Parametrically amplified super-radiance towards hot big bang universe

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We propose a mechanism of preheating stage after inflation, using a new idea of parametrically amplified super-radiance. Highly coherent state, characterized by macro-coherence of scalar field coupled to produced massless particle in pairs, is created by parametric resonance effects associated with field oscillation around its potential minimum, within a Hubble volume. The state is described effectively by the simple Dicke-type of super-radiance model, and super-radiant pulse is emitted within a Hubble time, justifying neglect of cosmic expansion. Produced particles are shown to interact to change their energy and momentum distribution to realize thermal hot big bang universe. A long standing problem of heating after inflation may thus be solved. A new dark matter candidate produced at the emergence of thermalized universe is suggested as well.

Introduction Inflationary universe scenario solves outstanding problems that face the beginning of hot big bang, notably flatness and horizon [1] [2]. At the end of inflation there is essentially nothing left except a causally connected large space-time. The inflation thus requires further a mechanism that connects it to hot thermal universe dominated by essentially massless particles, fostering important events of generating the baryon asymmetry and nucleo-synthesis.

A standard mechanism towards hot big bang after inflation is copious particle production due to parametric resonances towards the end of oscillatory phase of scalar field that governs inflation. Despite of many interesting theoretical attempts and numerical simulations, [3], [4], [5], [6], [7], [8], [9], there is no convincing picture of how a hot big bang is realized after inflation [10].

Our present work clarifies the presence of highly coherent state characterized by macro-coherence [11]. This coherence is realized by induced polarization of massless particle pairs due to parametric resonance, and super-radiant emission of massless particle pairs releases nearly all stored energy at an instant after macro-coherence is fully developed. The macro-coherence works in the entire Hubble volume, hence much more efficient than the usual Dicke super-radiance limited by wavelength of emitted photon [12], [13], [14]. Momentum distribution of produced massless particles at this stage of preheating is far away from thermal distribution, and re-distribution of momenta is shown to occur by interaction among produced particles, which require conditions readily realizable.

Throughout this paper we use the unit of $\hbar = c = k_B = 1$.

Inflaton field and its coupling to matter
While our results may be applied to a broader class of models, we focus on essential features of the present work, and specify the Lagrangian of the scalar field χ, called inflaton, and its coupling to the matter Lagrangian, $\mathcal{L}_m$, of the Standard Model or Grand Unified Theory (necessary if lepto- or baryo-genesis is to be implemented). In doing so, we resort to the principle of conformal coupling of χ to the trace of the energy-momentum tensor determined by $\mathcal{L}_m$. Its simplest realization is provided by Jordan-Brans-Dicke gravity [15] of the Lagrangian form, $-\sqrt{g_J} M_{\rm P}^2 F(\chi)\, R_J/2$, (with $M_{\rm P}$ the reduced Planck mass), transformed by Weyl rescaling, $g_{J\mu\nu} = g_{\mu\nu} F^{-1}(\chi)$, to the Einstein metric frame. We further add a potential term for inflaton, extending the original theory [15], thus called extended Jordan-Brans-Dicke gravity (eJBD) [16]. eJBD field appears naturally from quantum gravity theory of super-string in higher dimensional space-time. Our field-redefined form of Lagrangian in the Einstein metric is thus

$$\mathcal{L} = \sqrt{-g}\left(-\frac{M_{\rm P}^2}{2} R + \frac{1}{2}(\partial\chi)^2 - V_{\rm HO}(\chi) + F^2(\gamma\chi)\mathcal{L}_m(\varphi, g_{\mu\nu}F^{-1}(\gamma\chi)) \right) , \qquad (1)$$

with φ generically denoting matter fields.

The potential $V_{\rm HO}(\chi)$ is harmonic oscillation (HO) approximation to a more complicated inflaton potential V_χ, on which we elaborate later. The coupling constant γ of mass dimension -1 is taken of order $1/M_{\rm P}$, hence scalar-matter field coupling is of gravitational strength.

