu A^\mu/2$ to regularize infrared divergences, and the limit $m_\gamma \to 0$ should be performed at the end. However, as it turns out after the calculation, there are actually no IR divergences and this term is thus of no relevance here. In Eqs. (2.4) and (2.5), z_n is an off-shell parameter, which we choose equal to zero in our calculations, and we have set the point-transformation parameter $A = -1$ [50]. The building blocks of the effective Lagrangian are given as follows:

$$\begin{aligned}\overset{\leftrightarrow}{\nabla}_\mu &= \frac{1}{2}(\overset{\rightarrow}{\nabla}_\mu-\overset{\leftarrow}{\nabla}_\mu),\\ \overset{\rightarrow}{\nabla}_\mu\Psi^i_\nu &= \nabla^{ij}_\mu\Psi^j_\nu = \left[\delta^{ij}\partial_\mu+\delta^{ij}\Gamma_\mu-i\delta^{ij}v^{(s)}_\mu-i\epsilon^{ijk}\mathrm{Tr}\left(\tau^k\Gamma_\mu\right)+\frac{i}{2}\delta^{ij}\omega^{ab}_\mu\sigma_{ab}\right]\Psi^j_\nu-\Gamma^\alpha_{\mu\nu}\Psi^i_\alpha,\\ \bar\Psi^i_\nu\overset{\leftarrow}{\nabla}_\mu &= \nabla^{ij}_\mu\Psi^j_\nu = \bar\Psi^j_\nu\left[\delta^{ij}\partial_\mu-\delta^{ij}\Gamma_\mu+i\delta^{ij}v^{(s)}_\mu+i\epsilon^{ijk}\mathrm{Tr}\left(\tau^k\Gamma_\mu\right)-\frac{i}{2}\delta^{ij}\omega^{ab}_\mu\sigma_{ab}\right]-\bar\Psi^i_\alpha\Gamma^\alpha_{\mu\nu},\\ \overset{\rightarrow}{\nabla}_\mu\Psi &= \partial_\mu\Psi+\frac{i}{2}\,\omega^{ab}_\mu\sigma_{ab}\Psi+\left(\Gamma_\mu-iv^{(s)}_\mu\right)\Psi,\\ \bar\Psi\overset{\leftarrow}{\nabla}_\mu &= \partial_\mu\bar\Psi-\frac{i}{2}\,\bar\Psi\,\sigma_{ab}\,\omega^{ab}_\mu-\bar\Psi\left(\Gamma_\mu-iv^{(s)}_\mu\right),\end{aligned}$$

$$
\begin{aligned}
\omega_\mu^{ab} &= -\frac{1}{2}\, g^{\nu\lambda} e_\lambda^a \left(\partial_\mu e_\nu^b - e_\sigma^b \Gamma^\sigma_{\mu\nu}\right), \\
\Gamma^\lambda_{\alpha\beta} &= \frac{1}{2}\, g^{\lambda\sigma} \left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta}\right), \\
f_+^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger + u^\dagger F_R^{\mu\nu} u\,, \\
F_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu]\,, \\
F_L^{\mu\nu} &= \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu]\,, \\
f_+^{i\mu\nu} &= \frac{1}{2} \mathrm{Tr}\left(f_+^{\mu\nu} \tau^i\right), \\
\chi_+ &= u^\dagger \chi u^\dagger + u \chi^\dagger u, \\
\hat{\chi}_+ &= \chi_+ - \frac{1}{2}\langle \chi_+ \rangle, \\
\chi &= 2B_0(s + ip), \\
u_{\mu,i} &= \frac{1}{2} \mathrm{Tr}\left(u_\mu \tau^i\right).
\end{aligned} \tag{2.6}
$$

Notice that since the gravitational interaction respects the isospin symmetry, the amplitude of the $p \to \Delta^+$ transition receives non-vanishing contributions only via the isospin-symmetry breaking effects. In Appendix A, the above action is re-written in particle basis and the corresponding EMT is also specified.

3 Gravitational transition form factors to one loop

Below, we calculate the leading one-loop contributions to the matrix elements of the EMT for the one-particle states of the delta resonance and the nucleon. These matrix elements are extracted from the residues of Green's functions, which have complex poles corresponding to the unstable Δ states [51]. To organize different contributions according to a systematic expansion we employ the so-called ϵ-counting scheme (also referred to as the small scale expansion) [45] [1], i.e. the pion lines count as of chiral order Q^{-2}, where Q denotes the soft scale of the order of the pion mass. Further, the nucleon and delta lines count as Q^{-1}, interaction vertices originating from the effective Lagrangian of order N count also as of chiral order Q^N, while the vertices generated by the EMT, which are listed in Appendix B, have the orders corresponding to the number of the quark mass insertions and derivatives acting on the pion fields. Derivatives acting on the nucleon and delta fields count as of chiral order Q^0. The momentum transfer between the initial and final states counts as of chiral order Q, therefore in those terms of the EMT which involve full derivatives, these derivatives also count as chiral order Q. Integration over loop momenta is counted as chiral order Q^4. Furthermore, the delta-nucleon mass difference also counts as order Q within the ϵ-counting scheme. In diagrams involving electromagnetic radiative corrections, we assign the chiral order Q^{-2} to the photon line and count the electric charge e as chiral order Q. It is understood that the above described power counting for loop diagrams is realized in the results of manifestly Lorentz-invariant calculations only after performing an appropriate renormalization. We apply the EOMS scheme of Refs. [53, 54].

