

Local projective measurement-enhanced quantum battery capacity

Tinggui Zhang^{1,†}, Hong Yang² and Shao-Ming Fei³

¹ *School of Mathematics and Statistics,
Hainan Normal University, Haikou, 571158, China*

² *School of Physics and Electronic Engineering,
Hainan Normal University, Haikou, 571158, China*

³ *School of Mathematical Sciences,
Capital Normal University, Beijing 100048, China*

[†] *Correspondence to tinggui333@163.com*

Quantum battery is of significant potential applications in future industry and daily life. The battery capacity is an important indicator of a battery. How to improve the capacity of quantum batteries is of importance. We consider quantum batteries given by bipartite quantum systems, and study the enhancement of the battery capacity under local projective measurements on a subsystem of the quantum state. By using the two-qubit Bell diagonal states and the X-type states as examples, we show that the quantum battery capacity with respect to the whole system or the subsystem could be improved by local projective measurements. Our theoretical analysis will provide new ideas for the experimental development of quantum battery.

Keywords: quantum measurement; quantum battery capacity; quantum entanglement

PACS numbers: 04.70.Dy, 03.65.Ud, 04.62.+v

INTRODUCTION

As an energy storage system, the battery plays significant roles in both industry and daily life. With the development of quantum technology and information science, the quantum battery has emerged and been expected to balance the advantages of small size, large capacity, portability, rapid charging etc. R. Alicki and M. Fannes first formally introduced the idea of quantum batteries in an informational theoretic context by characterizing the maximum amount of energy that can be extracted from a quantum system under unitary operations [1]. Since then there have been extensive theoretical [2–53] and experimental[54–58] studies on quantum batteries. For example, many theoretical protocols have been proposed, such as charging by entangling operations[6, 7, 9, 14, 29, 36], charging with dissipative[29, 30], charging collectively and in parallel[10, 30]. Many-body interaction and energy fluctuation were also explored[11, 13, 25–27, 37, 38, 48]. The first model of a quantum battery that could be engineered in a solid-state architecture, was proposed by Ferraro et al.[10]. Many other concrete quantum battery models have been also proposed[11, 19, 52]. However, many aspects of the physics of quantum batteries remain unexplored, experimental work on quantum batteries is still in its infancy and a fully-operational proof of principle is yet to be demonstrated. For a review of quantum batteries, we refer to Ref.[50].

An important quantitative indicator of the quality of quantum batteries is the capacity (charging power, work storage, work extraction) [31, 42]. The commonly used

quantity is the ergotropy functional [31],

$$E(\rho, H) = \max_{U \in \mathbb{U}(d)} \{Tr[\rho H] - Tr[U\rho U^\dagger H]\},$$

where ρ is the quantum state of a d -dimensional quantum system Q with Hamiltonian H , the maximum takes over the states under all unitary evolution $\rho \rightarrow U\rho U^\dagger$, with $\mathbb{U}(d)$ the set of the unitary operators acting on Q . More recently, in Ref. [42] the authors provided a new definition of quantum battery capacity,

$$C(\rho, H) := \sum_{i=0}^{d-1} \epsilon_i (\lambda_i - \lambda_{d-1-i}), \quad (1)$$

where $\lambda_0 \leq \lambda_1 \leq \dots \leq \lambda_{d-1}$ denote the eigenvalues of the quantum state ρ and $\epsilon_0 \leq \epsilon_1 \leq \dots \leq \epsilon_{d-1}$ denote the eigenenergies of the Hamiltonian $H = \sum_i \epsilon_i |\epsilon_i\rangle\langle\epsilon_i|$. $C(\rho, H)$ is a Schur-convex functional of the quantum state and does not change when the battery is unitarily charged or discharged.

Quantum measurements, particularly the projections onto a chosen state, could change the transition rate of the measured system [59]. Numerous measurement-based control schemes have been applied to state purification [60, 61], information gain [62] and entropy production [63]. The measurement on auxiliary systems may help to control the target systems, serving as state-engineering scheme through a nonunitary procedure [64, 65]. In deed, the idea of improving the desired effect of the target system by measuring the auxiliary system has recently been applied to quantum battery charging [41] and power extraction [49]. In [41], the authors established for quantum battery a charging by-measurement framework based on rounds of joint evolution and partial-projection. Starting from a thermal state, the battery could also achieve a

near-unit ratio of ergotropy and energy through less than N measurements, when a population inversion is realized by measurements. In [49] the authors focused on the energy extraction instead of charging, that is, the protocol includes only one measurement on the auxiliary qubit, but not a sequence of measurements. Scenarios are investigated for both that the initial state of the battery and the auxiliary is a product and an entangled one. It is shown that the measurement-based protocol provides significantly better energy extraction from a quantum battery than that based on unitary operations.