Since at early epochs right after inflation all particle masses in the Standard Model effectively vanish, direct coupling to the matter fields are absent in the trace of energy-momentum tensor at the classical level. There is however quantum correction, known as the trace anomaly, giving rise to [17]

$$-\gamma\chi\, T^{\mu}_{\mu}\,, \quad T^{\mu}_{\mu} = -\frac{b_i\alpha_i}{16\pi}(F^i_{\rho\sigma})^2\,, \tag{2}$$

with $\alpha_i = g_i^2/4\pi$. Here b_i's are coefficients of the renormalization group β-functions. For instance, $b_{\rm em} = 11/3$ for the photon. Matter fields that enter in this trace anomaly are squared field strength of electroweak gauge bosons, QCD gluons in standard model and plus lepton- and baryon-number violating gauge bosons in grand unified theories, all denoted by the index i. We ignored coupling to fermion pairs since parametric resonance effect is negligible due to the Pauli blocking, and further omitted possible longitudinal Higgs boson contribution for simplicity.

Relevant energy scale for the use of the trace anomaly is at the expected reheat temperature $\sim 10^{16}$GeV, hence the renormalization group analysis for $g_i^2/4\pi$ is needed. We shall take the popular unified coupling $\alpha_{\rm G} \sim 1/35$ in SO(10) grand unified models [18], [19]). SO(10) fermion (including right-handed neutral lepton N_R) loop contributions give

$$\sum_i \frac{b_i\alpha_i}{16\pi} \equiv c_t\frac{\alpha_{\rm G}}{16\pi} \sim -0.014\,, \tag{3}$$

to be multiplied by a common squared gauge field strength $F^2_{\rho\sigma}$.

Great merits of this eJBD approach [20] are (1) a cosmological constant present in the Jordan-frame may be suppressed in the Einstein frame by a judicious choice of $F(\chi)$ [21], thus evading its fine-tuning problem, and (2) extension of two field eJBD theory may lead to simultaneous solution to inflation and accelerating universe near the present epoch.

The rest of our analysis holds in a more general class of models that give inflaton coupling to the trace of energy-momentum tensor.

Macro-coherent state due to parametric amplification At the end of inflation, inflaton field χ starts to oscillate around its potential minimum which is well described in the harmonic oscillator (HO). HO approximation gives in the flat FLRW metric a damped oscillation due to the Hubble friction $\sim 3H\chi'$ in the $\chi-$ evolution equation, which takes a simple sinusoidal form $\chi_0\cos(m_\chi t)$ within the Hubble time $1/H$, where $\chi' \equiv d\chi/dt$. We shall confine, for the moment, to the case within Horizon scales.

Due to the space-translational invariance of background inflaton field, the quantum system of gauge bosons may be Fourier-decomposed into momentum $\vec{k}-$modes and treat each mode independently.

Gauge fields in the radiation gauge of $A_0 = 0, \vec{\nabla}\cdot\vec{A} = 0$ satisfy

$$\frac{d^2}{dt^2}\vec{A}^i_k + k^2\vec{A}^i_k + \frac{K_i\chi'}{1+K_i\chi}\frac{d}{dt}\vec{A}^i_k = 0\,, \tag{4}$$

$$K_i = \gamma\frac{b_i\alpha_i}{4\pi}\,, \tag{5}$$

neglecting small tri-linear coupling terms for non-Abelian gauge fields. This equation is recast to, by denoting two components of vector fields, $\vec{D}^i_k = (1+K_i\chi)^{-1/2}\vec{A}^i_k$ by a single u_k,

$$\frac{d^2}{dt^2}u_k + k^2 u_k - \frac{K_i}{2}\chi'' u_k = 0\,, \tag{6}$$

with the approximation $\frac{K_i\chi''}{1+K_i\chi} \approx K_i\chi''$ and dropping higher order terms $3\left((K_i\chi')/(1+K_i\chi)\right)^2/4$. When $\chi(t)$ is of the sinusoidal form in the HO region, the differential equation for u_k has a periodic function $\chi(t)''$ as a coefficient.

