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# Some statistical properties of spiral galaxies

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**Abstract.** This paper presents some statistical correlations of 72 northern spiral galaxies. The results show that early-type spirals that are brighter, and thicker, and the axis ratios ( $H_z/H_r$ ) of the disk tend to be smaller along the Hubble sequence. We also find that  $H_z/H_r$  correlates strongly with the galaxy's color, and early-type spirals have larger values of  $H_z/H_r$ . The inclinations obtained by fitting the pattern of a spiral structure with a logarithmic spiral form are nearly the same as those obtained by using the formulas of Aaronson et al. (1980). Finally, the mean measured pitch angles for the different Hubble sequences in the Third Catalogue of Bright Galaxies by de Vaucouleurs et al. (1991) are derived.

**Key words:** galaxies:fundamental parameter-galaxies:spiral-galaxies:structure

## 1. Introduction

The inclination of a spiral galaxy ( i.e., the angle between the galactic plane and the tangent plane of the celestial sphere) is not only an important parameter, but also difficult to determine. A spiral galaxy consists of a thin disk, a bulge and spiral arms that are thought to be situated in the disk. If we assume that the thickness of the spiral plane is rather negligible in comparison to its extension, and that when a spiral galaxy is inclined moderately to the plane of sky, the thickness of the nucleus can be omitted, the inclination ( $\gamma$ ) can be obtained by:

$$\gamma = \arccos\left(\frac{d}{D}\right), \quad (1)$$

where  $D$  and  $d$  are the apparent major and minor isophotal diameters respectively. When a spiral galaxy is seen edge-on, it is not possible to consider the thickness of the nuclear part as negligible. Thus, Eq. (1) cannot be used to calculate the inclination. The reason for this is that the

apparent minor isophotal-diameter consists of two parts. One is attributed by the disk and, another by the bulge, the latter of which will decrease the real value of the inclination.

Considering that the disk is not infinitely thin, Aaronson et al. (1980) corrected Eq. (1) by

$$\gamma = \arccos \sqrt{1.042\left(\frac{d}{D}\right)^2 - 0.042 + 3^\circ}. \quad (2)$$

The constant of  $3^\circ$  is added in accordance with an empirical recipe. A more elaborate specification of the axial ratio for an edge-on system that depends on the Hubble type could be justified. The thinnest galaxies are Sc spirals', earlier types have larger bulges. Giovanelli et al. (1997) provided an example to justify why they assumed the axial ratio of Sc galaxies to be 0.13. A smaller value of the axial ratio for an edge-on system results in smaller derived inclinations, where the spirals are more face-on. Besides, if the values  $D$  and  $d$  are approximations due to errors, the inclination obtained by Eq. (2) is not an exact value. Ma et al. (1997, 1998) proposed a method to determine the inclination of a spiral galaxy by fitting a spiral arm with a logarithmic spiral form with constant pitch angle. They obtained the inclinations of 72 northern spiral galaxies.

The question of the mathematical form of spiral arms was recognized at the beginning of this century (von der Pahlen, 1911; Groot, 1925). Then, Danver (1942), Kennicutt (1981) and Kennicutt & Hodge (1982) systematically studied the shapes of spiral arms. Using the method of the least squares and as many points as possible situated on the spiral arm in question, Danver (1942) studied a sample of 98 nearby spirals by drawing the projected images on white paper and then, by copying it on the paper to be used for the measurement thanks to transparent illumination. Kennicutt (1981) measured the shapes of spiral arms in 113 nearby Sa-Sc galaxies by disposing directly of photographic enlargement and using an iterative procedure to correct for inclination effects. He gave an initial estimate of the inclination and pitch angle to orient the spiral to

a face-on geometry, and then used any residual sinusoidal deviations in the arm shapes to make small corrections to the derived orientation. Using the IRAF software, Ma et al. (1997, 1998) fitted the shapes of spiral arms on the images, so that they could show clearly whether the fitting was good or not. The DISPLAY program of IRAF software can enlarge the image and change its grey scale to minimize any personal prejudice about the regularity and prominence of arms. But we must emphasize that the DISPLAY program of IRAF has many variables, so the results are not always objective. In our program, we modify z1 (minimum greylevel to be displayed) and z2 (maximum greylevel to be displayed) in the DISPLAY program in order to display the images clearly. In the procedure of fitting, we emphasize the global spiral structure, where, except for the small-scale distortions, the arms can be represented by the logarithmic spiral forms.