Consider a bipartite system composed of the target quantum state A and the auxiliary system B . Our question is how the quantum measurements on the systems B would affect the quantum battery capacity of the system A or the whole bipartite system. In this paper, we formulate a unilateral measurement protocol and study such local projective measurement-enhanced quantum battery capacity by starting directly from general bipartite quantum states. In general, a complete quantum battery consists of a battery part and a charger part. Here, we only consider the battery part, that is, the whole system battery is composed by two-particle quantum state ρ_{AB} and the system Hamiltonian H_{AB} . And the battery of the subsystem is composed by the reduced state ρ_A and the Hamiltonian H_A corresponding to the subsystem A. We use the definition of quantum battery capacity given by Eq.(1). We find that for two-qubit Werner states, the local measurements can enhance the overall or the subsystem battery capacity.

MEASUREMENT-BASED PROTOCOL

Let ρ_{AB} be an $m \otimes n$ bipartite state in Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$. Let $H_A = \sum_{i=0}^{m-1} \epsilon_i^A |\epsilon_i\rangle^A \langle \epsilon_i|$ and $H_B = \sum_{i=0}^{n-1} \epsilon_i^B |\epsilon_i\rangle^B \langle \epsilon_i|$ be the Hamiltonian associated with the systems A and B, respectively. The Hamiltonian of the joint system is $H_{AB} = H_A \otimes I_n + I_m \otimes H_B$, where I_n (I_m) denotes the $n \times n$ ($m \times m$) identity matrix. The battery capacity $C(\rho_{AB}, H_{AB})$ can be calculated by using Eq.(1). Let $\rho_A = \sum_{i=0}^{m-1} \lambda_i |\psi_i\rangle \langle \psi_i|$ be the reduced state of the subsystem A. We have the battery capacity with respect to the subsystem A, $C(\rho_A, H_A) = \sum_{i=0}^{m-1} \epsilon_i^A (\lambda_i - \lambda_{d-1-i})$.

Let $\{B_k\}_{k=0}^{n-1}$ be a rank-1 local projective measurement on the subsystem B. Conditioned on the measurement outcome k , the quantum state ρ_{AB} changes to be

$$\rho_{AB}^k = \frac{1}{P_k} (I_m \otimes B_k) \rho_{AB} (I_m \otimes B_k) \quad (2)$$

with probability $P_k = \text{Tr}(I_m \otimes B_k) \rho_{AB} (I_m \otimes B_k)$ for $k = 0, 1, 2, \dots, n-1$. The final state ρ'_{AB} depends on the measurement basis as well as the outcomes. Under average probability (the group of measurements selected and the corresponding outputs are equally distributed),

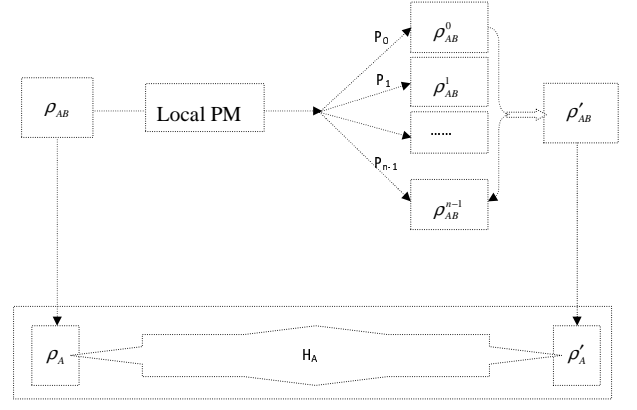


FIG. 1: A initially shared bipartite quantum state ρ_{AB} undergoes local projective measurements $\{B_k\}_{k=0}^{n-1}$ on the subsystem B. States ρ_{AB}^k are obtained with probability P_k . From the final state ρ'_{AB} and its reduced density matrix ρ'_A , we study the changes of the battery capacities before and after the measurement.

the final state is given by

$$\rho'_{AB} = \frac{1}{n} \sum_{k=0}^{n-1} P_k \rho_{AB}^k. \quad (3)$$

With arbitrary randomness probability, namely, the measurement that one chooses to use is according to the idea of freedom and the probability of different outputs of the same measurement is also different, one has

$$\rho'_{AB} = \sum_{k=0}^{n-1} \mu_k \rho_{AB}^k, \quad (4)$$

where $\sum_{k=0}^{n-1} \mu_k = 1$, $\mu_k \geq 0$. From the spectral decomposition of the reduced density matrix $\rho'_A = \text{Tr}_B(\rho'_{AB}) = \sum_{i=0}^{m-1} \lambda'_i |\psi_i\rangle \langle \psi_i|$, we obtain the capacity associated with the subsystem A after projective measurement, $C(\rho'_A, H_A) = \sum_{i=0}^{m-1} \epsilon_i^A (\lambda'_i - \lambda'_{d-1-i})$, see Fig. 1. for the schematic diagram. In the following we focus on two particular classes of initial states.

Two-qubit Bell-diagonal states

We first consider the initial states to be the two-qubit Bell-diagonal ones:

$$\rho_{AB} = \frac{1}{4} (I_2 \otimes I_2 + \sum_{i=1}^3 c_i \sigma_i \otimes \sigma_i),$$

where $\sigma_1, \sigma_2, \sigma_3$ are the standard Pauli matrices, c_i are real constants such that ρ_{AB} is a well defined density matrix. ρ_{AB} have eigenvalues $\lambda_0 = (1 - c_1 - c_2 - c_3)/4$, $\lambda_1 = (1 - c_1 + c_2 + c_3)/4$, $\lambda_2 = (1 + c_1 - c_2 + c_3)/4$ and $\lambda_3 = (1 + c_1 + c_2 - c_3)/4$. The coefficients c_i are so chosen that $\lambda_j \in [0, 1]$ for $j = 0, 1, 2, 3$. For the convenience of future analysis, we assume that $|c_1| \geq |c_2| \geq |c_3|$.