The standard form of Mathieu equation [23] is defined by scaling shifted time and dimensional parameters in (6), and is written as

$$\left(\frac{d^2}{d\tau^2} + (a - 2q\cos 2\tau)\right)u_k = 0\,, \tag{7}$$

$$\tau = \frac{m_\chi}{2}(t+\pi)\,, \quad a = \frac{4k^2}{m_\chi^2}\,, \quad q = K_i\chi_0\,. \tag{8}$$

χ_0 is the initial amplitude of sinusoidal function $\chi(\tau)$. The energy k of massless gauge bosons is equal to the half of the parent $\chi-$particle at rest. This gives the parameter in the Mathieu equation; $a = 1$.

Solutions of Mathieu equation (7) are characterized in terms of band structure in the (q,a) plane where bounded Bloch-type solutions (important in condensed matter physics when time is replaced by spatial coordinate) and exponentially unstable solutions alternate with their boundaries given by $a(q)$. The exponential growth is called parametric amplification in our terminology, also called broad resonance in other literatures. The small amplitude region of $q \to 0$ gives discrete unstable narrow bands around $a \sim n^2\,, n = 0,1,2,\cdots$, and is interpreted as a collapsed n-body χ decay; $n\chi \to A_k + A_{-k}$ [24]. On the other hand, the largest unstable bands are in the large $q-$amplitude with $a \le O(2)q$. The band structure and detailed behaviors of solutions are semi-analytically or numerically analyzed as in [23]. Mathematical softwares are available for unstable Mathieu solutions as well.

The phase coherence of produced gauge fields is best studied in the Schroedinger picture of quantum gauge system introduced in [22]. We use wave functional of each momentum-mode $\Psi(q_k;t)$ that satisfies

$$i\frac{\partial}{\partial t}\Psi(q_k;t) = -\frac{1}{2}\frac{\partial^2}{\partial^2 q_k}\Psi(q_k;t) + \frac{k^2}{2}q_k^2\Psi(q_k;t) \tag{9}$$

The total wave functional is the direct product of each momentum-mode. Variable q_k corresponds to momentum decomposed inflaton χ field and may be regarded as HO coordinate.

The gaussian ansatz for variable q_k of the wave functional leads to

$$\Psi(q_k;\tau) = \frac{1}{|u_k|}\, \exp[-\frac{q_k^2}{2|u_k|^2}\left(\pi - \frac{i}{2}\frac{d}{d\tau}|u_k|^2\right)](10)$$

where u_k is proved to satisfy the Mathieu equation of type (7) written in terms of rescaled time $\tau = (m_\chi t + \pi)/2$. This is exact result when the Hubble friction can be neglected in the HO region. When the parameter (a, q) of solution u_k belongs to a instability band, magnitudes of the gaussian width $\sim |u_k|^2$ of the wave functional $|\Psi(q_k;\tau)|^2$ exponentially grows in time, indicating copious gauge boson production.

A striking feature of wave functional is a constant phase given by $d\ln|u_k|^2/d\tau/2$ when the Mathieu solution is purely exponentially increasing within instability bands. The phase thus determined may differ in different momentum-mode of $\vec{k}$, but the Mathieu chart has degeneracy in direction of vector $\vec{k}$, hence modes of the same vector magnitude have a common phase.