There has been much interest concerning the separation of disk and bulge components in the observed surface brightness distribution of spiral galaxies. de Vaucouleurs (1958), for instance, established an isophotic map of M 31 in blue light by means of direct photoelectric scans, spaced at 10' intervals in declination from +39°31' to 42°30'. From photoelectric photometry, he determined that the thickness of the flat component is about 0.8 kpc. By assuming that a galaxy has an infinitesimally thin disk, Freeman (1970) and Sandage et al. (1970) collected and studied the radial distribution of the surface brightness  $I(r)$  for thirty-six S0 and spiral galaxies, and showed that  $I(r)$  distribution for these galaxies can be presented by two main components: an inner spheroidal component which follows the law of

$$\log I(r) \propto r^{1/4} \quad (3)$$

and an outer exponential component (disk), with

$$\log I(r) = I_0 e^{-r/h_r}, \quad (4)$$

where  $h_r$  is defined as a radial scale length. Van der Kruit and Searle (1981a) proposed a model for the light distribution in the disks of edge-on spiral galaxies, assuming that a galaxy has a locally isothermal, self-gravitating and truncated exponential disk. This model has the feature of being isothermal in  $z$  at all radii with a scale parameter  $z_0$  and has an exponential dependence of surface brightness upon  $r$  with a scale length  $h_r$ . The space-luminosity of this model can be described by

$$L(r, z) = L_0 e^{-r/h_r} \text{sech}^2(z/z_0). \quad (5)$$

With this model, van der Kruit & Searle (1981a, 1981b, 1982a, 1982b) determined  $h_r$  and  $z_0$  for seven disk-dominated and one spheroid-dominated spiral galaxies by using three-color surface photometry. Peng et al (1979) investigated three-dimensional disk galaxies, based on the fundamental assumption by Parenago that the density distribution along  $z$ -direction for a finite thickness disk is

$$\rho(r, \phi, z) = \frac{1}{H_z} \sigma(r, \phi) e^{-|z|/h}, \quad (6)$$

where  $h$  is defined as an exponential scale height,  $H_z$  is defined as a thickness of disk and equals  $2h$ , and  $\sigma(r, \phi)$  is the surface density. By solving Poisson's equation for a logarithmic density perturbation, Peng et al. (1979) obtained a criterion for density waves to appear, which is

$$r > r_0 = \frac{H_z \sqrt{m^2 + \Lambda^2}}{2}, \quad (7)$$

where  $(r_0, \phi_0)$  is the polar coordinate of the starting point from which arms of a galaxy stretch outward on the galactic plane, and  $m$  is the number of the arms in a spiral galaxy. Based on this criterion, Peng (1988) proposed a method for estimating the thickness of a non-edge-on spiral, and derived the thicknesses of four galaxies.

Guthrie (1992) derived the axial ratios  $R$  of disc components for 262 edge-on spiral galaxies on print copies of the blue Palomar Sky Survey plates by using a microscope fitted with a micrometer eyepiece. He then analyzed the distribution of isophotal axial ratios for 888 diameter-limited normal Sa-Sc galaxies to give information on the true axial ratios  $R_0$ , and at last presented the mean value of  $\log R_0$  is  $0.95 \pm 0.03$ .

Ma et al. (1997, 1998) derived the thicknesses of 72 spirals by using Peng's proposal (Peng, 1988) and presented some statistical correlations between thickness or flatness and other parameters.

The value of  $R_0$  derived by Guthrie (1992) is based on observational work and, should be much more reliable. So, it is important to compare Ma et al.'s results (1997, 1998) to Guthrie's (1992). In Ma et al. (1997, 1998), the flatnesses for 72 galactic disks  $(H_z/D_0)^1$  were given, and the mean value is  $0.033 \pm 0.002$ . Suppose that the value of ratio of radial scale length ( $h_r$ ) over exponential scale height ( $h$ ) for an average exponential galactic disk is equal to  $R_0$  from Guthrie (1992), which is 9. From Freeman (1970), we can derive  $D_0/(2h_r) \approx 5$ . Thus, we obtain  $H_z/D_0 \approx 0.023$ , which is in relative agreement with the mean value of Ma et al. (1997, 1998). At the same time, we calculate the values of  $\log R_0$  for spirals of various types T and list them in Table 1. T, from the RC3, is morphological types, and  $\sigma$  is the dispersion. From this table and Table 3 of Guthrie (1992), we can see that our results are in agreement with Guthrie's (1992). Except for the data concerning Scd galaxies, our results also agree with de Grijs' (1998).