[1] For an alternative power counting in ChPT with delta resonances see Ref. [52].

The matrix element of the total EMT for the transition $p \to \Delta^+$ can be parameterized in terms of five form factors as follows [29]:

$$
\begin{aligned}
&\langle \Delta, p_f, s_f | T^{\mu\nu} | N, p_i, s_i \rangle \\
&= \bar{u}_\alpha(p_f, s_f) \Bigg\{ F_1(t) \left(g^{\alpha\{\mu} P^{\nu\}} + \frac{m_{\Delta^+}^2 - m_p^2}{\Delta^2} g^{\mu\nu} \Delta^\alpha - \frac{m_{\Delta^+}^2 - m_p^2}{2\Delta^2} g^{\alpha\{\mu} \Delta^{\nu\}} - \frac{1}{\Delta^2} P^{\{\mu} \Delta^{\nu\}} \Delta^\alpha \right) \\
&+ F_2(t) \left(P^\mu P^\nu \Delta^\alpha + \frac{(m_{\Delta^+}^2 - m_p^2)^2}{4\Delta^2} g^{\mu\nu} \Delta^\alpha - \frac{m_{\Delta^+}^2 - m_p^2}{2\Delta^2} P^{\{\mu} \Delta^{\nu\}} \Delta^\alpha \right) \\
&+ F_3(t) \left(\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu} \right) \Delta^\alpha \\
&+ F_4(t) \left(g^{\alpha\{\mu} \gamma^{\nu\}} + \frac{2(m_p + m_{\Delta^+})}{\Delta^2} g^{\mu\nu} \Delta^\alpha - \frac{m_p + m_{\Delta^+}}{\Delta^2} g^{\alpha\{\mu} \Delta^{\nu\}} - \frac{1}{\Delta^2} \gamma^{\{\mu} \Delta^{\nu\}} \Delta^\alpha \right) \\
&+ F_5(t) \Bigg(P^{\{\mu} \gamma^{\nu\}} \Delta^\alpha + \frac{(m_{\Delta^+}^2 - m_p^2)(m_p + m_{\Delta^+})}{\Delta^2} g^{\mu\nu} \Delta^\alpha - \frac{m_p + m_{\Delta^+}}{\Delta^2} P^{\{\mu} \Delta^{\nu\}} \Delta^\alpha \\
&- \frac{m_{\Delta^+}^2 - m_p^2}{2\Delta^2} \gamma^{\{\mu} \Delta^{\nu\}} \Delta^\alpha \Bigg) \Bigg\} \gamma^5 u(p_i, s_i) \,,
\end{aligned}
\tag{3.1}
$$

where m_p and m_{Δ^+} are the proton and the Δ^+ masses, respectively, $P = (p_f + p_i)/2$, $\Delta = p_f - p_i$ and $t = \Delta^2$. The curly brackets in the superscripts stand for symmetrization of the involved indices, e.g., $P^{\{\mu}\gamma^{\nu\}} = P^\mu \gamma^\nu + P^\nu \gamma^\mu$. As mentioned in the introduction, if the isospin symmetry is not broken, the above amplitude is zero.

3.1 One-loop contributions of the strong interaction to the gravitational transition form factors

To obtain the one-loop contributions to the GTFFs due to strong isospin-breaking interactions one has to compute 25 diagrams, where there are only 10 topologically differing diagrams and the rest can be obtained by just changing the masses and overall factors. These 10 diagrams are depicted in Fig. 1. The isospin symmetry breaking terms of the effective Lagrangian, which contribute to these one-loop diagrams, are specified in Appendix A. We performed the calculations in the particle basis. In the limit of the exact isospin symmetry, the contributions of the different diagrams exactly cancel each other. Taking into account the dominant isospin breaking effect, we find that obtained form factors are proportional to the mass differences within iso-multiplets of nucleons, pions and delta resonances. The leading contributions are given by terms proportional to the pion mass differences. This is because these contributions involve integrals, whose integrands are proportional to

$$
\sim \frac{1}{p^2 - M_{\pi^+}^2} - \frac{1}{p^2 - M_{\pi^0}^2} \simeq \frac{M_{\pi^+}^2 - M_{\pi^0}^2}{\left(p^2 - M_{\pi^+}^2\right)\left(p^2 - M_{\pi^0}^2\right)} \,, \tag{3.2}
$$

where each of the propagators originates from different diagrams that would cancel each other in the isospin limit. As the mass difference $M_{\pi^+}^2 - M_{\pi^0}^2$ counts as of chiral order two, the right-hand side of Eq. (3.2) has the same order as each of the terms in the left-hand side. That is, the total contribution of these diagrams, which is proportional to the pion mass difference squared, has the same order as the individual diagrams. On the other hand, the contributions proportional to the proton-neutron mass difference are given by integrals, whose integrands are proportional to

$$\sim \frac{1}{\not p - m_p} - \frac{1}{\not p - m_n} \simeq \frac{m_p - m_n}{(\not p - m_p)\,(\not p - m_n)}\,. \tag{3.3}$$

As the mass difference $m_p - m_n$ counts as chiral order two, and the nucleon propagators as order minus one, the right-hand side of Eq. (3.3) has one order higher than each of the terms on the left-hand side. That is, the total contribution of diagrams is suppressed by Q relative to the contributions of the individual diagrams. Analogous power counting holds also for the contributions proportional to the mass differences of the delta resonances. Notice further that isospin breaking vertices other than the mass terms start contributing at higher orders.

