

# Non-linear inhibitory responses enhance performance in collective decision-making

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The precise modulation of activity through inhibitory signals ensures that both insect colonies and neural circuits operate efficiently and adaptively, highlighting the fundamental importance of inhibition in biological systems. Modulatory signals are produced in various contexts and are known for subtly shifting the probability of receiver behaviors based on response thresholds. Here we propose a non-linear function to introduce inhibitory responsiveness in collective decision-making inspired by honeybee house-hunting. We show that, compared with usual linear functions, non-linear responses enhance final consensus and reduce deliberation time. This improvement comes at the cost of reduced accuracy in identifying the best option. Nonetheless, for value-based tasks, the benefits of faster consensus and enhanced decision-making might outweigh this drawback.

The behavioral and signaling patterns of social animal groups [1, 2] have sparked extensive research into collective behavior and decision-making, primarily to understand the underlying mechanisms that drive these emergent properties [3]. Inhibitory signals, in particular, play an essential role in social insects, fine-tuning collective decision-making and coordinating critical tasks such as house-hunting and foraging [4–8]. These inhibitory signals, often communicated through vibrations or tactile interactions, allow colonies to efficiently allocate resources and labor. For instance, in honeybees, stop signals can prevent the recruitment of additional foragers to poor or perilous food sources, thereby optimizing foraging efforts [4–7]. Similarly, during nest site selection, bees use stop signals to halt the promotion of less suitable sites, ensuring that the colony converges on the best available option [8]. By integrating these stop signals, social insects enhance their ability to make adaptive and robust decisions, ultimately supporting the survival and success of the colony. The fascinating social behavior of honeybees, including their intricate recruiting signaling patterns such as the waggle dance [9], has inspired the design of decentralized decision-making algorithms [10–14], and their application to robotic systems [15].

According to Nieh [4, 7] and Pastor *et al.* [5], during foraging tasks, honeybees’ stop signals can alter the probability of waggle dancers ceasing their dance and leaving the nest, thereby reducing recruitment. However, dancers do not exhibit an immediate response to these signals. This feature is characteristic of modulatory signals, which are known for subtly shifting the probability of receiver behaviors based on their response thresholds. Lau *et al.* [6] further suggested that, depending on receiver response thresholds, stop signals do not exert a strong colony-wide effect until signaling levels are sufficiently elevated. A similar mechanism has for long also been observed in brain neuronal activity [16], where

balance between excitation and inhibition is critical for processes such as sensory perception, motor control or cognitive functions. Recent efforts have been made in establishing the similarities between individual decision-making in primate brains and collective decision-making in social insect colonies [17–19].

Field experiments on honeybee house-hunting [8] introduced the term *cross-inhibition*, as it was demonstrated that stop signals were predominantly exchanged between agents promoting competing options. Cross-inhibition has proven essential for resolving deadlocks in decisions between very similar alternatives [8, 10, 20–22]. However, as argued in Ref. [22], cross-inhibition trades accuracy for stability. This means the system can confidently make a decision for any option, regardless of whether it is the highest quality one or not. In value-based tasks, this trade-off may not necessarily be detrimental, as the system prioritizes making a choice that yields a sufficiently high reward within a limited time, thus balancing the speed-value trade-off [22–25]. Furthermore, depending on the intensity of cross-inhibition, this mechanism may pause the decision-making process if the qualities of the available options are not deemed high enough, allowing the system to wait for a potentially better option to appear [10, 12]. Such a system transitions from indecision to decision through pitchfork or saddle-node bifurcations [14, 26], controlled by the model parameters.