The structure of this paper is as following: in Sect. 2, we outline the principles of obtaining an inclination and a pitch angle; Sect. 3 presents some statistical properties; and conclusions will be shown in Sect. 4.

<sup>1</sup>  $D_0$ , which is measured at or reduced down to the surface brightness level  $\mu_B = 25.0B$  magnitudes per square arcsecond, and corrected to the "face-on" ( $\gamma = 0^\circ$ ). For the Galactic extinction, but not for redshift,  $D_0$  is from the Third Catalogue of Bright Galaxies by de Vaucouleurs et al. (hereafter RC3).

**Table 1.** Values of  $\overline{\log R_0}$  for spirals of various types T.

T	$\overline{\log R_0}$	$\sigma$
2	$0.65 \pm 0.04$	0.09
3	$0.79 \pm 0.06$	0.21
4	$0.79 \pm 0.04$	0.21
5	$0.90 \pm 0.04$	0.18
6	$1.20 \pm 0.15$	0.33

## 2. Principles of obtaining an inclination and a pitch angle

As we know, when the line of intersection (namely, the major axis of the image) between the galactic plane and tangent plane is taken as the polar axis, it is easily proved that

$$r = \rho \sqrt{1 + \tan^2 \gamma \sin^2 \theta} \quad (8)$$

and

$$\tan \phi = \frac{\tan \theta}{\cos \gamma}, \quad (9)$$

where  $r$  and  $\phi$  are the polar co-ordinates in the spiral plane and  $\rho$ ,  $\theta$  are the corresponding co-ordinates in the tangent-plane, and  $\gamma$  is the inclination. If it is possible to represent arms by equiangular spirals as

$$r = r_0 e^{\lambda(\phi - \phi_0)} \quad (10)$$

and

$$\mu = \arctan \lambda, \quad (11)$$

where  $r$  and  $\phi$  are the polar co-ordinates on the spiral arm in the spiral plane, and  $\mu$ , defined as a pitch angle, is the angle between the tangent to the arm and the circle with the constant  $r$ . The mathematical form of Eq. (10) in the tangent plane of the celestial sphere is

$$\rho(\theta, \gamma) = \rho_0 \frac{f(\theta_0, \gamma)}{f(\theta, \gamma)} e^{\lambda B(\theta, \gamma)}, \quad (12)$$

here

$$f(\theta, \gamma) = \sqrt{\sin^2 \theta + \cos^2 \theta \cos^2 \gamma} \quad (13)$$

and

$$B(\theta, \gamma) = g(\theta, \gamma) - g(\theta_0, \gamma), \quad (14)$$

where

$$g(\theta, \gamma) =$$

$$\begin{cases} \arctan(\tan \theta / \cos \gamma), & k\pi - \frac{\pi}{2} < \theta < k\pi + \frac{\pi}{2} \\ \pi + \arctan(\tan \theta / \cos \gamma), & k\pi + \frac{\pi}{2} < \theta < k\pi + \frac{3\pi}{2}, \end{cases} \quad (15)$$

where  $k$  is an integer. Supposing that  $(\rho_i, \theta_i)$  are the measured co-ordinates of points of the arms, and making use of the least squares method, we set

$$\sum_{i=1}^n [\rho_i - \rho(\theta_i, \gamma)] = \text{minimum}. \quad (16)$$

By direct differentiation with respect to  $\lambda$ , we obtain the equation for determining the parameter  $\lambda$  of

$$\sum_{i=1}^n B(\theta_i, \gamma) \rho(\theta_i, \gamma) [\rho_i - \rho(\theta_i, \gamma)] = 0. \quad (17)$$

This is a transcendental equation. Using the least squares method, we can obtain  $\gamma$  and  $\mu$ . Details have been shown in Ma et al. (1997, 1998).