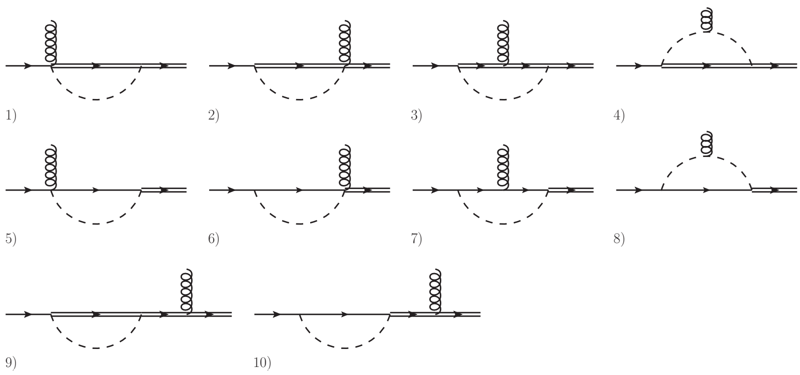

Figure 1. Strong contributions to the gravitational transition form factors. Solid and double lines correspond to nucleons and Δ resonances, respectively. Dashed lines represent the pions, while the curly lines correspond to gravitons. Initial and final states refer to p and Δ^+, respectively, while the baryon lines inside loops refer to propagators of one of the following particles: $\{\Delta^{++}, \Delta^{+}, \Delta^{0}, p, n\}$. Notice that the total contribution of these diagrams vanishes in the limit of exact isospin symmetry.

Diagrams $1, 2, 4, 5, 6$ and 8 in Fig. 1 start contributing at chiral order three while the diagrams $3, 7, 9$ and 10 start contributing at chiral order two. This is because the leading-order contribution to the gravitational-source-baryon-baryon vertex has order zero.[2] Thus the diagrams in Fig. 1 give contributions of orders two and three. We have verified that the one-loop order result of diagrams in Fig. 1 does not contain power counting violating contributions and all ultraviolet divergences can be absorbed into redefinition of the low-energy coupling constants of the effective Lagrangian. The obtained results for the form

[2]Actually the gravitational-source-baryon-baryon vertex originating from the leading-order Lagrangian has two contributions, one of the order zero and the other of the order one. This means that diagrams $3, 7, 9$ and 10 contribute to two different chiral orders (2 and 3). These two contributions cannot be considered separately, because otherwise the current will not be conserved. This needs to be carefully taken into account when specifying the (possible) power-counting violating terms.

factors are too involved to be given as analytic expressions but are available from the authors upon request in the form of a *Mathematica* notebook. The same applies also to the results of the radiative corrections considered in the next subsection.

3.2 One-loop radiative corrections to the gravitational transition form factors

To obtain the one-loop electromagnetic corrections to the transition form factors one has to compute the diagrams contributing up to order four, shown in Fig. 2. The chiral power counting for Fig. 2 is similar to that for Fig. 1. In this calculation we do not distinguish between the masses of the Δ states and between the masses of the proton and the neutron, i.e. we set $m_{\Delta^{++}} = m_{\Delta^+} = m_{\Delta^0} = m_{\Delta^-}$ and $m_p = m_n$.

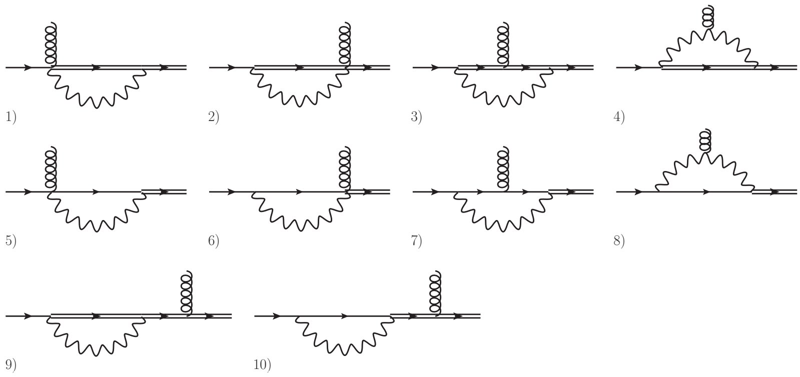

Figure 2. Electromagnetic contributions to the transition form factors. Solid and double solid lines correspond to nucleons and Δ resonances, respectively. Wavy lines denote photons, while curly lines represent gravitons.

Analogously to the strong-interaction contributions, we found that the one-loop order result of diagrams shown in Fig. 2 does not involve power-counting violating terms, and all ultraviolet divergences can be absorbed into redefinition of the low-energy coupling constants of the most general effective Lagrangian.