The exponential growth does not continue indefinitely due to dissipative decay of parent field χ. Let us discuss the back reaction of particle pair production against the χ system. The effective lagrangian $\mathcal{L}_{\rm parametric}$ should contain dissipation term due to parametrically amplified two-particle decay and quantum fluctuation $\langle A_k^2\rangle$ necessarily related to the dissipation due to the fluctuation-dissipation theorem of statistical mechanics. The effective lagrangian generally takes the form,

$$\mathcal{L}_{\rm parametric} = \int \frac{d^3k}{(2\pi)^3} \left(\frac{1}{2}\Gamma_k^{\rm R}(\chi)\chi\chi' + c_t\frac{\alpha_{\rm G}}{16\pi}\,\gamma m_\chi^2\,\langle A_k^2\rangle(t)\chi\right)\,. \quad (11)$$

Quantum fluctuation is typically related to Rayleigh dissipation $\Gamma_k^{\rm R}(\chi)$ [25] by $\langle A_k^2\rangle(t) \propto e^{\Gamma_k^R t}$.

Lagrangian of Rayleigh dissipation $\propto \chi\chi'$ is necessarily time-reversal violating, while quantum fluctuation generates a kind of induced polarization $\propto \langle A_k^2\rangle$ in medium within the system initially dominated by χ field alone. In this $\sim \chi A_k^2$ interaction model the induced polarization acts as a time dependent external force to the χ system. The coherence of produced gauge bosons for definite momentum-pair mode $(\vec{k}, -\vec{k})$ is maintained during the parametric amplification within instability bands.

Rayleigh dissipation rate Γ_k^R is given by the decay rate in perturbation theory times the enhancement factor $\Delta k/m_\chi$, which is calculated to give

$$\Gamma_k^R = \Gamma_B\frac{\Delta k}{m_\chi}\,, \quad \Gamma_B = (c_t\frac{\alpha_{\rm G}}{16\pi})^2\,\frac{\gamma^2 m_\chi^3}{2}\,, \quad (12)$$

with $\Delta\chi \gg m_\chi$ the field dependent width of relevant instability band, which may be numerically estimated.

We note on how parametric amplification may affect the inflationary stage. A large exponential amplification and copious particle production require a sizable oscillation amplitude in HO region. Outside HO region the potential form may differ. We shall take a single field version of potential advocated in [20], which reads as

$$V_\chi = V_0\,(\lambda_0 + c\chi^2)\,e^{-\gamma\chi}\,, \quad \lambda_0 > 0\,, \quad c > 0\,. \; (13)$$

Inflation in this potential occurs slightly below a potential maximum around $2/\gamma$ (assuming a small λ_0), but much above the HO region around $\lambda_0\gamma/2c$. Hence, in the early phase of inflation there is no substantial effect of parametric amplification. It is true however that chaotic type inflation of quadratic potential is much affected by parametrically amplified super-radiance.

Application of Dicke super-radiance model In order to make relation of parametric amplification to super-radiance clearer, we extend the density matrix extending given in [22]. The bilinear projection operator $|\Psi(q_k;\tau)\rangle\langle\Psi(q_k;\tau)|$ onto pure quantum states of [22] is first expanded in terms of most convenient basis, for which we take the HO energy eigen-states $|n\rangle \sim (a^\dagger)^n\,|0\rangle\,, \langle q_k|0\rangle \propto e^{-m_\chi q_k^2/2}$. $a, a^\dagger$ are annihilation and creation operators of HO and $|0\rangle$ is the ground state of zero-point energy. The off-diagonal matrix elements, $\langle n|\Psi(q_k;\tau)\rangle\langle\Psi(q_k;\tau)|n'\rangle\,, n \neq n'$, rapidly oscillates in time, and its short time average nearly vanishes. Thus, a classical system of finite non-vanishing entropy emerges.

The total number of χ particles within the Hubble horizon is

$$\mathcal{N}_\chi = H^{-3}n_\chi \sim H^{-3}\frac{\rho_\chi}{m_\chi}\,, \quad (14)$$

with n_χ, ρ_χ the number and the energy densities of χ particles. In HO region the non-relativistic relation $\rho_\chi = n_\chi m_\chi$ is valid.