## 3. Statistical properties

### 3.1. Sample

Our statistical sample contains 72 northern spiral galaxies for which the values of disk thickness and inclination and pitch angle of individual spiral arms are from Ma et al. (1997, 1998). Except for M 31, the images, which have distinguishable spiral arms, are from the Digitized Palomar Sky Survey. The image of M 31 is digitized from POSSII (field No. 295) via PDS measurement at Purple Mountain Observatory of China. Most galaxies in this sample have the grand design structure. All the galaxies have the total color indexes  $((B - V)_T^0)$ , which are corrected for the differential Galactic and internal extinction to “face-on”, and for redshift between  $B$  and  $V$  bands. The mean numerical Hubble stage indexes (T) of the galaxies, which are quoted in the RC3, are from 2 to 6, and 2, 3, 4, 5 and 6 correspond to Sab, Sb, Sbc, Sc and Scd respectively. This is an inclination-limited sample, i.e., the values of  $\log(D_{25}/d_{25})$  for these galaxies are smaller than the limited-value (0.76), where  $D_{25}$  and  $d_{25}$ , taken from the RC3, are the apparent major and minor isophotal diameters measured at or reduced down to the surface brightness level  $\mu_B = 25.0B$  magnitudes per square arcsecond. Clearly, this is not a diameter-limited sample.

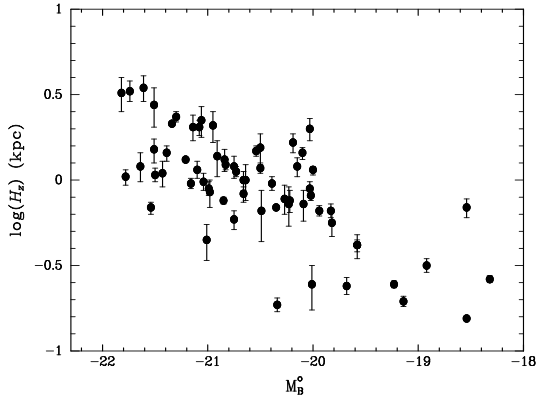
### 3.2. Thickness vs absolute magnitude

Ma et al. (1998) have presented some statistical correlation between thickness or flatness and other parameters. In this section, we will investigate some other statistical correlations.

Fig. 1 plots the correlation between the thickness and the total (“face-on”) absolute magnitude in the B band. The latter is derived by using the formula

$$M_B^o = B_T^o + 5 - 5 \log d, \quad (18)$$

here  $B_T^o$  is the total “face-on” magnitude from the RC3, corrected for the Galactic and internal extinction, as well as for redshift,  $d$  the distance, in units of pc, from Ma et al. (1997, 1998). One can find that there is a significant correlation between thickness and B-band absolute magnitude, i.e., thicker galaxies are brighter.



**Fig. 1.** Thickness of spiral galaxy plotted against the total (“face-on”) absolute magnitude in the B-band.

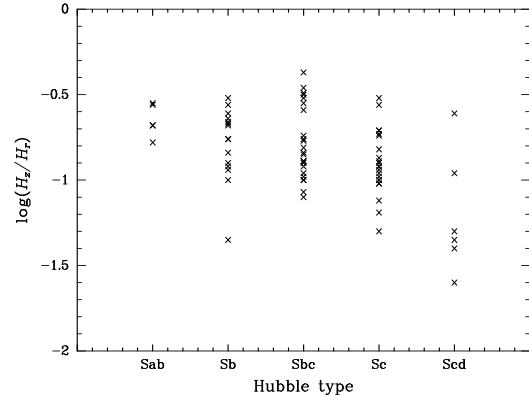
Based on the careful studies of Binggeli et al. (1985) about the Virgo cluster, Binggeli et al. (1988) have thoroughly reviewed what is currently known about the luminosity function  $\Phi(L)$ . They have derived the luminosity function  $\Phi(L, T)$  for each morphological type separately in the Virgo cluster. Furthermore, according to the detailed analysis of the member galaxies for the Virgo cluster (Zhao & Shao, 1994; Shao & Zhao, 1996), Shu et al. (1995) also investigated its LF for individual morphological types. Combining with the luminosity function  $\Phi(L, T)$  of the Virgo cluster (Binggeli et al., 1988; Shu, et al., 1995), Fig. 1 automatically implies that early-type spirals, which are on average brighter than late-type ones, are also thicker.