3.3 Numerical results for the gravitational transition form factors

In Figs. 3 and 4, we present the numerical results of the obtained strong and electromagnetic contributions to the real and imaginary parts of the transition form factors, respectively. Notice here that the imaginary parts of the calculated form factors are generated solely by the loop contributions with internal nucleon lines. For the numerical results, we used the following values of the involved parameters:

$$\begin{aligned} g_A &= 1.289, \quad g = 1.35, \quad m_{\pi^0} = 0.135, \quad m_p = 0.938, \quad m_n = 0.940, \\ m_\Delta &= 1.232, \quad F = 0.092, \quad m_{\pi^+} = 0.140, \quad m_{\Delta^{++}} = 1.231, \quad m_{\Delta^+} = m_\Delta, \\ m_{\Delta^0} &= 1.233, \quad g_1 = 9g_A/5, \quad e = 0.303\,, \quad d_3^{(2)} = 2.72\,\text{GeV}^{-1}\,, \end{aligned} \tag{3.4}$$

where the various masses and the pion decay constant F are given in GeV. We used the $SU(6)$ symmetry estimation for the coupling constants g_1, g_A and g taken from Ref. [55], for the masses of delta resonances we used estimations of Refs. [56, 57], $d_3^{(2)}$ corresponds to $b_1/2$ of Ref. [58], while the remaining values have been taken from the PDG [59].

The plots demonstrate that the diagrams with radiative corrections give smaller contributions than the ones with pion loops in line with the power counting estimations. On the other hand the groups of diagrams with internal nucleon and delta lines give comparable contributions.

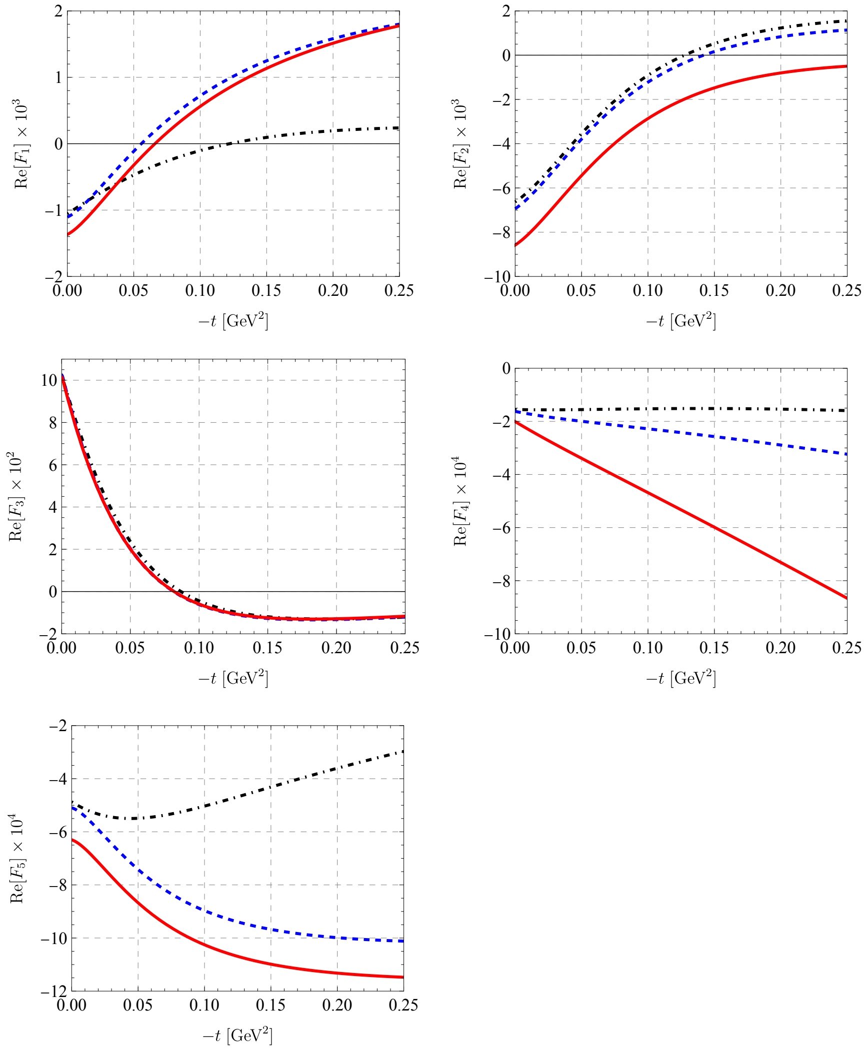

Figure 3. The real parts of the $p \to \Delta^+$ transition form factors. Dash-dotted (black), dashed (blue) and solid (red) lines correspond to the form factors containing contributions of loop diagrams with inner pion and nucleon lines only, diagrams with inner pion and nucleon lines plus radiative corrections, and all loop contributions, respectively.

4 Conclusions and outlook

In the framework of manifestly Lorentz-invariant ChPT for pions, nucleons, photons and the delta resonances interacting with an external gravitational field, we calculated the