The state after parametric resonance effects are fully developed is well described by Dicke model based on the algebra of angular momentum [12], [13], [14]. There are however crucially important differences: (1) There are only two states in the Dicke model, linear combinations of the ground and some excited state, while ours has states of order $M_{\rm P}/m_\chi$, typically as large as $O(1 \sim 10^5)$, (2) inflaton decay products in our model are correlated two massless particle pairs unlike a single photon in the Dicke case.

In the original Dicke model of super-radiance it is supposed that laser irradiation generates a coherent state within the light wavelength region, the region limitation caused by a common phase factor of incident radiation $e^{i\vec{k}\cdot\vec{x}}$ with $k \sim 1/$wavelength. Our case of pair production has the vanishing phase $e^{i(\vec{k}-\vec{k})\cdot\vec{x}} = 1$ for relevant back to back pair emission.

The enhanced coherence without the wavelength limitation was theoretically suggested and is called macro-coherence [11]. The macro-coherent two-photon emission was experimentally verified in [26] for para-H_2 molecular transition, with rate enhancement of order 10^{15}.

Despite of these differences one can take over nice features of the Dicke model provided that the wavelength limitation of coherent region is lifted.

Phase development and enhanced super-radiant decay rate Following Dicke [12], one can introduce the algebra of the total angular momentum satisfying

$$[R_i\,, R_j] = i\epsilon_{ijk} R_k\,. \tag{15}$$

The value J of the total angular momentum is related to product $\mathcal{N}$ of the $\chi-$particle number and the number of levels $M_{\rm P}/m_\chi$; $\mathcal{N} = 2J+1 \approx 2J$. This product is further related to the χ energy within the Horizon volume, hence $\mathcal{N} m_\chi = H^{-3}\rho_\chi$. By using (14), we have

$$\mathcal{N} = \mathcal{N}_\chi \frac{M_{\rm P}}{m_\chi}\,. \tag{16}$$

Projection of angular momentum onto an axis has component values M ranging from $M = J$ to $M = -J$, hence the entire range being $\Delta M = 2J+1 \approx 2J$. Despite that more than two levels are introduced in our case excitation and de-excitation occur between two energetically adjacent levels, hence $\Delta M = \pm 1$. Hence one can use the lowering R_- operator for pair emission and the raising R_+ operator for pair excitation between $\Delta M = \pm 1$. This makes our case and the Dicke case essentially equivalent.

Different momentum $\vec{k}-$modes evolve independently and one must sum over these contributions in the end. But if a particular momentum mode evolution is predominantly fast, this mode alone may take over the entire evolution to dominantly realize hot thermal universe at the next stage.

The population probability denoted below by $P_M(t)$ of the state $|J, M\rangle$ follows the evolution equation

$$\frac{dP_M}{dt} = \Gamma_k^R \left(|\langle J, M+1|R^+|J,M\rangle|^2 P_{M+1} \right.$$
$$\left. -|\langle J, M-1|R^-|J,M\rangle|^2 P_{M-1} \right)\,. \tag{17}$$

supplemented by well-known relations, $|\langle J, M + 1|R^+|J,M\rangle|^2 = (\frac{N}{2} - M)(\frac{N}{2} + M + 1)$ etc. One can prove the law of probability conservation,

$$\frac{d}{dt}\sum_{M=-J}^{J} P_M = 0\,. \tag{18}$$

This is a coupled set of linear differential equations of a gigantic size $2J = \mathcal{N}_\chi$ (the quantity given by (16)). After the end of inflation, $\mathcal{N}_\chi$ number of χ particles evolve independently within the horizon, in particular they may decay at different times. One needs to match the stage of parametric amplification to the super-radiant stage smoothly.