### 3.3. Dependence of $H_z/H_r$ on morphological type, luminosity and color

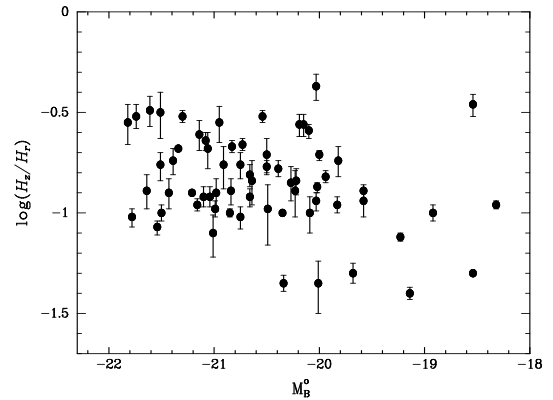
In this section, we will present some statistical correlations between  $H_z/H_r$  and morphological type, luminosity and color for spiral galaxies.

Ma et al. (1997, 1998) derived the flatnesses ( $H_z/D_0$ ) for 72 spiral galaxies. When we take  $D_0/H_r \approx 5$  ( $H_r$  is defined as  $2h_r$ ), we can derive the values of  $H_z/H_r$  for these spirals. Fig. 2 shows  $H_z/H_r$  as a function of different Hubble types. It can be seen that  $H_z/H_r$  of spiral galaxies decreases smoothly an average along the Hubble type, but the dispersion in  $H_z/H_r$  among galaxies of the same Hubble sequence is very large. This, perhaps, reflects that the intrinsic flattening of spiral disks is smaller for later type galaxies. de Grijs (1998) also shows that galaxies become systematically thinner when going from type S0 to Sc.

Fig. 3 plots  $H_z/H_r$  as a function of total (“face-on”) absolute magnitude in the B band. In Fig. 3, we can see a clear trend reflecting an increase in the values of  $H_z/H_r$  as the galaxies become brighter for all but a few galaxies.



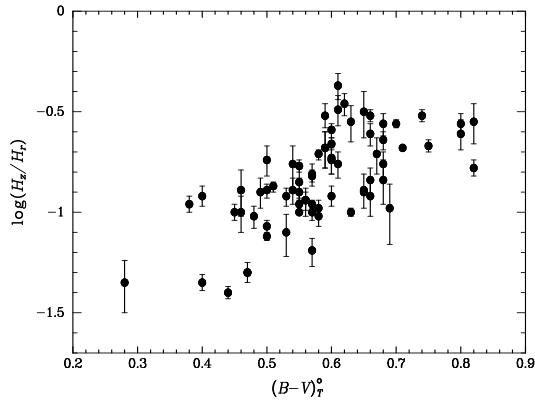
**Fig. 2.**  $H_z/H_r$  of spiral galaxy plotted against the Hubble type.



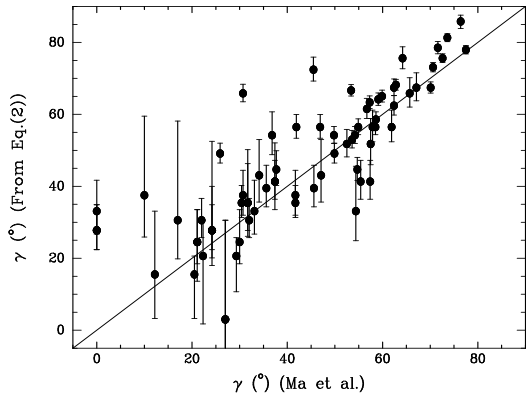
**Fig. 3.**  $H_z/H_r$  of spiral galaxy plotted against magnitude in the B-band.

Combining this data with the luminosity function  $\Phi(L, T)$  of the Virgo cluster (Binggeli et al., 1988; Shu et al., 1995), we find that early-type spirals, which are brighter on average than late-type ones, have larger values of  $H_z/H_r$ .

