

APPLYING AUTOMATIC DIFFERENTIATION TO OPTIMIZE DIFFERENTIAL MICROPHONE ARRAY DESIGNS

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ABSTRACT

This paper introduces a novel methodology leveraging differentiable programming to design efficient, constrained adaptive non-uniform Linear Differential Microphone Arrays (LDMA) with reduced implementation costs. Utilizing an automatic differentiation framework, we propose a differentiable convex approach that enables the adaptive design of a filter with a distortionless constraint in the desired sound direction, while also imposing constraints on microphone positioning to ensure consistent performance. This approach achieves the desired Directivity Factor (DF) over a wide frequency range and facilitates effective recovery of wide-band speech signals at lower implementation costs.

Index Terms— Differentiable programming, LDMA, Directivity factor.

1. INTRODUCTION

Derivatives play a vital role in machine learning, especially in optimization processes. Automatic Differentiation (AD) has revolutionized the computation of these derivatives, and its incorporation into machine learning frameworks as differentiable programming has opened new avenues for optimizing complex functions with greater efficiency and accuracy [1, 2, 3].

In the domain of acoustics and sound processing, differentiable programming has shown great promise. For instance, paper [4] introduced a differentiable DSP module-based synthesizer, significantly improving spectral similarity in sound-matching tasks. Extending this work, [5] utilized neural networks to optimize synthesis parameters for real-world sounds, while [6] developed the Differentiable DX7 model, enhancing both spectral optimization and audio quality. Lastly, [7] explores using differentiable methods to optimize room design for achieving desired acoustic outcomes.

Building on these advancements, we propose a novel method for designing constrained adaptive non-uniform LDMA using differentiable programming. Differential Microphone Array (DMA) design often involves a cumbersome multistage process. Our approach streamlines this by directly

optimizing array parameters under specific constraints, resulting in a more efficient and adaptive design process, validated through performance metrics.

Several studies have laid the groundwork for LDMA design. For example, [8] tackled the white noise amplification in wideband signals, improving performance with frequency-invariant beampatterns via Maclaurin's series expansion. Building on this, [9] provides a theoretical comparison, introduces a two-stage robust DMA beamforming approach to maximize white noise gain (WNG), addresses extra-null issues at high frequencies, and offers a solution for frequency-independent beampatterns. Similarly, considering key performance metrics like DF and WNG, [10] optimized LDMA geometry by dividing the frequency band into subbands and using particle swarm optimization, and expanding on this, [11] explored nonuniform LDMA by using spatial difference operators and a two-stage design process. Taking steering limitations into account, [12] addressed these challenges in the differential beamformers with linear microphone arrays, proposing new strategies for designing steerable beamformer. Similarly, [13] proposed a fully steerable design using both omnidirectional and bidirectional microphones, validated through simulations.

Our key contribution is introducing a method for designing optimal and fully steerable nonuniform LDMA using differentiable programming. Unlike previous approaches to LDMA design, we achieve the adaptive LDMA configuration by directly optimizing the cost function under specific constraints: 1) ensuring distortion-free signal reception, and 2) limiting the distance between microphones. This approach achieves the desired DF across different frequencies and outperforms existing methods in terms of cost and implementation time.

This paper is structured as follows: Section 2 defines the problem and presents our methodology. Section 3 discusses the simulation results, demonstrating the effectiveness of our approach. We conclude with a summary of our findings and their implications for future research.

2. SYSTEM MODEL AND PROBLEM DEFINITION

2.1. System Model of the Nonuniform LDMA

We consider a plane wave coming from a far-field sound source with the speed of $c = 340 \text{ m/s}$ encountering an adaptive nonuniform LDMA with M omnidirectional microphones. The distances between microphones are not uniform, with the distance between the m th microphone and $m + 1$ th microphone denoted δ_m for $m = 1, \dots, M - 1$. The position of each microphone can be adjusted to achieve the desired beampattern. Assuming the desired sound signal coming from the direction which is defined by the relative azimuthal angle θ_d where the main lobe is oriented towards that angle. In this context, the steering vector is as follows

$$d(w, \theta_d) = [e^{-jw\tau_1 \cos \theta_d} \ e^{-jw \sum_{i=1}^2 \tau_i \cos \theta_d} \ \dots \ e^{-jw \sum_{i=1}^{M-1} \tau_i \cos \theta_d}]^T \quad (1)$$

where the superscript^T denotes the transpose operator and j represents the imaginary unit., $w = 2\pi f$ is the angular frequency, and $\tau_i = \delta_i/c$ for $i = 1, \dots, M - 1$ is the delay between microphone i and microphone $i + 1$. Fig. 1 shows a non-uniform linear differentiable microphone array. As it can be seen, the delay between m th microphone and the first microphone is $\sum_{i=1}^{m-1} \tau_i$ which is the sum of the delays from the first microphone to the m th microphone and the respective phase difference would be $\sum_{i=1}^{m-1} w \frac{\delta_i}{c} \cos \theta_d = \sum_{i=1}^{m-1} w \tau_i \cos \theta_d$.