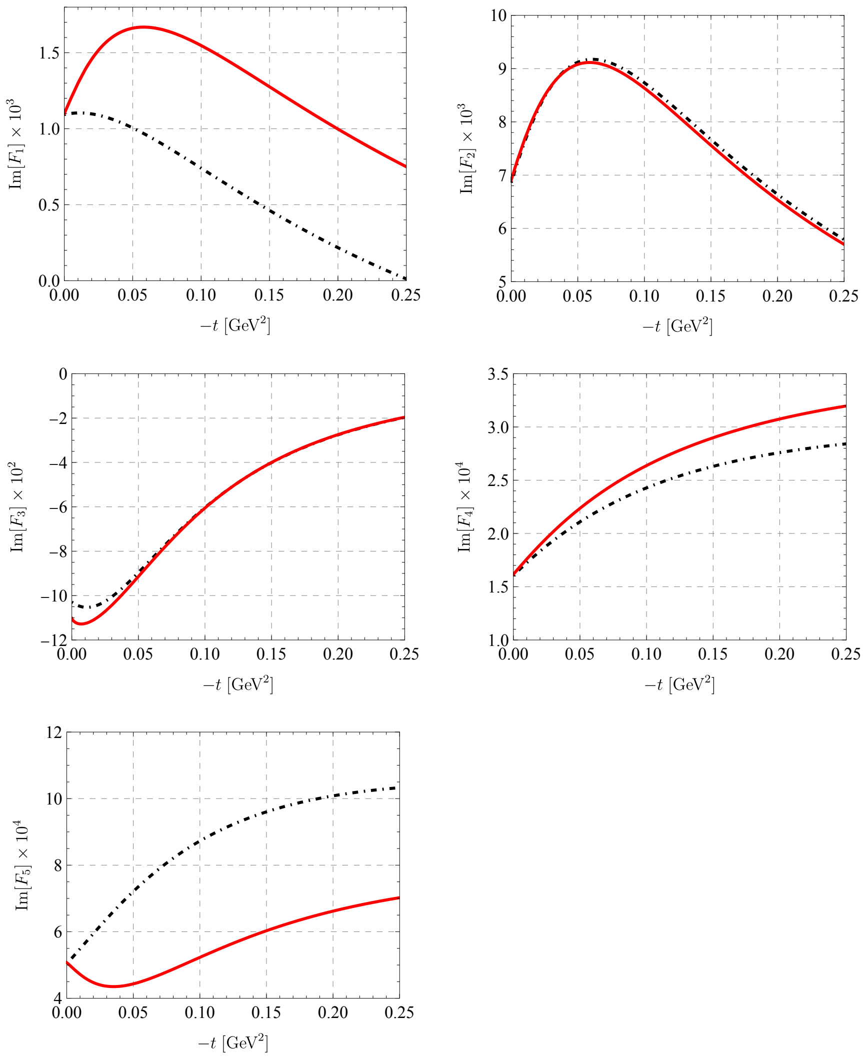

Figure 4. Imaginary parts of the $p \to \Delta^+$ transition form factors. Dash-dotted (black), and solid (red) lines correspond to the form factors containing contributions of loop diagrams with inner pion and nucleon lines only, and diagrams with inner pion and nucleon lines plus radiative corrections, respectively.

leading one-loop contributions to the matrix element of the EMT corresponding to the $p \to \Delta^+$ transition and extracted the resulting gravitational transition form factors. As the gravitational interaction respects the isospin symmetry, the amplitude of the $p \to \Delta^+$ transition receives non-vanishing contributions due to isospin symmetry breaking. The results of the current work take into account the leading-order electromagnetic and strong isospin-breaking effects. Ultraviolet divergences and power counting violating pieces generated by loop diagrams in the manifestly Lorentz-invariant formulation of ChPT can be treated using the EOMS renormalization scheme of Refs. [53, 54]. However, at the order of our calculations, the one-loop contributions to the form factors are found to be free of contributions that violate the chiral power counting. This is consistent with the absence of tree-level contributions at the considered order. For this reason, our results involve no free parameters and can be regarded as predictions of ChPT. Notice, however, that the empirical information on the mass splittings between the Δ resonance states, which enters as an input in our calculations, is presently rather poor. Numerical results for the obtained transition form factors demonstrate that the electromagnetic and strong isospin violating effects give contributions of comparable sizes. This holds true for contributions with both internal nucleon and delta lines.

Acknowledgments

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A Isospin-symmetry breaking terms

To obtain the leading isospin breaking effects due to the strong interaction we distinguish between the masses of the delta resonances, and also between the masses of the proton and the neutron, and the charged and neutral pions. We rewrite the action using the physical basis, instead of the isospin basis, by writing the fields explicitly as follows:

$$
\begin{aligned}
\Psi &= \begin{pmatrix} \Psi_p \\ \Psi_n \end{pmatrix}, \\
\Psi_{\mu,1} &= \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{3}}\Delta^0_\mu - \Delta^{++}_\mu \\ \Delta^-_\mu - \frac{1}{\sqrt{3}}\Delta^+_\mu \end{pmatrix}, \; \Psi_{\mu,2} = -\frac{i}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{3}}\Delta^0_\mu + \Delta^{++}_\mu \\ \Delta^-_\mu + \frac{1}{\sqrt{3}}\Delta^+_\mu \end{pmatrix}, \; \Psi_{\mu,3} = \sqrt{\frac{2}{3}} \begin{pmatrix} \Delta^+_\mu \\ \Delta^0_\mu \end{pmatrix}, \\
\pi^1 &= \frac{1}{\sqrt{2}}\left(\pi^+ + \pi^-\right), \; \pi^2 = \frac{i}{\sqrt{2}}\left(\pi^+ - \pi^-\right), \; \pi^3 = \pi^0.
\end{aligned} \tag{A.1}
$$