In honeybee-inspired collective decision-making models, the cross-inhibition rate has usually been considered a linear function of the population sending the stop signals. This choice represents the simplest modeling assumption, where the abandonment of one’s opinion is linearly proportional to the accumulation of stop signals received from peers with opposing options. However, similar to the foraging behavior of bees discussed earlier, Seeley *et al.* also suggested that nest-site scout waggle dances are likely terminated when stop-signal in-

hibition surpasses a certain threshold [8]. Motivated by this experimental evidence, here we investigate the impact of non-linear inhibitory responsiveness [14, 26, 27] within honeybee-inspired decision-making models. The response depends on the amount of stop signals received and diminishes or becomes negligible when stop signals are sparse, see Fig. 1(a). This approach also aligns with the concept of complex social contagion models [28, 29], which posits that multiple exposures to a given opinion are required to trigger a shift in belief. Similarly, our model assumes that a minimum threshold of stop signals must be reached before cross-inhibition acts.

Focusing on binary decision tasks, we demonstrate that our approach enhances the consensus formation capabilities of decentralized systems compared to linear cross-inhibition models, particularly when dealing with options of similar qualities. The benefits are twofold: first, the final decision is achieved with virtually no bees committed to the less favored option; second, the time to reach a stationary state is significantly reduced.

*Model* – We use a simplified version of the honeybee inspired LES model [30], which features the typical transition rates of collective decision making models [10–13]. In the LES model, a swarm of  $N$  scout bees evaluates  $k$  potential nest sites, indexed by  $\alpha = 1, \dots, k$ . Each site  $\alpha$  is characterized by an intrinsic quality  $q_\alpha \geq 0$  and a spontaneous discovery probability  $\pi_\alpha \geq 0$ .

Bees can be in any of  $k + 1$  states: uncommitted or committed to one of the  $k$  available sites. The transitions from uncommitted to committed state are governed by discovery and recruitment rates, representing individual and social behavior, and balanced by the interdependence parameter,  $\lambda$ . Likewise, the transitions from committed to uncommitted states are governed by abandonment and cross-inhibition rates, which reflect individual and socially motivated behaviors, respectively.

The model's mean-field rate equations for the fractions of agents committed to each site,  $f_\alpha(t)$ , can be derived using the master equation formalism [31]. Including cross-inhibition, these equations are:

$$\begin{aligned} \dot{f}_\alpha(t) = & f_0(t) [(1 - \lambda)\pi_\alpha + \lambda f_\alpha(t)] \\ & - r_\alpha f_\alpha(t) - \lambda' f_\alpha(t) \sum_{\beta \neq \alpha} \sigma(f_\beta), \quad \alpha = 1, \dots, k \end{aligned} \quad (1)$$

where  $f_0(t) = 1 - \sum_{\alpha=1}^k f_\alpha(t)$  is the fraction of uncommitted bees. The discovery rate,  $(1 - \lambda)\pi_\alpha$ , refers to the rate at which uncommitted bees discover and commit to site  $\alpha$ , and the recruitment rate  $\lambda f_\alpha$  represents the rate at which uncommitted bees are recruited by peers already committed to option  $\alpha$ . The rate  $r_\alpha$  at which bees stop advertising a site is inversely proportional to its quality,  $r_\alpha = 1/q_\alpha$ . Finally, the cross-inhibition rate,  $\lambda' f_\alpha \sigma(f_\beta)$  ( $\beta \neq \alpha$ ), is the rate at which bees abandon their options after receiving stop signals from those advocating for competing options. Here,  $\lambda'$  regulates the intensity