We consider the following Hamiltonian for the two-qubit system,

$$H_{AB} = \epsilon^A \sigma_3 \otimes I_2 + \epsilon^B I_2 \otimes \sigma_3, \quad (5)$$

where $\epsilon^A \geq \epsilon^B \geq 0$. The eigenvalues of H_{AB} are $-\epsilon^A - \epsilon^B$, $-\epsilon^A + \epsilon^B$, $\epsilon^A - \epsilon^B$ and $\epsilon^A + \epsilon^B$ in ascending order. Therefore, we have

$$C(\rho_{AB}, H_{AB}) = (|c_1| + |c_2|)(\epsilon^A + \epsilon^B) + (|c_1| - |c_2|)(\epsilon^A - \epsilon^B).$$

The reduced states of ρ_{AB} are $\rho_A = I_2/2$ and $\rho_B = I_2/2$. Consequently, we have $C(\rho_A, H_A) = 0$.

Consider local projective measurement given by the computational base $\{|k\rangle\}$, $\{\Pi_k = |k\rangle\langle k|, k = 0, 1\}$. The measurement gives rise to the ensemble $\{\rho_{AB}^k, P_k\}$. We have

$$\begin{aligned} P_k \rho_{AB}^k &= (I_2 \otimes \Pi_k) \rho_{AB} (I_2 \otimes \Pi_k) \\ &= \frac{1}{4} [I_2 \otimes \Pi_k + \sum_{i=1}^3 c_i \sigma_i \otimes (\Pi_k \sigma_i \Pi_k)] \\ &= \frac{1}{4} [I_2 \otimes \Pi_k + (-1)^k c_3 \sigma_3 \otimes \Pi_k], \end{aligned} \quad (6)$$

where we have used the relations $\Pi_k \sigma_j \Pi_k = 0$ for $k = 0, 1, j = 1, 2$, $\Pi_0 \sigma_3 \Pi_0 = \Pi_0$ and $\Pi_1 \sigma_3 \Pi_1 = -\Pi_1$. From Eq.(6) we get

$$\begin{aligned} \rho_{AB}^0 &= \frac{1}{2} \begin{pmatrix} 1 + c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 - c_3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\ \rho_{AB}^1 &= \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 - c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 + c_3 \end{pmatrix} \end{aligned}$$

as $P_0 = P_1 = \frac{1}{2}$.

By straightforward calculation we have the following observations.

(i) Taking the final state to be the one given by (3), we have $\rho_{AB}^k = \frac{1}{2} \sum_{k=0}^1 \rho_{AB}^k$. Then $C(\rho_{AB}^k, H_{AB}) = 2|c_3|\epsilon^A$ and $C(\rho_A^k, H_A) = 0$. Therefore, $F := C(\rho_{AB}^k, H_{AB}) - C(\rho_A^k, H_A) \leq 0$, namely, the local measurement weakens the battery capacity of the whole bipartite system, but does not change the battery capacity of the subsystem A .

(ii) Taking the final state to be the one given by (4), we have $\rho_{AB}^k = \mu_0 \rho_{AB}^0 + \mu_1 \rho_{AB}^1$. Without loss of generality, let $\mu_0 > \mu_1$. We obtain $C(\rho_{AB}^k, H_{AB}) = (\mu_0 - \mu_1 + |c_3|)(\epsilon^A + \epsilon^B) + (\mu_1 - \mu_0 + |c_3|)(\epsilon^A - \epsilon^B)$ and $C(\rho_A^k, H_A) = 2(\mu_0 - \mu_1)\epsilon^A |c_3|$. In this case, the local measurement could either reduce or increase the battery capacity of the whole system, but always increase the battery capacity of the subsystem A .

(iii) When $c_3 = 0$, $f := C(\rho_{AB}^k, H_{AB}) - C(\rho_A^k, H_A)$ is always zero. However, for the ρ_{AB}^k given in (ii), one has $C(\rho_{AB}^k, H_{AB}) = 2(\mu_0 - \mu_1)\epsilon^B$.

The above observations show that the unilateral measurement on subsystem B may increase the battery capacity of the subsystem A , but not always the entire system.

Example 1: Let us consider a specific Bell-diagonal state with $c_1 = c_2 = c_3 = -a$, i.e., the two-qubit Werner state [66],

$$\rho_W = a|\psi^-\rangle\langle\psi^-| + \frac{1-a}{4}I_4,$$

where $|\psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$ is the maximally entangled state and $0 \leq a \leq 1$. ρ_W is separable when $a \leq \frac{1}{3}$ and entangled otherwise. From the observation (ii), we see that after the projective measurement on subsystem B , the battery capacity of the subsystem A increases as long as $a \neq 0$ and $\epsilon^A > 0$, no matter if the original state is separable or entangled. When $\mu_0 - \mu_1 > a$, the measurement on the subsystem B can also increase the battery capacity of the whole system. When $\mu_0 - \mu_1 = a$, the battery capacity of the whole system keeps unchanged. Therefore, for a random probability combination Eq.(4), by choosing appropriate parameters the local projective measurement may enhance the battery capacity of the overall or the individual subsystem.