We substitute the above definition of the fields into Eqs. (2.2), (2.3), (2.4) and (2.5). The terms relevant for the leading one-loop order contributions to the $p \to \Delta^+$ transition are given by

$$
S^{(2)}_\pi = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\partial_\mu\pi^0\partial^\mu\pi^0 - \frac{1}{2}M_0^2\pi^0\pi^0 + \partial_\mu\pi^+\partial^\mu\pi^- - M^2_{\pi^+}\pi^+\pi^-\right\}, \tag{A.2}
$$

$$
S^{(1)}_{\mathrm{N}\pi} = \int d^4x\sqrt{-g}\left\{\bar\Psi_p\, i\gamma^\mu \overset{\leftrightarrow}{\nabla}_\mu\Psi_p - m_p\bar\Psi_p\Psi_p + \bar\Psi_n\, i\gamma^\mu\overset{\leftrightarrow}{\nabla}_\mu\Psi_n - m_n\bar\Psi_n\Psi_n\right.
$$

$$+ \frac{g_A}{2F}\left(\partial_\mu\pi^0\left[\bar\Psi_n\gamma^\mu\gamma^5\Psi_n - \bar\Psi_p\gamma^\mu\gamma^5\Psi_p\right] - \sqrt{2}\left[\partial_\mu\pi^-\bar\Psi_n\gamma^\mu\gamma^5\Psi_p + \partial_\mu\pi^+\bar\Psi_p\gamma^\mu\gamma^5\Psi_n\right]\right)\Big\}\,, \tag{A.3}$$

$$\begin{aligned}
S^{(1)}_{\Delta\pi} = &-\int d^4x\sqrt{-g}\Big\{\Big[\sum_{i\in\{++,+,0,-\}}\bar\Delta^{i\mu}\, i\gamma^\alpha\overset{\leftrightarrow}{\nabla}_\alpha\Delta^i_\mu - m_{\Delta^i}\,\bar\Delta^i_\mu\Delta^{i\mu}\\
&-g^{\lambda\sigma}\left(\bar\Delta^i_\mu i\gamma^\mu\overset{\leftrightarrow}{\nabla}_\lambda\Delta^i_\sigma + \bar\Delta^i_\lambda i\gamma^\mu\overset{\leftrightarrow}{\nabla}_\sigma\Delta^i_\mu\right) + i\bar\Delta^i_\mu\gamma^\mu\gamma^\alpha\gamma^\nu\overset{\leftrightarrow}{\nabla}_\alpha\Delta^i_\nu + m_{\Delta^i}\bar\Delta^i_\mu\gamma^\mu\gamma^\nu\Delta^i_\nu\Big]\\
&+\frac{1}{6F}\Big[\bar\Delta^0_\mu O_1^{\mu\nu\alpha}\Delta^0_\nu\partial_\alpha\pi^0 - \sqrt{6}\bar\Delta^-_\mu O_1^{\mu\nu\alpha}\Delta^0_\nu\partial_\alpha\pi^- - 2\sqrt{2}\bar\Delta^+_\mu O_1^{\mu\nu\alpha}\Delta^0_\nu\partial_\alpha\pi^+\\
&-\sqrt{6}\bar\Delta^0_\mu O_1^{\mu\nu\alpha}\Delta^-_\nu\partial_\alpha\pi^+ - 2\sqrt{2}\bar\Delta^0_\mu O_1^{\mu\nu\alpha}\Delta^+_\nu\partial_\alpha\pi^- + 3\bar\Delta^-_\mu O_1^{\mu\nu\alpha}\Delta^-_\nu\partial_\alpha\pi^0\\
&-\bar\Delta^+_\mu O_1^{\mu\nu\alpha}\Delta^+_\nu\partial_\alpha\pi^0 - \sqrt{6}\bar\Delta^+_\mu O_1^{\mu\nu\alpha}\Delta^{++}_\nu\partial_\alpha\pi^- - \sqrt{6}\bar\Delta^{++}_\mu O_1^{\mu\nu\alpha}\Delta^+_\nu\partial_\alpha\pi^+\\
&-3\bar\Delta^{++}_\mu O_1^{\mu\nu\alpha}\Delta^{++}_\nu\partial_\alpha\pi^0\Big]\Big\}\,,
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
S^{(1)}_{\Delta N\pi} = &\int d^4x\sqrt{-g}\,\frac{g_{\pi n\Delta}}{F}\Big\{\bar\Psi_n\partial_\mu\pi^+O_2^{\mu\nu}\Delta^-_\nu - \bar\Psi_p\partial_\mu\pi^-O_2^{\mu\nu}\Delta^{++}_\nu + \frac{1}{\sqrt{3}}\Big(\sqrt{2}\,\bar\Psi_n\partial_\mu\pi^0O_2^{\mu\nu}\Delta^0_\nu\\
&-\bar\Psi_n\partial_\mu\pi^-O_2^{\mu\nu}\Delta^+_\nu + \bar\Psi_p\partial_\mu\pi^+O_2^{\mu\nu}\Delta^0_\nu + \sqrt{2}\,\bar\Psi_p\partial_\mu\pi^0O_2^{\mu\nu}\Delta^+_\nu\Big)\Big\}\,,
\end{aligned} \tag{A.5}$$

where $O_1^{\mu\nu\alpha} = g_1\gamma^\alpha\gamma^5g^{\mu\nu} + g_2(g^{\mu\alpha}\gamma^\nu\gamma^5 + g^{\nu\alpha}\gamma^\mu\gamma^5) + g_3\gamma^\mu\gamma^\alpha\gamma^5\gamma^\nu$ and $O_2^{\mu\nu} = g^{\mu\nu} - \gamma^\mu\gamma^\nu$. To arrive at these results, we expanded the matrix u of pion fields and kept only the first nontrivial term, i.e. $u = 1 + i/(2F)\tau^i\pi^i + \mathcal{O}(1/F^2)$.