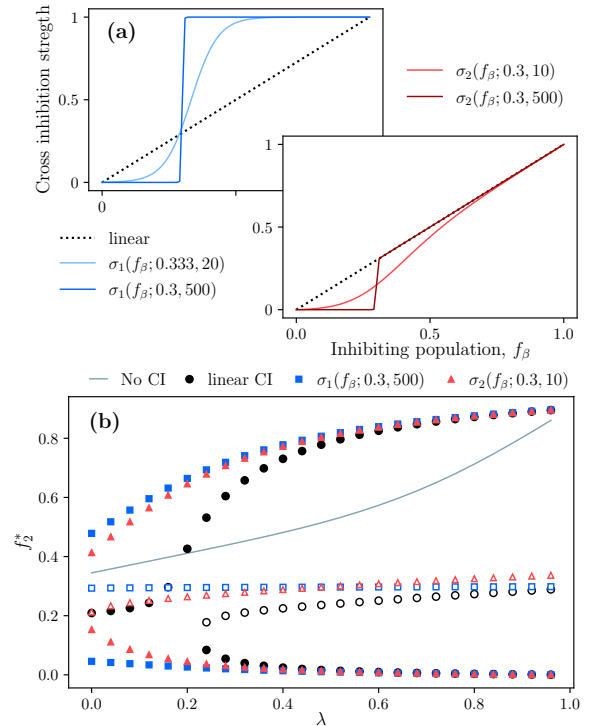


FIG. 1. (a): Examples of inhibitory responses (see Eq. 2). (b): Bifurcation diagrams on increasing interdependence  $\lambda$  for linear (black circles), sharp sigmoid  $\sigma_1(f_\beta; 0.3, 500)$  (blue squares), and smooth bounded sigmoid  $\sigma_2(f_\beta; 0.3, 10)$  (red triangles) cross-inhibition responses. The case  $\lambda' = 0$  is included as a continuous line for comparison. Other model parameters are  $\pi_1 = \pi_2 = 0.1$ ,  $q_1 = 9$ ,  $q_2 = 10$ .

of cross-inhibition interactions. For the purposes of this study, we will set  $\lambda' = 1$  in the following.

The stationary points of the system can be determined by numerically solving the equations obtained by setting  $\dot{f}_\alpha(t) = 0$  [32]. Without cross-inhibition ( $\lambda' = 0$ ), the system simplifies to the expressions derived in [31], thoroughly analyzed in [32, 33].

*Cross-inhibition response* – The function  $\sigma(f_\beta)$  determines the actual strength of the cross-inhibition based on the fraction of bees  $f_\beta$  sending stop signals. Traditionally, cross-inhibition, similarly to recruitment interactions, has been modeled as a proportional response to the fraction of adversary population, i.e.  $\sigma(f_\beta) = f_\beta$ . Here we consider a non-linear cross-inhibition response. Specifically, we propose two sigmoid-like test functions, where the cross-inhibition strength remains weak for small values of the inhibiting population:

$$\begin{aligned} \sigma_1(f_\beta; x_0, a) &= \frac{1}{1 + e^{-a(f_\beta - x_0)}}, \\ \sigma_2(f_\beta; x_0, a) &= \frac{f_\beta}{1 + e^{-a(f_\beta - x_0)}}. \end{aligned} \quad (2)$$

The parameter  $a$  controls the steepness of these functions, and  $x_0$  is a threshold controlling the sigmoid's ascent position. Some instances of these functions are depicted in