Two-qubit X-states

We consider the initial states to be the two-qubit X-type ones,

$$\rho_X = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}, \quad (7)$$

where $\sum_{i=1}^4 \rho_{ii} = 1$, $\rho_{22}\rho_{33} \geq |\rho_{23}|^2$ and $\rho_{11}\rho_{44} \geq |\rho_{14}|^2$. ρ_X is entangled if and only if either $\rho_{11}\rho_{44} \leq |\rho_{14}|^2$ or $\rho_{22}\rho_{33} \leq |\rho_{23}|^2$ [67]. The eigenvalues of the two-qubit X-states are given by

$$\begin{aligned} \lambda_0 &= \frac{1}{2}[(\rho_{11} + \rho_{44}) + \sqrt{(\rho_{11} - \rho_{44})^2 + 4|\rho_{14}|^2}], \\ \lambda_1 &= \frac{1}{2}[(\rho_{11} + \rho_{44}) - \sqrt{(\rho_{11} - \rho_{44})^2 + 4|\rho_{14}|^2}], \\ \lambda_2 &= \frac{1}{2}[(\rho_{22} + \rho_{33}) + \sqrt{(\rho_{22} - \rho_{33})^2 + 4|\rho_{23}|^2}], \\ \lambda_3 &= \frac{1}{2}[(\rho_{22} + \rho_{33}) - \sqrt{(\rho_{22} - \rho_{33})^2 + 4|\rho_{23}|^2}]. \end{aligned}$$

The reduced density matrix of the subsystem A ,

$$\rho_X^A = \begin{pmatrix} \rho_{11} + \rho_{22} & 0 \\ 0 & \rho_{33} + \rho_{44} \end{pmatrix}, \quad (8)$$

has eigenvalues $\lambda_0^A = \rho_{11} + \rho_{22}$ and $\lambda_1^A = \rho_{33} + \rho_{44}$. With respect to the Hamiltonian (5), one has $C(\rho_X, H_{AB})$ and $C(\rho_X^A, H_A)$.

ρ_X can be rewritten as in Bloch representation,

$$\rho_X = \frac{1}{4}(I_2 \otimes I_2 + a_3 \sigma_3 \otimes I_2 + b_3 I_2 \otimes \sigma_3 + \sum_{i=1}^3 c_i \sigma_i \otimes \sigma_i),$$

where $a_3 = \rho_{11} + \rho_{22} - (\rho_{33} + \rho_{44})$, $b_3 = \rho_{11} + \rho_{33} - (\rho_{22} + \rho_{44})$, $c_1 = 2(\rho_{33} + \rho_{44})$, $c_2 = 2(\rho_{23} - \rho_{14})$ and $c_3 = \rho_{11} + \rho_{44} - (\rho_{22} + \rho_{33})$. After the measurement $\Pi_k = |k\rangle\langle k|$ on subsystem B , we obtain

$$\begin{aligned} P_k \rho_X^k &= (I_2 \otimes \Pi_k) \rho_X (I_2 \otimes \Pi_k) \\ &= \frac{1}{4} [I_2 \otimes \Pi_k + a_3 \sigma_3 \otimes \Pi_k + b_3 I_2 \otimes (\Pi_k \sigma_3 \Pi_k) \\ &\quad + \sum_{i=1}^3 c_i \sigma_i \otimes (\Pi_k \sigma_i \Pi_k)] \\ &= \frac{1}{4} [I_2 \otimes \Pi_k + a_3 (\sigma_3 \otimes \Pi_k) \\ &\quad + (-1)^k b_3 (I_2 \otimes \Pi_k) + (-1)^k c_3 (\sigma_3 \otimes \Pi_k)]. \end{aligned} \quad (9)$$

Taking $k = 0, 1$ into Π_k , we obtain specific probabilities and corresponding quantum states. Namely, $P_0 = \frac{1+b_3}{2}$, $P_1 = \frac{1-b_3}{2}$ and

$$\rho_X^0 = \frac{1}{2(1+b_3)} \begin{pmatrix} 1+b_3+a_3+c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1+b_3-a_3-c_3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\rho_X^1 = \frac{1}{2(1-b_3)} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1-b_3+a_3-c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-b_3-a_3+c_3 \end{pmatrix}.$$

From Eq.(3) or Eq.(4) we can investigate the projective measurement impact on the battery capacity of subsystem A and the whole bipartite system.