Fig. 4 plots  $H_z/H_r$  as a function of total galactic color index  $((B - V)_T^0)$  in the RC3 and shows a strong trend which suggests that a bluer galaxy has a smaller value of  $H_z/H_r$ . Roberts & Haynes (1994) studied the physical parameters along the Hubble sequence systematically by making use of two primary catalogues, the RC3 and a private catalogue maintained by R. Giovanelli & M. Haynes. He presented the well-established trend between morphology and mean color which reveals that the E and S0 galaxies are clearly redder than spirals and that late-type spirals are bluer on average than early-type ones. So, Fig. 4 also implies that later-type galaxies have smaller values of  $H_z/H_r$ .



**Fig. 4.**  $H_z/H_r$  of spiral galaxies plotted versus color.



**Fig. 5.** Comparison of inclinations derived from Eq. (2) and Ma et al. (1997, 1998).

### 3.4. Statistical property of inclination

Tully & Fisher (1977) showed that if a spiral structure is well defined, the opening of the spiral structure could be used to define the inclination of the disk. It has been shown by Danver (1942), Kennicutt (1981) and Kennicutt & Hodge (1982) that the spiral arm can be represented by a logarithmic spiral form with a constant pitch angle. Grosbøl (1985) studied 605 galaxies with inclination angles inferior to  $56^\circ$ , and estimated the position and inclination angles of these galaxies based on the bisymmetric intensity distribution in their outer parts applying a one dimensional Fourier transform method. Ma et al. (1997, 1998) obtained the inclinations of 72 northern spiral galaxies by fitting the arms on the images. The RC3 lists the values of  $D_{25}/d_{25}$  and their errors,  $D_{25}$  and  $d_{25}$  are the apparent major and minor isophotal diameters measured at or reduced down to the surface brightness level

$\mu_B = 25.0$  B magnitudes per square arcsecond. We calculate the inclinations by use of Eq. (2). Fig. 5 presents the correlation between them and those obtained by Ma et al. (1997, 1998). This figure shows that, when we take into account the errors in the inclination due to errors in  $D_{25}/d_{25}$ , and although there is some scatter, Ma et al.'s result (1997, 1998) is consistent with what is derived from Eq. (2).

### 3.5. Mean value of the pitch angle along the Hubble sequences in the RC3

Hubble (1926, 1936) introduced an early scheme to classify galaxies. Its concepts are still in use, a sequence starting from elliptical to spiral galaxies, and including lenticular. This scheme has been extended by some astronomers (Holmberg, 1958; de Vaucouleurs, 1956, 1959; Morgan, 1958, 1959; van den Bergh, 1960a, b, 1976; Sandage, 1961; Sandage & Tammann, 1981, 1987; Sandage & Bedke, 1993) over the years, who tried to employ multiple classification criteria. The two main systems commonly used are derived from Hubble's original classification criteria. One is the Hubble system as explained in detail by Sandage (1961), Sandage & Tammann (1981, 1987) and Sandage & Bedke (1993), and another, developed by de Vaucouleurs (1956, 1959), adds more detailed descriptions to the notation and makes a division and extension of the Sc and SBc families by introducing the Scd, Sd, Sdm, Sm and Im subdivisions.

Galaxy morphological classification is still mainly done visually by dedicated astronomers, based on the Hubble's original scheme and its modification. It is possible for each observer to give slightly different weights to various criteria, although the criteria for classification are accepted generally by them. Lahav et al. (1995) and Naim et al. (1995) investigated the consistency of visual morphological classifications of galaxies from six independent observers. They found that individual observers agree with one another with combined rms dispersions of between 1.3 and 2.3 type units, typically about 1.8 units of the revised Hubble numerical type index T, although there are some cases where the range of types given to it was superior to 4 units in width.