One may assign HO state $|n\rangle$ of gauge bosons to each χ particle for this purpose. Due to energy exchange of χ and gauge boson, one can derive the relation $n = J - M$. The initial state in the Dicke picture may be identified as distributed $M-$states by the relative weight of density matrix elements $\propto \rho_k\,(M, M; \tau = 0)$,

$$|J, M\rangle_i = \left(\sum_{M'} (\rho_k(M', M'; 0)\right)^{-1/2}$$
$$\sqrt{\rho_k\,(M, M; 0)}\,|J, M\rangle \equiv P_M(0)|J,M\rangle\,. \tag{19}$$

When the parametric amplification is fully developed, density matrix elements $\rho_k\,(M, M; 0)$ are calculable.

A systematic method to determine the weight factors $\rho_k\,(M, M; 0)$ in (19) shall be worked out in our subsequent longer paper. Here we are content to concentrate on qualitative features that arise from much simplified initial state.

Since the fundamental Dicke equation (17) is a set of linear differential equations, one can decompose this linear system and later take linear combinations of decomposed components. That this procedure is useful is justified due to a common phase in $\rho_k\,(M, M; t_i)$ for each M. The simplest initial condition is the Dicke initial condition by $P_M(0) = \delta_{M,J}$, a single state for a specific J value which may be different from the estimated $J = \mathcal{N}/2$. This would clarify qualitatively how a collective decay of the χ system may occur depending on J.

Chain states connected by operation R^- satisfy permutation symmetry of different $\chi-$particles. Degeneracy increases each time a chain decay occurs. This may be interpreted as a further coherence development of the system.

The concept of delayed time in super-radiance [13], [14], needs to be stressed. The spontaneous decay governed by the exponential law $e^{-\gamma_B t}$ is characterized by the lifetime of averaged decay time $t_B = 1/\gamma_B$. In the parametrically amplified decay the perturbative decay rate γ_B is replaced by the Rayleigh dissipation rate Γ_k^R. On the other hand, the super-radiant decay depletes almost all excited states at what is called delayed time given by $\ln\mathcal{N}/\mathcal{N}\times$ the lifetime, which is much shorter than the lifetime of spontaneous decay for a very large $\mathcal{N} = 2J$. The delayed time t_d is given by

$$t_d = \frac{\ln\mathcal{N}}{\mathcal{N}}\,\frac{1}{\Gamma_k^R}\,. \tag{20}$$

The depletion leaves behind χ field corresponding to zero-point oscillation.

A simple estimate of delayed time is possible by taking the Hubble rate as $H = \Gamma_k^R$ in the formula of $\mathcal{N}$, (16), to

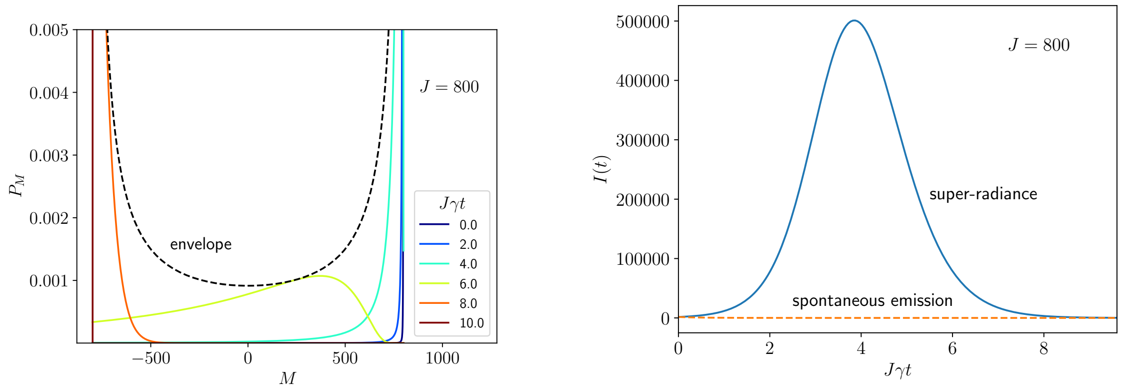

FIG. 1. **(left panel)** Time evolution of P_M as a function of M. The black dashed line labeled by envelope is the envelope of largest peak values at each M. **(right panel)** Time evolution of the radiation intensity, the one predicted by super-radiance in sold blue and by spontaneous emission in dashed red.