Example 2: Consider the following specific state,

$$\rho_{AB} = \frac{1}{3} [(1-x)|00\rangle\langle 00| + 2|\psi^+\rangle\langle \psi^+| + x|11\rangle\langle 11|],$$

where $x \in [0, 1/2]$ and $|\psi^+\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$. ρ_{AB} has eigenvalues $\lambda_0 = 0$, $\lambda_1 = x/3$, $\lambda_2 = (1-x)/3$ and $\lambda_3 = 2/3$. The reduced density matrix ρ_A has eigenvalues $\delta_0 = (1+x)/3$ and $\delta_1 = (2-x)/3$. Therefore, $C(\rho_{AB}, H_{AB}) = \frac{2}{3}(\epsilon^A + \epsilon^B) + \frac{1-2x}{3}(\epsilon^A - \epsilon^B)$ and $C(\rho_A, H_A) = 2\frac{1-2x}{3}\epsilon^A$. Adopting Eq.(3) for ρ'_{AB} , we obtain

$$\rho'_{AB} = \begin{pmatrix} \frac{1-x}{4-2x} & 0 & 0 & 0 \\ 0 & \frac{1}{2+2x} & 0 & 0 \\ 0 & 0 & \frac{1}{4-2x} & 0 \\ 0 & 0 & 0 & \frac{x}{2+2x} \end{pmatrix}.$$

Therefore, $C(\rho'_{AB}, H_{AB}) = \frac{1-x}{2+2x}(\epsilon^A + \epsilon^B) + \frac{x}{4-2x}(\epsilon^A - \epsilon^B)$ and $C(\rho'_A, H_A) = 2\frac{1-2x}{(2-x)(1+x)}\epsilon^A$. Thus, $f = C(\rho'_A, H_A) - C(\rho_A, H_A) \geq 0$ for all $x \in [0, \frac{1}{2})$, namely, the battery capacity of the subsystem A will be always enhanced, see Fig. 2.

If we adopt Eq.(4) for ρ'_{AB} , similarly we obtain

$$\rho'_{AB} = \begin{pmatrix} \frac{\mu_0(1-x)}{2-x} & 0 & 0 & 0 \\ 0 & \frac{\mu_1}{1+x} & 0 & 0 \\ 0 & 0 & \frac{\mu_0}{2-x} & 0 \\ 0 & 0 & 0 & \frac{\mu_1 x}{1+x} \end{pmatrix}.$$

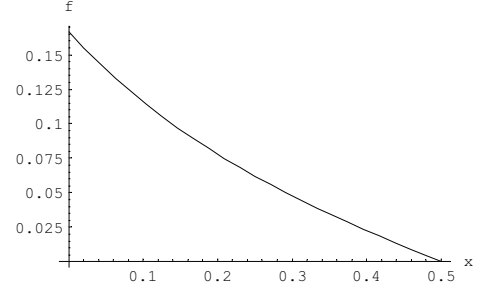


FIG. 2: The variation of the battery capacity of the subsystem A for ρ_{AB} in Example 2, where $f = C(\rho'_A, H_A) - C(\rho_A, H_A)$ is a function of x for $\epsilon^A = 0.5$.

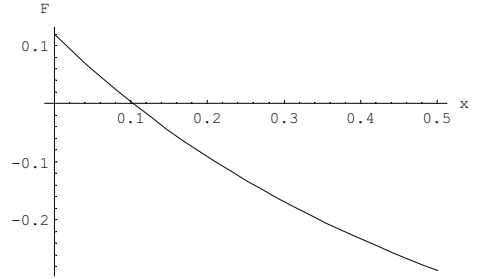


FIG. 3: The variation of the battery capacity of the whole system for ρ_{AB} in Example 2. Here, $F = C(\rho'_{AB}, H_{AB}) - C(\rho_{AB}, H_{AB})$ is a function of x for $\epsilon^A = 0.5$, $\epsilon^B = 0.3$, $\mu_1 = 0.9$ and $\mu_0 = 0.1$, where μ_1 and μ_2 correspond to the probabilities of choosing different measurement operators, respectively.

Therefore, $C(\rho'_{AB}, H_{AB}) = \frac{(1-x)\mu_1}{1+x}(\epsilon^A + \epsilon^B) + \frac{x\mu_0}{2-x}(\epsilon^A - \epsilon^B)$ and $C(\rho'_A, H_A) = 2\frac{(\mu_1 - \mu_0)x^2 + 2\mu_1 - (3\mu_1 + \mu_0)x}{(2-x)(1+x)}\epsilon^A$. Here, we have assumed that $\mu_1 > \mu_0 > 2x\mu_1$. Taking $\epsilon^A = 0.5$, $\epsilon^B = 0.3$, $\mu_1 = 0.9$ and $\mu_0 = 0.1$ with $\mu_0 > 2x\mu_1$ as an example, we get $x < 0.056$. It is seen from Fig. 3 that $F = C(\rho'_{AB}, H_{AB}) - C(\rho_{AB}, H_{AB})$ is greater than 0, namely, the battery capacity of the entire system also increases. On the other hand, as long as x tends to 0 and μ_1 tends to 1, we have $C(\rho'_A, H_A) - C(\rho_A, H_A) > 0$, i.e., the battery capacity of the subsystem A increases too.