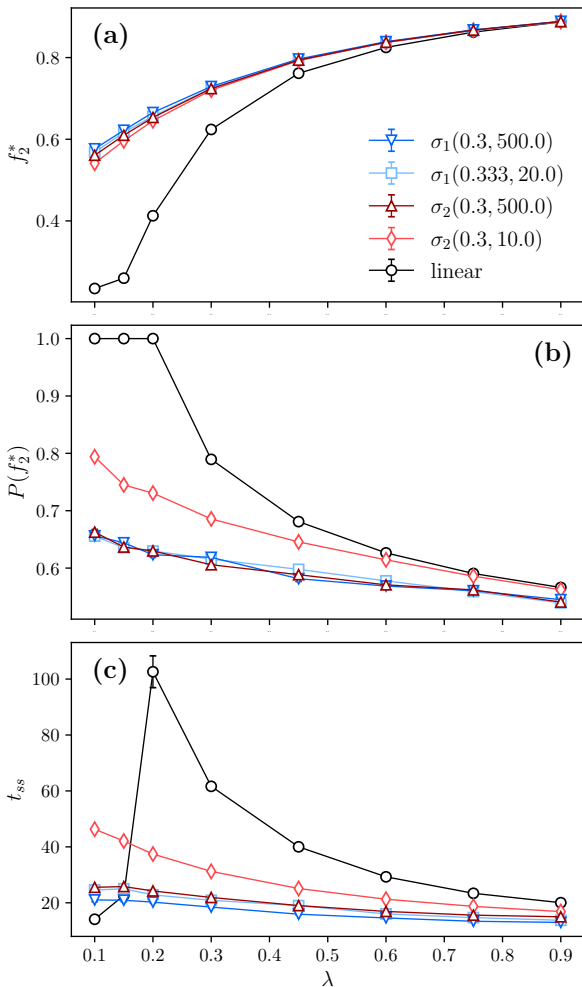


FIG. 2. Linear vs. non-linear cross-inhibition results in (a) binary) decision scenario. (a): Occupation fraction for the best-quality site,  $f_2^*$ . (b): Probability of reaching the best option,  $P(f_2^*)$ . (c): Time to settle into the stationary state,  $t_{ss}$ . Other parameters are  $\pi_1 = \pi_2 = 0.1$ ,  $q_1 = 9$ ,  $q_2 = 10$ .

Fig. 1(a). The function  $\sigma_1$  captures the scenario where the cross-inhibition strength increases sharply, similar to a step function, once the threshold population  $x_0$  is approached. On the other hand,  $\sigma_2$  assumes that the cross-inhibition strength grows sub-linearly below this threshold and transitions to a limiting linear regime above it.

In the following, we will focus on the simplest case of a binary decision between two sites that differ in quality ( $q_1 < q_2$ ). The system's dynamics display a different number of stable points for different values of the model's parameters. For instance, increasing the strength of the social interactions leads to an (unfolded) pitchfork bifurcation [34]. This behavior is shown in Fig. 1(b) for linear cross-inhibition (black-circles), and has been previously observed in similar models [10–12]. When switching to non linear cross-inhibition, a bifurcation still occurs, but its position depends on the particular response choice. Two examples of this are also shown in Fig. 1(b). The curves with blue squares and

red triangles represent results for a sharp sigmoid function  $\sigma_1(f_\beta; 0.3, 500)$  and a smooth linearly-bounded sigmoid function  $\sigma_2(f_\beta; 0.3, 10)$ , respectively. Results obtained for other non-linear functions displayed in Fig 1(a) are shown in Supplemental Figure SF1 [35]. Reducing the strength of cross-inhibition, or slightly varying the threshold parameter  $x_0$ , produces qualitatively similar results, though the positions of the bifurcations are shifted. For bifurcation plots at  $\lambda' = 0.5$ , see Supplemental Figure SF2 [35].

*Performance measure* – To assess the model's performance with non-linear cross-inhibition interactions, we numerically evaluate the stationary fixed point values, focusing on the occupation fraction for the best-quality site,  $f_2^*$ . This quantity represents the decision accuracy of the system. However, as previously discussed, decision accuracy alone is not the only relevant variable a system seeks to maximize, especially in value-based decisions [22, 24, 36]. In scenarios where the available sites are similar in quality, it may be preferable to make a quick decision rather than spending a large amount of time to choose a slightly better site. Therefore, in addition to accuracy, we use agent-based stochastic simulations to measure two additional performance metrics: (i) the probability  $P(f_2^*)$  of reaching the best option; and (ii) the time  $t_{ss}$  required to settle into this stationary state. These complementary quantities provide a comprehensive evaluation of the system's decision accuracy and speed performance.