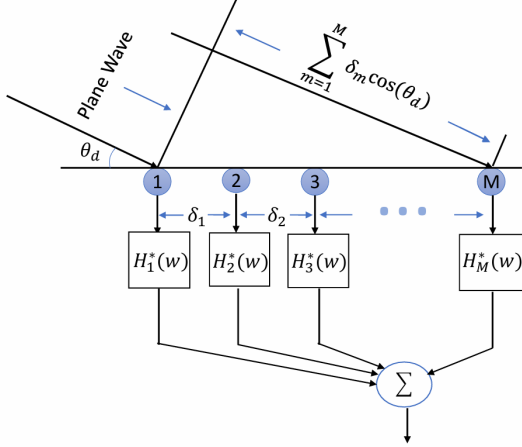


Fig. 1. An adaptive non-uniform linear microphone array.

2.2. Beampatterns

we adopt an adaptive filter design where the filter coefficients $H_m^*(w)$ (Note that the superscript $*$ represents the complex conjugate), interelement spacing δ_m , are programmatically determined. Our approach focuses on optimizing these two

variables to design adaptive non-uniform LDMA of varying orders using automatic differentiation to ensure that the beampattern of the proposed method aligns with the desired beampattern of the same order LDMA.

The weight $H_m^*(w)$ is applied to the output of the microphone m ; the complete set of these weights is represented in the form of a vector containing all individual weights, thereby defining the overall response of the microphone array to incoming sound waves in a vector as follows

$$h(w) = [H_1(w) \ H_2(w) \ \dots \ H_M(w)]^T \quad (2)$$

Beampattern or directional sensitivity, which represents the sensitivity of microphone arrays to the direction of the incoming sound, is an essential aspect of the array signal processing. It indicates how well the microphone array can capture or reject sound from various directions. Beampattern of the proposed nonuniform LDMA is mathematically defined as follows

$$\begin{aligned} \mathcal{B}_M[h(w), \theta] &= d^H(w, \theta)h(w) \\ &= \sum_{m=1}^M H_m(w) e^{\sum_{i=1}^{m-1} w \tau_i \cos \theta} \end{aligned} \quad (3)$$

where $d^H(w, \theta)$ is the steering vector and defined in Eq. 1. LDMA are known for their frequency-independent characteristics and the desired beampattern of the N th-order DMA is as follows:

$$\mathcal{B}_{d,N}[\theta] = \sum_{n=0}^N a_n \cos^n(\theta - \theta_d), \quad \text{where} \quad \sum_{n=0}^N a_n = 1. \quad (4)$$

where a_n for $n = 0, \dots, N$ are real numbers and θ_d is the steering direction of the LDMA.

2.3. Optimization Problem

The objective is to find the optimum weights and positions of the microphones in our beamforming system to minimize the mean squared error (MSE) between the desired beampattern and the proposed one. This optimization is subject to two constraints: 1) ensuring that the distance δ_m is within the range of δ_{\min} and δ_{\max} , and 2) ensuring that the desired sound signal from the direction θ_d is received without distortion. The MSE optimization problem is formulated as follows:

$$\begin{aligned} \min_{\delta_m, H_m(w)} \quad & E[|\mathcal{B}_{d,N}[h(w), \theta] - \mathcal{B}_M[h(w), \theta]|^2] \\ \text{s.t.} \quad & d(w, \theta_d)^H h(w) = 1 \\ & \delta_{\min} \leq \delta_m \leq \delta_{\max} \end{aligned} \quad (5)$$

where $d(w, \theta_d)$ is a steering vector (the source signal comes from the direction θ_d) defined in Eq.1, then the distortionless constraint at θ_d is as

$$d(w, \theta_d)^H h(w) = \sum_{m=1}^M H_m(w) e^{\sum_{i=1}^{m-1} w \tau_i \cos \theta_d} = 1.$$

Note: In LDMA, spatial processing is critical. To avoid spatial aliasing, it is essential that the interelement spacing, δ_m , is less than half the wavelength ($\lambda/2$). This constraint ensures that the system can uniquely determine the direction of incoming sound waves without ambiguity [8]. Accordingly, we constrain the maximum interelement spacing δ_{max} to be less than $\lambda/2$. In our computational model, we have the flexibility to set $\delta_{min} = 0$, allowing for enhanced adaptability in our system's configuration.