CONCLUSIONS AND OUTLOOK

We have studied the enhancement of the quantum battery capacities for bipartite systems under local projective measurements. In particular, we have investigated the cases that the initial states are two-qubit Bell diagonal and X-type ones. We have provided analytical re-

sults for these general initial states, which indicated that our scheme can improve the battery capacity of the entire or the subsystems, regardless of whether the original quantum state is entangled or separable. There are also other studies on quantum batteries by using quantum measurements [7, 9, 41, 43, 49], but with different extraction protocols. Our study has some advantages compared with, for example, the one in the latest Ref.[41]. Firstly, we adopted the latest definition of quantum battery capacity, and our protocol starts directly from the bipartite quantum states, without considering the auxiliary systems. Secondly, Ref.[41] requires multiple measurements and the assistance of POVM measurements. Here, our protocol only involves one projective measurement. Moreover, we have provided the analytical results when the initial state is a general 2-qubit quantum state.

The quantum battery capacity is unitary invariant and solely given by the eigenvalues of the quantum states and the Hamiltonian. There are relationships between the battery capacity and the quantum resources such as quantum entropy, entanglement and coherence [36, 42]. We have shown that the improvement of quantum battery capacity is related to both the initial states and the combination forms of the final states. Our results indicate that the local projection measurement is a kind of important resource for improving the capacity of quantum batteries. It would also be interesting to investigate the impact of general positive operator-valued measures on the improvement of quantum battery capacities.

Acknowledgments: This work is supported by the Hainan Provincial Natural Science Foundation of China under Grant No.121RC539; the National Natural Science Foundation of China (NSFC) under Grant Nos. 12204137, 12075159 and 12171044; the specific research fund of the Innovation Platform for Academicians of Hainan Province under Grant No. YSPTZX202215 and Hainan Academician Workstation (Changbin Yu).

[1] R. Alicki and M. Fannes, Entanglement boost for extractable work from ensembles of quantum batteries, *Phys. Rev. E* **87**, 042123 (2013).
 [2] M. Frey, K. Funo, and M. Hotta, Strong local passivity in finite quantum systems, *Phys. Rev. E* **90**, 012127 (2014).
 [3] M. Perarnau-Llobet, K. V. Hovhannisyan, M. Huber, P. Skrzypczyk, J. Tura, and A. Acín, Most energetic passive states, *Phys. Rev. E* **92**, 042147 (2015).
 [4] E.G. Brown, N. Friis, and M. Huber, Passivity and practical work extraction using gaussian operations, *New. J. Phys.* **18**, 113028 (2016).
 [5] C. Sparaciari, D. Jennings, and J. Oppenheim, Energetic instability of passive states in thermodynamics, *Nature Communications* **8**, 1895 (2017).
 [6] F. Campaioli, F. A. Pollock, F. C. Binder, L. Céleri, J. Goold, S. Vinjanampathy, and K. Modi, Enhancing the

charging power of quantum batteries, *Phys. Rev. Lett.* **118**, 150601 (2017).
 [7] G. Francica, J. Goold, F. Plastina, and M. Paternostro, Daemonic ergotropy: enhanced work extraction from quantum correlations, *njp Quant. Info.* **3**, 12 (2017).
 [8] G. M. Andolina, D. Farina, A. Mari, V. Pellegrini, V. Giovannetti, and M. Polini, Charger-mediated energy transfer in exactly solvable models for quantum batteries, *Phys. Rev. B* **98**, 205423 (2018).
 [9] G. Manzano, F. Plastina, and R. Zambrini, Optimal work extraction and thermodynamics of quantum measurements and correlations, *Phys. Rev. Lett.* **121**, 120602 (2018).
 [10] D. Ferraro, M. Campisi, G. M. Andolina, V. Pellegrini, and M. Polini, High-Power collective charging of a solid-state quantum battery, *Phys. Rev. Lett.* **120**, 117702 (2018).
 [11] T. P. Le, J. Levinsen, K. Modi, M. M. Parish, and F. A. Pollock, Spin-chain model of a many-body quantum battery, *Phys. Rev. A* **97**, 022106 (2018).
 [12] Á. M. Alhambra, G. Styliaris, N. A. R. Briones, J. Sikora, and E. M. Martínez, Fundamental limitations to local energy extraction in quantum systems, *Phys. Rev. Lett.* **123**, 190601 (2019).
 [13] D. Rossini, G. M. Andolina, and M. Polini, Many-body localized quantum batteries, *Phys. Rev. B* **100**, 115142 (2019).
 [14] G. M. Andolina, M. Keck, A. Mari, V. Giovannetti, and M. Polini, Quantum versus classical many-body batteries, *Phys. Rev. B* **99**, 205437 (2019).
 [15] R. Alicki, A quantum open system model of molecular battery charged by excitons, *Jour. of Chem. Phys.* **150**, 214110 (2019).
 [16] A. Crescente, M. Carrega, M. Sassetti, and D. Ferraro, Ultrafast charging in a two-photon Dicke quantum battery, *Phys. Rev. B* **102**, 245407 (2020).
 [17] S. Ghosh, T. Chanda, and A. Sen(De), Enhancement in the performance of a quantum battery by ordered and disordered interactions, *Phys. Rev. A* **101**, 032115 (2020).
 [18] S. Gherardini, F. Campaioli, F. Caruso, and F. C. Binder, Stabilizing open quantum batteries by sequential measurements, *Phys. Rev. Res.* **2**, 013095 (2020).
 [19] D. Rossini, G. M. Andolina, D. Rosa, M. Carrega, and M. Polini, Quantum advantage in the charging process of Sachdev-Ye-Kitaev batteries, *Phys. Rev. Lett.* **125**, 236402 (2020).
 [20] M. T. Mitchison, J. Goold, and J. Prior, Charging a quantum battery with linear feedback control, *Quantum* **5**, 500 (2021).
 [21] S. Ghosh, T. Chanda, S. Mal, and A. Sen(De), Fast charging of a quantum battery assisted by noise, *Phys. Rev. A* **104**, 032207 (2021).
 [22] F. Zhao, F.Q. Dou, and Q. Zhao, Quantum battery of interacting spins with environmental noise, *Phys. Rev. A* **103**, 033715 (2021).
 [23] K. Sen and U. Sen, Local passivity and entanglement in shared quantum batteries, *Phys. Rev. A* **104**, L030402 (2021).
 [24] S. Tirone, R. Salvia, and V. Giovannetti, Quantum energy lines and the optimal output ergotropy problem, *Phys. Rev. Lett.* **127**, 210601 (2021).
 [25] Y. Yao and X.Q. Shao, Stable charging of a Rydberg