Figure 2 represents the behavior of these three quantities as a function of the interdependence  $\lambda$  for close values of the sites' qualities,  $q_1 = 9$  and  $q_2 = 10$ , for the different cross-inhibition responses. In the non-linear case, we use the same parameters as in Fig. 1. We can observe that all non-linear cross-inhibition functions tested outperform the linear cross-inhibition in terms of pure consensus accuracy. However, the linear approach provides a higher probability of selecting the better option. These differences are particularly relevant for small to moderate values of the interdependence. In this regime, not triggering cross-inhibition unless an option has gained some representation allows the system to build a stronger consensus, albeit at the risk of less reliably choosing the better option. Nonetheless, this comes with the benefit of making a decision in a much shorter time, as shown in Fig. 2(c). This can be a significant advantage when choosing between similarly valued options. As reported in [27, 37], quicker consensus can be achieved by allowing the system to first build sub-populations of comparable sizes before triggering competition between them. In those works, this is achieved by time-varying social interaction rates, including recruitment and cross-inhibition. In contrast, we propose a simpler time-invariant mechanism that weakens the perception of cross-inhibition signals unless they are received from a significant portion of the population. This approach allows both populations

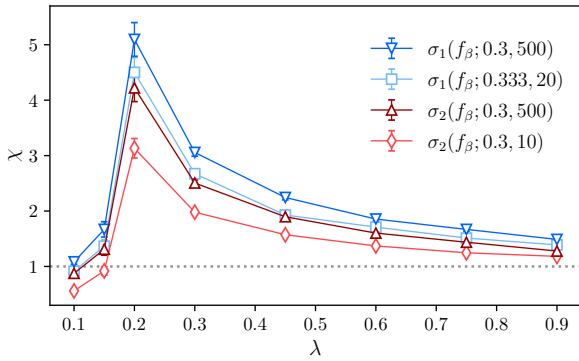


FIG. 3. Performance ratio  $\chi$  of non-linear cross-inhibitory responses on increasing interdependence  $\lambda$ . Other parameters are ( $q_1 = 9.0, q_2 = 10$ ) and  $\pi_1 = \pi_2 = 0.1$ .

to grow without interference from stop signals. Once the populations reach substantial sizes, cross-inhibition is triggered, and a faster decision is made.

Each type of sigmoid function is tested with both a sharp response (high  $a = 500$  value), where the cross-inhibition rapidly shifts from no effect to maximum or linear bound, and a smooth response (low  $a \in [10, 20]$  value), where the transition to the final bound is more gradual. Interestingly, our results for the best-quality site occupation fraction show a remarkable insensitivity to the specific details of the sigmoid functions (see Fig. 2(a)). Moreover, these results are significantly higher for low and moderate interdependence compared to those of the linear cross-inhibition model. On the other hand, the probability of reaching the best option is considerably reduced for sharper cross-inhibitory responsiveness, independently of the function selected, Fig. 2(b). This is due to the indiscriminate action of inhibition on the option that first reaches the activation threshold  $x_0$ , irrespective of its quality. While the smooth sigmoid also yields probabilities similar to the sharp functions, due to the over-representation of the inhibiting population when the threshold is trespassed, approaching smoothly the linear bound grants an intermediate result.

In order to encapsulate the effect of these three measures in a single quantity we define the objective performance,

$$\psi_\sigma = \frac{f_2^* P(f_2^*)}{t_{ss}}, \quad (3)$$

weighting the three quantities at stake. To assess how non-linear cross-inhibition compares to linear cross-inhibition, we also introduce the performance ratio  $\chi = \psi_\sigma / \psi_{lin}$ . Fig. 3 depicts this performance ratio for the same quality pair as in Fig. 2. We observe a performance ratio  $\chi > 1$  for nearly all values of  $\lambda$ . Moreover, as depicted in Supplemental Figure SF3 [35] this ratio increases as the site qualities become closer, indicating a more significant performance improvement when using non-linear cross-inhibition.

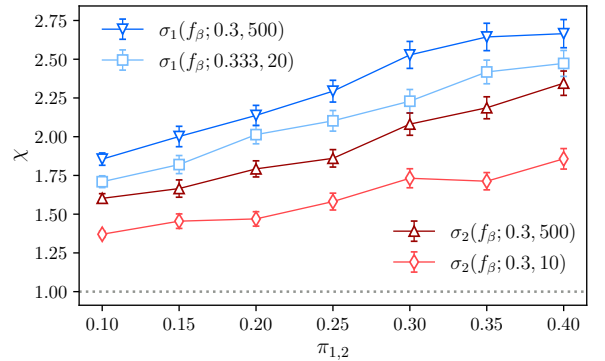


FIG. 4. Performance ratio  $\chi$  of non-linear cross-inhibitory responses on increasing the discovery probabilities  $\pi_1 = \pi_2 \equiv \pi_{1,2}$ . Other parameters are  $q_1 = 9, q_2 = 10$  and  $\lambda = 0.6$ .