give a crudely estimated quantity,

$$\mathcal{N} \sim (\frac{16\pi}{|c_t|\alpha_{\rm G}})^6 \frac{M_P \rho_\chi}{m_\chi^8 \Delta\chi^3 \gamma^6} \sim 4\times 10^7 (\frac{M_{\rm P}}{m_\chi})^8 \,, \quad (21)$$

highly sensitive to the much unknown m_χ value. The energy density ρ_χ is taken $(10^{16}{\rm GeV})^4$ from anticipated thermal energy that subsequently follows. Other adopted parameter values are $\gamma = 0.1/M_{\rm P}, \Delta\chi = M_{\rm P}$.

We illustrate in Fig(1) result of numerical integration of rate equations (17) under the Dicke initial condition. Similar numerical computation has been performed in [27]. As time proceeds, distribution of state changes in time scale of a fraction $O(1/J\gamma)$, which means after a few successive spontaneous emissions the phase coherence is developed. At the vicinity of $M \sim 0$, all the states are nearly degenerate in M, at which the pulse-like radiation is emitted as shown in the right panel. The radiation intensity is estimated by $I(t) = -\frac{d}{dt}\sum_{M=-J}^{M=J} MP_M(t)$, to give a result in the right panel of Fig(1). In this example of a modest value of $J = 800$, the delayed time occurs at $\sim 4/(800\times$the Rayleigh dissipation rate). As clear from the right panel of Fig(1), the macro-coherent super-radiance terminates copious particle production at times when spontaneous emission is hardly working.

The condition of neglecting Hubble expansion is justified if the inequality is satisfied; $\Gamma_k^R \mathcal{N}^2 > H$, incorporating macro-coherent amplified rate $\propto \mathcal{N}^2$. At the expected time scale of $H = \Gamma_k^R$, this condition is equivalent to $\mathcal{N}^2 > 1$, which is readily satisfied as indicated in the $\mathcal{N}$ estimate of (21).

The elegant Dicke's algebraic method is simple and convincing. Our parametrically amplified super-radiance has an extraordinary merit in that this simplicity may be applied to the entire universe within the horizon. We have only sketched how the parametric amplification prepares macro-coherent super-radiance without giving details of the initial state for the Dicke algebraic model. Going beyond this oversimplified approach is numerically demanding, but is worth of detailed study, for instance for the purpose of exploring the possibility of primordial black hole formation.

We note a striking difference of parametrically amplified super-radiance from laser irradiated super-radiance. Our case is a phenomenon of self-organized macro-coherence evolution by inflaton oscillation, unlike the other case of laser-triggered evolution. It would be of great interest if one can find this kind of self-organized phenomenon in laboratories.

Thermalization The state of universe within the Horizon volume right after macro-coherent super-radiance may be described by massless pairs of many different momenta $\vec{k}$ with different spread $\Delta^3 k$ (arising from regions deep in instability bands of the Mathieu chart) at different sites $\vec{x}$ of spread $\Delta^3 x$. This is far from the thermal distribution described by a single temperature. The universe is however spatially homogeneous, and particle momenta are randomly distributed. Thermalization may occur during the next stage of interaction among produced particles. A question now arises whether thermalization is fast enough, namely, whether thermalization rate is larger than the Hubble rate.