- quantum battery in an open system, *Phys. Rev. E* **104**, 044116 (2021).
- [26] S. Zakavati, F. T. Tabesh, and S. Salimi, Bounds on charging power of open quantum batteries, *Phys. Rev. E* **104**, 054117 (2021).
- [27] K. Xu, H.J. Zhu, G.F. Zhang, and W.M. Liu, Enhancing the performance of an open quantum battery via environment engineering, *Phys. Rev. E* **104**, 064143 (2021).
- [28] S. Ghosh and A. Sen(De), Dimensional enhancements in a quantum battery with imperfections, *Phys. Rev. A* **105**, 022628 (2022).
- [29] F. Mayo and A. J. Roncaglia, Collective effects and quantum coherence in dissipative charging of quantum batteries, *Phys. Rev. A* **105**, 062203 (2022).
- [30] J. Carrasco, R. Maze, Jeronimo, C. Hermann-Avigliano, and F. Barra, Collective enhancement in dissipative quantum batteries, *Phys. Rev. E* **105**, 064119 (2022).
- [31] S. Tirone, R. Salvia, S. Chessa, and V. Giovannetti, Quantum work capacitances, arXiv:2211.02685 (2022).
- [32] F.Q. Dou, H. Zhou, and J.A. Sun, Cavity heisenbergspin-chain quantum battery, *Phys. Rev. A* **106**, 032212 (2022).
- [33] R. R. Rodriguez, B. Ahmadi, G. Suarez, P. Mazurek, S. Barzanjeh, and P. Horodecki, Optimal quantum control of charging quantum batteries, arXiv:2207.00094 (2022).
- [34] K. Xu, H.G. Li, Z.G. Li, H.J. Zhu, G.F. Zhang, and W.M. Liu, Charging performance of quantum batteries in a double-layer environment, *Phys. Rev. A* **106**, 012425 (2022).
- [35] T. K. Konar, L. G. C. Lakkaraju, S. Ghosh, and A. Sen(De), Quantum battery with ultracold atoms: Bosons versus fermions, *Phys. Rev. A* **106**, 022618 (2022).
- [36] H.L. Shi, S. Ding, Q.K. Wan, X.H. Wang, and W.L. Yang, Entanglement, coherence, and extractable work in quantum batteries, *Phys. Rev. Lett.* **129**, 130602 (2022).
- [37] F.Q. Dou, Y.Q. Lu, Y.J. Wang, and J.A. Sun, Extended Dicke quantum battery with interatomic interactions and driving field, *Phys. Rev. B* **105**, 115405 (2022).
- [38] F.Q. Dou, Y.J. Wang, J.A. Sun, Highly efficient charging and discharging of three-level quantum batteries through shortcuts to adiabaticity, *Front. Phys.* **17**(3), 31503 (2022).
- [39] R. Salvia, G. De Palma, and V. Giovannetti, Optimal local work extraction from bipartite quantum systems in the presence of Hamiltonian couplings, *Phys. Rev. A* **107**, 012405 (2023).
- [40] K. Xu, H.J. Zhu, H. Zhu, G.F. Zhang and W.M. Liu, Charging and self-discharging process of a quantum battery in composite environments. *Front. Phys.* **18**(3), 31301 (2023).
- [41] J. S. Yan and J. Jing, Charging by quantum measurement, *Phys. Rev. Appl.* **19**, 064069 (2023).
- [42] X. Yang, Y. H. Yang, M. Alimuddin, R. Salvia, S. M. Fei, L. M. Zhao, S. Nimmrichter, and M. X. Luo, The battery capacity of energy-storing quantum systems, *Phys. Rev. Lett.* **131**, 030402 (2023).
- [43] S. Tirone, R. Salvia, S. Chessa, and V. Giovannetti, Work extraction processes from noisy quantum batteries: the role of non local resources, *Phys. Rev. Lett.* **131**, 060402 (2023).
- [44] F.Q. Dou and F.M. Yang, Superconducting transmon qubit-resonator quantum battery, *Phys. Rev. A* **107**, 023725(2023).
- [45] K. Sen, U. Sen, Noisy quantum batteries, arXiv:2302.07166v1 (2023).
- [46] S. Tirone, R. Salvia, S. Chessa, and V. Giovannetti, Quantum work extraction efficiency for noisy quantum batteries: the role of coherence, arXiv:2305.16803 (2023).
- [47] B. A. Mohammad, H. Mohammad, A. Saguia, M. S. Sarandy, and A. C. Santos, Localization effects in disordered quantum batteries, arXiv:2306.13164 (2023)