Considering the directivity factor as a way to evaluate the performance of the proposed method showing how well an LDMA directs a sound signal to a specific direction, it is computed as follows

$$DF = \frac{|\mathcal{B}_M[h(w), \theta_d]|^2}{\frac{1}{2\pi} \int_0^{2\pi} |\mathcal{B}_M[h(w), \theta]|^2 d\theta}. \quad (6)$$

The Mean Square Error (MSE) between two beampatterns is another way of evaluating the performance of our system and it is defined as:

$$\epsilon_{\mathcal{B}_M[h(w), \theta]} = \frac{1}{2\pi} \int_0^{2\pi} |\mathcal{B}_M[h(w), \theta] - \mathcal{B}_{d,N}[\theta]|^2 d\theta. \quad (7)$$

It shows how well the beampattern of the proposed method is close to the desired beampattern of an LDMA.

3. NUMERICAL RESULTS

Considering the simulation setup as the order of LDMA is $N = 2$, the number of microphones is $M = 5$, sound speed is $c = 340m/s$, $f = 1kHz$, $\omega = 2\pi f = 2\pi \times 1000 \approx 6,283$, $\delta_{min} = 0$, and $\delta_{max} = 15$ cm. The optimization problem has been solved in Python using JAX and scipy.optimize. The numerical optimization technique used for this constraint problem is the Sequential Least Squares Programming (SLSQP) algorithm which is part of the minimization function in the scipy.optimize module of the SciPy library in Python. We did the simulation for the cases when the desired sound signal coming from θ_d equal to $0, \pi/3$, and π , and the results have been shown in Fig. 2, Fig. 3, and Fig. 4, respectively. These figures demonstrate that, in each case, the beampattern produced by the proposed method aligns with the desired beampattern. Although there are a few misfits between the proposed beampattern and the desired one in some frequencies, since it is highly dependent on the initial guess for the optimization parameters, $H_m(w)$ and τ_m , by adjusting them properly, we can achieve the desired performance with the proposed method. Fig. 5 and Fig. 6 show the MSE and the DF for the second order LDMA versus frequency from 0 to 4kHz, respectively. We can see from them that the DF for the proposed method is close to the DF of the desired beampattern with the error rate around 0.01 and the MSE is less than 0.005. Fig. 7, Fig. 8 and Fig. 9 show that the beampattern of

the proposed method compared with the desired one for the third order LDMA when θ_d is equal to $0, \pi/3$ and π , respectively. Setting $N = 3$ and $M = 4$, we can see even when M takes its smallest value which is equal to $N + 1$, we have the desired performance; therefore, considering the cost of implementation, it is suggested to use $M = N + 1$ and doing the optimization for that.

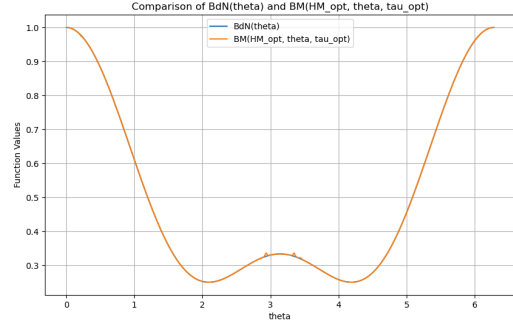


Fig. 2. Desired beam-pattern and the beam-pattern of the proposed method for the second order LDMA is compared versus incident angle when the angle of arrival is $\theta_d = 0$.

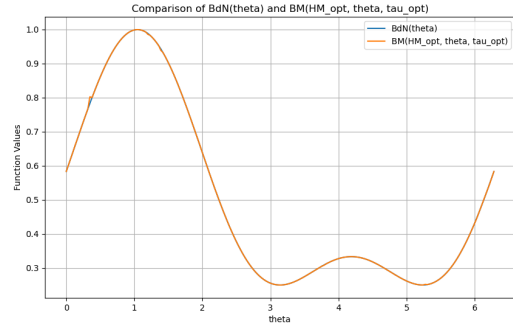


Fig. 3. Desired beam-pattern and the beam-pattern of the proposed method for the second order LDMA is compared versus the incident angle when the angle of arrival is $\theta_d = \pi/3$.

4. CONCLUSION

This paper proposes a novel approach based on differentiable programming for designing constrained adaptive non-uniform LDMA of any desired order. The optimization problem is solved by considering two variables: filter coefficients and microphone positions. This approach is significantly more efficient in terms of time and effort required for implementation. Using DF and MSE as the performance evaluation metrics, we demonstrate that the beampatterns of the second-and

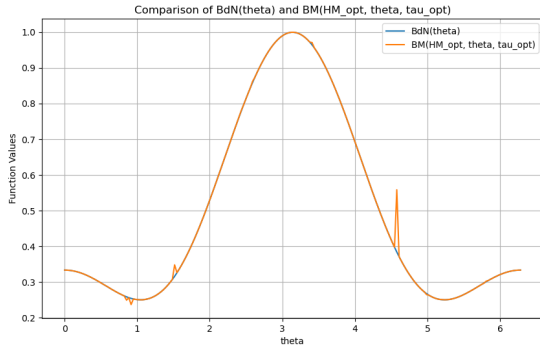


Fig. 4. Desired beam-pattern and the beam-pattern of the proposed method for the second order LDMA is compared versus the incident angle when the angle of arrival is $\theta_d = \pi$.