We shall check the consistency condition, $\Gamma > H$, taking first an average momentum T and confirming this to be regarded as a finite temperature. To estimate thermalization rate $\Gamma(T)$, let us take an example of Compton process given by total cross section $2\pi\alpha^2 \ln(s/m_e^2)/s$ in high energy limit of squared center-of-mass energy $s \gg m_e^2$. At finite temperatures the estimated cross section is of order, $\pi\alpha^2/T^2$. Multiplied by the photon number density $n_\gamma = 3\zeta(3)T^3/2\pi^2$, Compton rate at finite temperature is of order, $3\zeta(3)\alpha^2 T/2\pi$. There are as many as $O(N)$ colliding particles against, say, a fermion, which gives an effective rate,

$$\Gamma = \frac{3\zeta(3)}{2\pi} N\overline{\alpha^2} T \,. \tag{22}$$

This should be larger than the Hubble rate,

$$H = \frac{\pi}{3}\sqrt{\frac{N}{10}}\frac{T^2}{M_P} \,, \quad M_P = \frac{1}{\sqrt{8\pi G_N}} \sim 2.4 \times 10^{18}\,\text{GeV} \,. \tag{23}$$

The required inequality $\Gamma > H$ gives a condition,

$$T < \frac{9\sqrt{10}\,\zeta(3)}{2\pi^2}\sqrt{N}\,\overline{\alpha^2} M_P \sim 1.73\sqrt{N}\overline{\alpha^2} M_P \,. \tag{24}$$

We may take $N = O(100)$ in the Standard Model, $O(500)$ in SO(10) Grand-Unified-Theory models, and $\overline{\alpha^2} = O(10^{-4} \sim 10^{-3})$. The right hand side is expected to be slightly larger than 10^{16} GeV. This shows that thermalized universe of temperature $O(10^{16})$ GeV may be realized. It is fortunate that this temperature is sufficiently high such that subsequent baryo- or lepto-genesis may occur.

Dark matter candidate As a byproduct of the super-radiant decay of χ, a dark matter candidate naturally arises. Gauge boson pair at a site in our Dicke model forms a triplet due to the maximal symmetry of the angular momentum state [28]. Non-trivial topology may be defined by mapping the unit sphere in real sphere onto modulus of the 3-vector of triplet. It is more likely that topological objects are formed involving many different sites instead of a single site. Objects of finite discrete winding number of this mapping may survive till later epochs of cosmological evolution despite surface interaction of condensates with surrounding thermal medium. Remnant objects after recombination become a good candidate of dark matter. The dark matter thus constructed is made of $(\vec{k}, -\vec{k})$ modes belonging to stability bands of the Mathieu chart.

During thermal evolution after condensate formation at the end of inflation accretion of fermions may occur, and it is not clear what sort of constituents make up dark matter at recombination. Moreover, their masses may have a wide range. It is difficult at the moment to determine how to detect these dark matter. One cannot exclude the possibility that accreting condensates end with primordial black holes.

Unlike all other dark matter candidates considered in the literature dark matter objects in this scenario are produced simultaneous with emergence of the hot universe. Thus, more detailed investigation may make their cosmological relevance more evident.

Discussion and outlook During earlier phases of inflation the harmonic potential approximation is not valid and analysis based on the Mathieu equation needs to be modified for discussion in this stage. Non-harmonic pieces of the original potential, typically deviation from the eJBD potential, $\chi^2 e^{-\gamma\chi}$, introduces mode mixing and this may introduce phase de-coherence. The elegant technique introduced by Dicke is no longer applicable, and one has to set up the Maxwell-Bloch equation [14], [11] to deal with spatial de-coherence. The mode mixing is however present only in the earlier preheating stage, and one may forget about this de-phasing in the HO regime. If the phase coherence is fully developed in HO regime, our approach presented in this work should be approximately valid.

We assumed that inflation ends with parametrically amplified super-radiance leaving the scalar field at its potential minimum. It is not entirely clear whether remnant kinetic field energy is large enough to override a potential barrier in the eJBD type of potential. If the barrier crossover occurs, it may give a mechanism of transforming inflaton to dark energy quintessence field.

We estimated the reheat temperature and the validity of various approximations using the model of inflaton that couples to the trace of energy-momentum tensor. Other models of coupling can be dealt using the same method as given here, and these give different reheat temperature and their cosmology may be different.

Investigation of these and many other interesting problems such as new scenario of lepto-genesis are left to future works.

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