The performance improvement peaks around  $\lambda \sim 0.2$ , corresponding to the point where the difference in decision times between linear and nonlinear models is the greatest. The subsequent performance decrease, on increasing  $\lambda$ , is due to the combined effect of interdependence and cross-inhibition driving the losing population to very low fractions, while the winning population dominates. In this scenario, the cross-inhibition strength exerted by the winning population on its adversary becomes similar to that in the linear model, regardless of the specific non-linear response chosen. The advantage of the non-linear response is mainly due to the weaker effect of the losing population's cross-inhibition. Furthermore, as noted in [32], when  $\lambda \rightarrow 1$ , the system can make a strong decision without cross-inhibition, although incorporating it significantly reduces decision time.

Comparing different non-linear cross-inhibition functions, we find that their performances are relatively close, with the smooth, linearly bounded sigmoid being the only one that underperforms. The effectiveness of a strong, sudden activation of cross-inhibition was previously reported by Talamali et al. [27], mainly for the choosing accuracy. In contrast, our approach demonstrates a comprehensive enhancement in both overall consensus accuracy and decision time.

So far, we have tested consensus dynamics by fixing the discovery probabilities and varying the interdependence. Increasing interdependence reduces the amount of individual exploration by prioritizing peers' options. This strategy has been shown to optimize consensus accuracy, even in the absence of cross-inhibition [30, 32], although it may extend decision time [32]. When the discovery probabilities increase (with fixed  $\lambda$ ), the system more readily incorporates environmental information. This reduces decision time but leads to poorer final consensus, especially when options are of similar quality [32]. Thus,  $\pi_\alpha$  can also be viewed as a *noise* parameter. In such scenarios, cross-inhibition is crucial to avoid deadlocks, as reported in various case studies [10, 11, 20, 22].

Fig. 4 shows the performance ratio of non-linear cross-

inhibition responses as discovery probabilities increase. The corresponding performance variables are plotted in Supplemental Figure SF4 [35]. As the *noise* in the system increases, non-linear cross-inhibition yields better performance. Interestingly, examining the individual quantities  $f_2^*$  and  $t_{ss}$  on increasing  $\pi_1 = \pi_2 = \pi_{1,2}$ , we observe opposing trends for the linear and non-linear model. The linear response yields decreasing  $f_2^*$  while increasing  $t_{ss}$ ; in contrast, non-linear responses reverse this trend. Consequently, weakening the stop signals from the losing population becomes essential in this context. Comparing the different non-linear responses, we observe that the performance is consistently higher for the standard sigmoid functions than for the linearly bounded sigmoids. In each case, the sharp response also grants better performance.

*Conclusions* – In this letter, we investigate non-linear cross-inhibition interactions in decentralized decision making models inspired by house-hunting honeybees. The primary design goal is to weaken an individual’s response to stop signals when they are received from a small fraction of the population. We model this behavior using two non-linear functions, tested with different parameters (Fig. 1). Focusing on binary decision tasks, we demonstrate that non-linear cross-inhibition results in higher consensus (the fraction of the population committed to the chosen option) and quicker decisions. These two benefits come at the cost of reducing accuracy in reliably choosing the best quality option. Nonetheless, in decisions made among options with close qualities, a stronger and quicker decision for a “good enough” option may be more beneficial than a weaker consensus or a slower decision process that yields the absolute best option [22, 24, 36]. Our results thereby open promising avenues for future research in decentralized collective decision-making and practical applications in swarm robotics.

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