The tightness of the spiral arm, in addition to the degree of resolution in the arm and the relative size of the unresolved nuclear region, are the fundamental criteria in Hubble's (1926, 1936) classification of spiral galaxies. In order to better understand the nature and origin of the Hubble sequence and evaluate the difference of classification, Kennicutt (1981) compared the arm pitch angles as measured by him with the Hubble type as determined by Sandage & Tammann (1981, hereafter ST) and Yerkes class by Morgan (1958, 1959). From Figs. 7 and 8 in Kennicutt (1981), we can see that, although the ST's classification is based almost solely upon disk resolution and the Morgan's is based entirely on the central concentration of

**Table 2.** Mean values of the pitch angles  $\mu$  for different Hubble types.

Hubble type	$\bar{\mu}$	$\sigma$	N
S0/a	$6.25 \pm 1.08$	2.86	8
Sa	$4.60 \pm 1.40$	2.80	5
Sab	$9.36 \pm 1.03$	3.71	14
Sb	$11.82 \pm 0.81$	4.35	30
Sbc	$15.16 \pm 0.76$	5.36	51
Sc	$15.50 \pm 0.62$	4.25	48
Scd	$18.92 \pm 1.19$	5.34	21
Sd	$25.00 \pm 0.00$	0.00	2

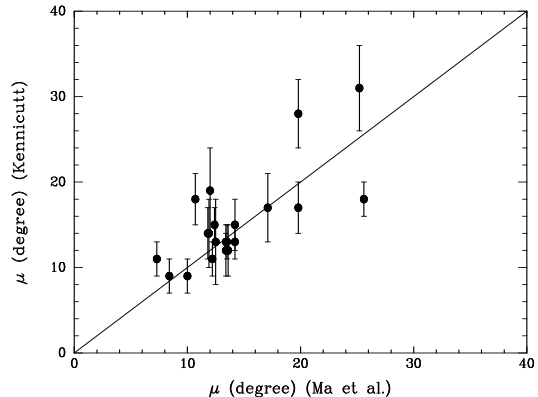
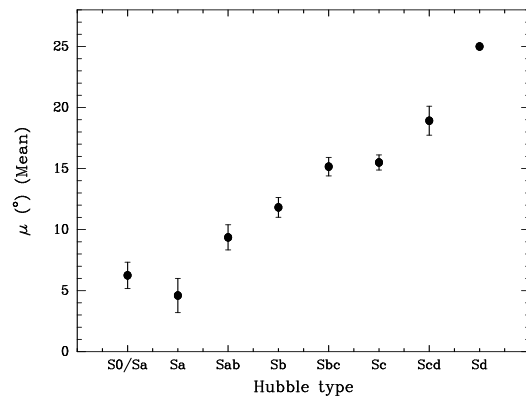
the galaxies, the trends between the arm pitch angles and the two types are almost the same. In the classifications by de Vaucouleurs et al. (1976, RC2; 1991, RC3), all these three criteria are considered.

A most important feature of spiral galaxies is that the stellar content between the spiral arms and the spheroidal component is very different, and the central part is redder than the arm region. The Yerkes system by Morgan (1958, 1959) contains the information on the state of stellar evolution in the central region of galaxies and it complements the information in the Hubble sequence by de Vaucouleurs (1956, 1959), which, in cases of conflicting criteria, emphasizes more strongly the strength of population in the arm. There are other sets of classifications in which disk-to-bulge is the primary discriminant (van den Bergh, 1976) or the ratio of disk-to-bulge to spiral arm morphology is primarily considered (Dressler, 1980; Wilkerson, 1980). Among these three criteria, perhaps, spiral arm morphology may be distance independent.

Until now, there are only a few papers dealing with pitch angles. So, the mean value of pitch angle for the different Hubble types, which is an important parameter in the decision of whether the WKB approximation can be satisfied (Binney & Tremaine, 1987), has not been presented. As we know, the WKB approximation is an indispensable tool for understanding the origin and evolution of a density wave in spiral galaxies.

Before deriving the mean value of pitch angle for the different types in the RC3, it is useful to compare the pitch angles of the common galaxies derived by Kennicutt (1981) and Ma et al. (1998). 22 of the spirals measured by Ma et al. (1998) were previously done by Kennicutt (1981), and the average pitch angles are compared in Fig. 6. Except for 5 galaxies, the results are consistent between Kennicutt (1981) and Ma et al. (1998). Some of the discrepancies can be attributed to our measured error, and the others may be expected from the galaxies themselves; for example, when a spiral galaxy has two asymmetric arms: different authors (Kennicutt, 1981; Ma et al. 1998) did not measure the same arm in the same galaxy.