- [48] A. G. Catalano, S. M. Giampaolo, O. Morsch, V. Giovannetti, and F. Franchini, Frustrating quantum batteries, arXiv:2307.02529 (2023).
- [49] P. Chaki, A. Bhattacharyya, K. Sen, U.Sen, Auxiliary-assisted stochastic energy extraction from quantum batteries, arXiv:2307.16856v1(2023).
- [50] F. Campaioli, S. Gherardini, J. Q. Quach, M. Polini, G. M. Andolina, Colloquium: Quantum batteries, arXiv:2308.02277 (2023).
- [51] D. Morrone, M. A. C. Rossi, A. Smirne, and M. G. Genoni, Charging a quantum battery in a nonmarkovian environment: a collisional model approach, *Quant. Science and Tech.* **8**, 035007 (2023).
- [52] H.Y. Yang, H.L. Shi, Q.K. Wan, K. Zhang, X.H. Wang, and W.L. Yang, Optimal energy storage in the Tavis-Cummings quantum battery, *Phys. Rev. A* **109**, 012204 (2024).
- [53] P. Bakhshinezhad, B. R. Jablonski, F. C. Binder, and N. Friis, Trade-offs between precision and fluctuations in charging finite-dimensional quantum batteries, *Phys. Rev. E* **109**, 014131 (2024).
- [54] J. Q. Quach, K. E. McGhee, L. Ganzer, D. M. Rouse, B. W. Lovett, E. M. Gauger, J. Keeling, G. Cerullo, D. G. Lidzey, and T. Virgili, Superabsorption in an organic microcavity: Toward a quantum battery, *Science Advances* **8**, 3160 (2022).
- [55] A. Delgado, P. A. M. Casares, R. dos Reis, M. S. Zini, R. Campos, N. Cruz-Hernandez, A. C. Voigt, A. Lowe, S. Jahangiri, M. A. Martin-Delgado, J. E. Mueller, and J. M. Arrazola, Simulating key properties of lithium-ion batteries with a fault-tolerant quantum computer, *Phys. Rev. A* **106**, 032428 (2022).
- [56] J. Joshi and T. S. Mahesh, Experimental investigation of a quantum battery using star-topology nmr spin systems, *Phys. Rev. A* **106**, 042601 (2022).
- [57] N. C. Rubin, D. W. Berry, F. D. Malone, A. F. White, T. Khattar, A. E. De Prince III au2, S. Siculo, M. Kühn, M. Kaicher, J. Lee, and R. Babbush, Fault-tolerant quantum simulation of materials using bloch orbitals, arXiv:2302.05531 (2023).
- [58] J. Franklin, J. Bedard, and I. Sochnikov, Versatile millikelvin hybrid cooling platform for superconductivity research, *IEEE Transactions on Applied Superconductivity* **33**, 1600303 (2023).
- [59] D. Home and M. Whitaker, A conceptual analysis of quantum zeno; paradox, measurement, and experiment, *Ann. Phys. (N.Y.)* **258**, 237 (1997).
- [60] H.M. Wiseman and J.F. Ralph, Reconsidering rapid qubit purification by feedback, *New J. Phys.* **8**, 90 (2006).
- [61] J. Combes, H. M. Wiseman, K. Jacobs, and A. J. O'Connor, Rapid purification of quantum systems by measuring in a feedback-controlled unbiased basis, *Phys. Rev. A* **82**, 022307 (2010).
- [62] J. Combes and H. M. Wiseman, Maximum information gain in weak or continuous measurements of qudits: Complementarity is not enough, *Phys. Rev. X* **1**, 011012 (2011).

- [63] G.T. Landi, M. Paternostro, and A. Belenchia, Informational steady states and conditional entropy production in continuously monitored systems, *PRX Quantum* **3**, 010303 (2022).
- [64] C. Elouard and A. N. Jordan, Efficient quantum measurement engines, *Phys. Rev. Lett.* **120**, 260601 (2018).
- [65] S. Rogers and A. N. Jordan, Postselection and quantum energetics, *Phys. Rev. A* **106**, 052214 (2022).
- [66] R.F. Werner, Quantum states with Einstein-Podolsky-Rosen correlations admitting a hidden-variable model, *Phys. Rev. A* **40**, 4277 (1989).
- [67] A. Sanpera, R. Tarrach, and G. Vidal, Local description of quantum inseparability, *Phys. Rev. A* **58**, 826 (1998).