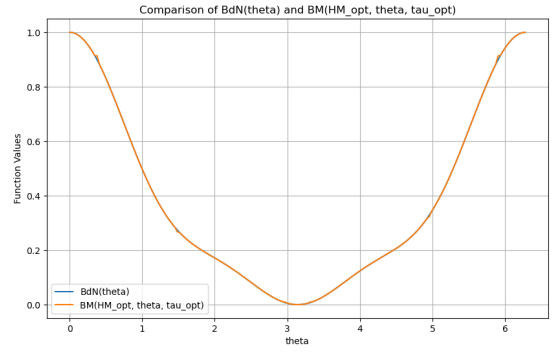


Fig. 7. Desired beam-pattern and the beam-pattern of the proposed method for third order LDMA is compared versus the incident angle when the angle of arrival is $\theta_d = 0$.

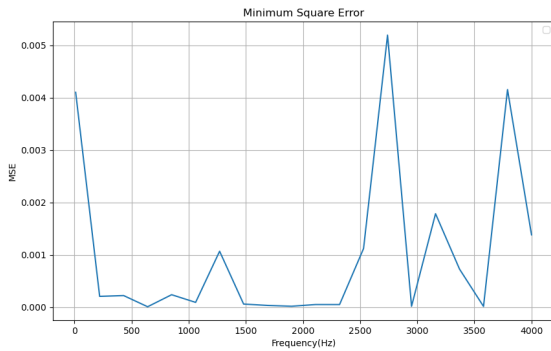


Fig. 5. Minimum mean square error of the proposed method for the second order LDMA versus frequency when the angle of arrival is $\theta_d = \pi$.

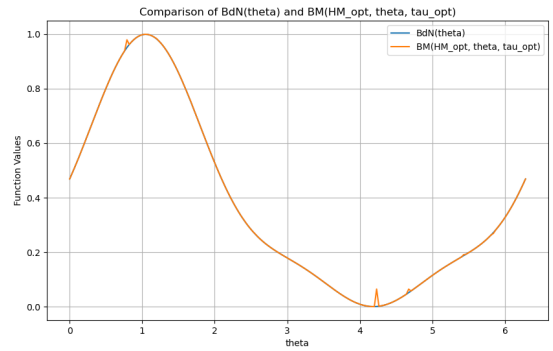


Fig. 8. Desired beam-pattern and the beam-pattern of the proposed method for the third order LDMA is compared versus the incident angle when the angle of arrival is $\theta_d = \pi/3$.

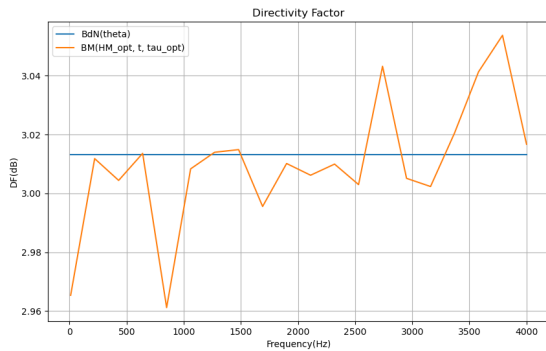


Fig. 6. Directivity Factor of the proposed method is compared with the desired one for the second order LDMA versus frequency when the angle of arrival is $\theta_d = \pi$.

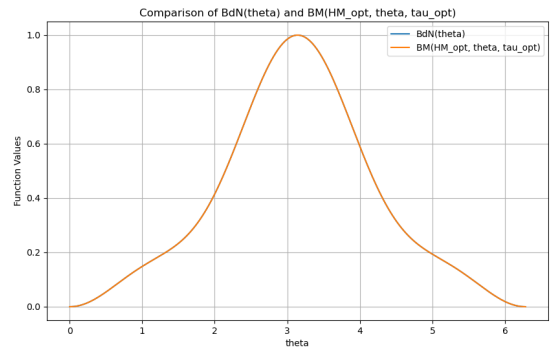


Fig. 9. Desired beam-pattern and the beam-pattern of the proposed method for the third order LDMA is compared versus the incident angle when the angle of arrival is $\theta_d = \pi$.

third-order LDMA's produced by the proposed method closely match the desired beam patterns with the respective orders.

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