The mass splittings within iso-multiplets are not just due to strong isospin breaking but also receive important contributions from the electromagnetic interaction. However, there is no point at separating these contributions here, and such a separation is anyway afflicted with some uncertainties, see e.g. the pedagogical discussion in Ref. [60].

To obtain the leading isospin breaking effects due to the diagrams with radiative corrections (i.e. with photon propagators) we do not distinguish between the masses of the isospin partners, i.e. we take $m_{\Delta^{++}} = m_{\Delta^+} = m_{\Delta^0} = m_{\Delta^-}$, and $m_p = m_n$. For the external sources, we take the following expressions:

$$r_\mu = l_\mu = -e\,A_\mu\frac{\tau_3}{2}, \tag{A.6}$$

$$v^s_\mu = -\frac{e}{2}\,A_\mu, \tag{A.7}$$

where e is the electric charge of the proton.

B The energy-momentum tensor

Using the definition of the EMT for bosonic matter fields interacting with the gravitational metric field,

$$T_{\mu\nu}(g,\psi) = \frac{2}{\sqrt{-g}}\frac{\delta S_{\rm m}}{\delta g^{\mu\nu}}\,, \tag{B.1}$$

we obtain in flat spacetime from the action terms of Eqs. (2.1) and (2.2):

$$T^{(2)}_{\gamma,\mu\nu} = F^{\alpha}_{\mu} F_{\alpha\nu} + m_{\gamma}^2 A_\mu A_\nu + \eta_{\mu\nu}\left(\frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - \frac{m_\gamma^2}{2}A_\alpha A^\alpha\right), \tag{B.2}$$

$$T^{(2)}_{\pi,\mu\nu} = \frac{F^2}{4}\,\mathrm{Tr}(D_\mu U (D_\nu U)^\dagger) - \frac{\eta_{\mu\nu}}{2}\left\{\frac{F^2}{4}\,\mathrm{Tr}(D^\alpha U(D_\alpha U)^\dagger) + \frac{F^2}{4}\,\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\right\} + (\mu\leftrightarrow\nu)\,, \tag{B.3}$$

where $\eta_{\mu\nu}$ is the Minkowski metric tensor with the signature $(+,-,-,-)$. For the fermionic fields interacting with the gravitational vielbein fields we use the definition [61]

$$T_{\mu\nu}(g,\psi) = \frac{1}{2e}\left[\frac{\delta S}{\delta e^{a\mu}}\, e^a_\nu + \frac{\delta S}{\delta e^{a\nu}}\, e^a_\mu\right]. \tag{B.4}$$

The action of Eq. (2.3) leads to the following expression for the EMT in flat spacetime:

$$T^{(1)}_{\mathrm{N},\mu\nu} = \frac{i}{2}\bar\Psi\,\gamma_\mu \overset{\leftrightarrow}{D}_\nu\Psi - \frac{\eta_{\mu\nu}}{2}\left(\bar\Psi\, i\gamma^\alpha\overset{\leftrightarrow}{D}_\alpha\Psi - m\bar\Psi\Psi\right) + (\mu\leftrightarrow\nu)\,, \tag{B.5}$$

while Eqs. (2.4) and (2.5) lead to the following expressions:

$$\begin{aligned}
T^{(1)}_{\Delta\pi,\mu\nu} = &-\bar\Psi^i_\mu\, i\gamma^\alpha\overset{\leftrightarrow}{D}_\alpha\Psi^i_\nu + \bar\Psi^i_\alpha\, i\gamma^\alpha\overset{\leftrightarrow}{D}_\mu\Psi^i_\nu + \bar\Psi^i_\mu\, i\gamma^\alpha\overset{\leftrightarrow}{D}_\nu\Psi^i_\alpha + m_\Delta\bar\Psi^i_\mu\Psi^i_\nu - \frac{i}{2}\,\bar\Psi^i_\alpha\,\gamma_\mu\overset{\leftrightarrow}{D}_\nu\Psi^{i\alpha}\\
&+\frac{i}{2}\Big(\bar\Psi^i_\mu\,\gamma_\nu\overset{\leftrightarrow}{D}_\alpha\Psi^{i\alpha} + \bar\Psi^{i\alpha}\,\gamma_\nu\overset{\leftrightarrow}{D}_\alpha\Psi^i_\mu - \bar\Psi^i_\mu\,\gamma_\nu\gamma^\alpha\gamma_\beta\overset{\leftrightarrow}{D}_\alpha\Psi^{i,\beta} - \bar\Psi^i_\alpha\gamma^\alpha\gamma_\nu\gamma^\beta\overset{\leftrightarrow}{D}_\mu\Psi^i_\beta\\
&-\bar\Psi^i_\alpha\gamma^\alpha\gamma^\beta\gamma_\nu\overset{\leftrightarrow}{D}_\beta\Psi^i_\mu\Big) + \frac{i}{4}\,\partial^\lambda\Big[\bar\Psi^{i,\alpha}\Big(\gamma_\mu\eta_{\lambda[\alpha}\eta_{\beta]\mu} + \eta_{\lambda\mu}\eta_{\nu[\alpha}\gamma_{\beta]} + \eta_{\mu\nu}\eta_{\lambda[\beta}\gamma_{\alpha]}\Big)\Psi^{i,\beta}\Big]\\
&-\frac{m_\Delta}{2}\,\left(\bar\Psi^i_\mu\,\gamma_\nu\gamma^\alpha\Psi^i_\alpha + \bar\Psi^i_\alpha\,\gamma^\alpha\gamma_\nu\Psi^i_\mu\right) - \frac{g_1}{4}\,\left[2\bar\Psi^i_\mu u_\alpha\gamma^\alpha\gamma_5\Psi^i_\nu + \bar\Psi^{i,\alpha}u_\mu\gamma_\nu\gamma_5\Psi^i_\alpha\right]\\
&-\frac{g_2}{4}\left[2\bar\Psi^i_\mu u_\nu\gamma^\alpha\gamma_5\Psi^i_\alpha + 2\bar\Psi^i_\alpha u_\nu\gamma^\alpha\gamma_5\Psi^i_\mu + \bar\Psi^{i,\alpha}u_\alpha\gamma_\nu\gamma_5\Psi^i_\mu + \bar\Psi^i_\mu u_\alpha\gamma_\nu\gamma_5\Psi^{i\alpha}\right]\\
&-\frac{g_3}{4}\left[\bar\Psi^i_\mu u_\alpha\gamma_\nu\gamma^\alpha\gamma_5\gamma^\beta\Psi^i_\beta + \bar\Psi^i_\beta u_\alpha\gamma^\beta\gamma^\alpha\gamma_5\gamma_\nu\Psi^i_\mu + \bar\Psi^i_\alpha u_\mu\gamma^\alpha\gamma_\nu\gamma_5\gamma^\beta\Psi^i_\beta\right]\\
&+\frac{\eta_{\mu\nu}}{2}\Big[\bar\Psi^i_\alpha\, i\gamma^\beta\overset{\leftrightarrow}{D}_\beta\Psi^{i\alpha} - m_\Delta\,\bar\Psi^i_\alpha\Psi^{i\alpha} - \bar\Psi^i_\alpha i\gamma^\alpha\overset{\leftrightarrow}{D}_\beta\Psi^{i\beta} - \bar\Psi^{i\alpha} i\gamma^\beta\overset{\leftrightarrow}{D}_\alpha\Psi^i_\beta\\
&+i\bar\Psi^i_\rho\gamma^\rho\gamma^\alpha\gamma^\lambda\overset{\leftrightarrow}{D}_\alpha\Psi^i_\lambda + m_\Delta\bar\Psi^i_\alpha\gamma^\alpha\gamma^\beta\Psi^i_\beta + \frac{g_1}{2}\,\bar\Psi^i_\beta u_\alpha\gamma^\alpha\gamma_5\Psi^{i\beta}\\
&+\frac{g_2}{2}\bar\Psi^{i\alpha}\left(u_\alpha\gamma_\beta + u_\beta\gamma_\alpha\right)\gamma_5\Psi^{i\beta} + \frac{g_3}{2}\bar\Psi^i_\alpha u_\beta\gamma^\alpha\gamma^\beta\gamma_5\gamma^\lambda\Psi^i_\lambda\Big] + (\mu\leftrightarrow\nu)\,,
\end{aligned} \tag{B.6}$$

$$\begin{aligned}
T^{(1,2)}_{\pi N\Delta,\mu\nu} = g_{\pi N\Delta}\Big\{&\frac{1}{2}\,\eta_{\mu\nu}\left[\bar\Psi^i_\alpha u^\alpha_i\Psi + \bar\Psi u^\alpha_i\Psi^i_\alpha - \bar\Psi^i_\alpha\gamma^\alpha\gamma^\beta u^i_\beta\Psi - \bar\Psi\gamma^\beta\gamma^\alpha u^i_\beta\Psi^i_\alpha\right] - \bar\Psi^i_\mu u^i_\nu\Psi\\
&-\bar\Psi u^i_\nu\Psi^i_\mu + \frac{1}{2}\,\left[\bar\Psi^i_\mu\gamma_\nu\gamma^\alpha u^i_\alpha\Psi + \bar\Psi^i_\alpha\gamma^\alpha\gamma_\mu u^i_\nu\Psi + \bar\Psi\gamma^\alpha\gamma_\nu u^i_\alpha\Psi^i_\mu + \bar\Psi\gamma_\mu\gamma^\alpha u^i_\nu\Psi^i_\alpha\right]\Big\}\\
&+\frac{i}{2}d_3^{(2)}\Big\{\bar\Psi f^i_{+,\mu\beta}\gamma_5\gamma_\nu\tilde\Psi^{i\beta} + 2\bar\Psi f^i_{+,\alpha\mu}\gamma_5\gamma^\alpha\Psi^i_\nu - \eta_{\mu\nu}\bar\Psi f^i_{+,\alpha\beta}\gamma_5\gamma^\alpha\tilde\Psi^{i\beta}\\
&-\left[z_n + \frac{1}{2}\right]\left(\bar\Psi f^i_{+,\alpha\mu}\gamma_5\gamma^\alpha\gamma_\nu\gamma^\beta\Psi^i_\beta + \bar\Psi f^i_{+,\alpha\beta}\gamma_5\gamma^\alpha\gamma^\beta\gamma_\mu\Psi^i_\nu\right)\Big\} + (\mu\leftrightarrow\nu)\,,
\end{aligned} \tag{B.7}$$

where the covariant derivatives D acting on spin-1/2 and spin-3/2 fields coincide with ∇ of Eq. (2.6) with $\Gamma^{\beta}_{\mu\nu} = \omega^{ab}_{\mu} = 0$. The superscripts in the expressions of EMT indicate the orders which are assigned to the corresponding terms of the action (effective Lagrangian).

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