Table 2 presents the mean values of pitch angle ( $\mu$ ) from the ones measured by Kennicutt (1981) and Ma et al. (1997, 1998) for the different Hubble sequences in the RC3,

**Fig. 6.** Comparison of pitch angles derived by Kennicutt (1981) and Ma et al. (1997, 1998), for galaxies in common.**Fig. 7.** Mean measured pitch angle plotted against different Hubble types in the RC3.

where  $\sigma$  is the dispersion and N the number of galaxies. The important result from Table 2 is that the mean pitch angles of Sa-Sc are not larger than  $15.5^\circ$ , so that  $\cotan\mu \geq 3.6$ . Thus the WKB approximation is satisfied at the mean pitch angles from Sa-Sc, but not by very much.

Fig. 7 presents the mean values versus the Hubble types in the RC3, and it shows that, from the early to late Hubble types, the mean values of the pitch angle increase, although the value for S0/a's is larger than that for Sa's. In fact, it is difficult to distinguish between S0/a and Sa.

Besides Hubble classification systems, Elmegreen & Elmegreen (1982a, b; 1987) introduced another classification system which is designed to emphasize arm continuity, length and symmetry, and it is related to the presence or strength of density waves. This system is not dependent on the galaxy Hubble sequence and contains 12 dis-

tinct arm classes, corresponding to a systematic change from the ragged and patchy arms in ‘flocculent’ galaxies to the two symmetric and continuous arms in ‘grand design’ ones. Intermediate arm classes show characteristics of both the ‘flocculent’ and ‘grand design’ types. Based on their spiral arm classification system, Elmegreen & Elmegreen (1982a, b; 1987) classified the spiral galaxies in the field, in binary systems, in groups and in clusters, and suggested that bars tend to correlate with spiral density waves, companions may influence (or generate) symmetric density waves, grand design galaxies are physically larger than flocculent ones by a factor of  $\sim 1.5$ , and grand design galaxies are also preferentially in dense groups.

#### 4. Conclusions

In this paper, we investigate some statistical correlations about the thickness of galactic disks and pitch angles of spiral arms. The main conclusions are: (1). Early-type spirals, that are brighter on the average, are thicker; (2). The axis ratio ( $H_z/H_r$ , here  $H_r$  and  $H_z$  are defined as two times the radial scale length ( $h_r$ ) and exponential scale height ( $h$ )) of galactic disk tends to be smaller along the Hubble sequence; (3). Except for a few galaxies, early-type spiral galaxies have larger values of  $H_z/H_r$ ; (4).  $H_z/H_r$  correlates strongly with galaxy color; (5). The inclinations obtained by fitting the pattern of spiral structure with a logarithmic spiral form are nearly the same as those obtained by using the formulas of Aaronson et al. (1980); (6). The mean measured pitch angles for different Hubble sequences in the RC3 are derived, and the results show that the mean pitch angles of Sa-Sc’s are not larger than  $15.5^\circ$ , so that  $\cotan\mu \geq 3.6$ . Thus the WKB approximation can be satisfied at the mean pitch angles from Sa-Sc’s, but not by very much; (7). From early to late Hubble types, the mean value of pitch angle increases, despite some scattering.

Although the method, proposed by Peng (1988) for deriving the thickness of a face-on disc, is effective and simple, it relies on a spiral structure theory that predicts that a spiral arm has to end somewhere in the disk. However, this theory might not be completely right. For example, Zaritsky et al. (1993) has presented K-band ( $2.2 \mu m$ ) images of M 51, which reveal remarkable dynamical structure not visible in the conventional optical observations, and show that the spiral arms extend significantly further towards the galaxy’s center than previously observed. In the optical images, the spiral arms begins at a radius of about  $30''$  from the center. But, the K-band residual image showed the spiral arms wind through an additional  $540^\circ$  beyond that seen in the B-band images of the entire galaxy, and end at about  $10''$  from the center of the galaxy. If spiral galaxies whose spiral arms do not stop anywhere in the disk exist, the parameters  $\rho_0$  and  $r_0$  (Peng, 1988; Ma et al., 1997, 1998) cannot be found. In the K-band images, which minimize the effect of dust and

maximize sensitivity to the dominant stellar population, we can derive our reliable values of  $\rho_0$  and  $r_0$ . As an example, we apply our method to the image of M 51 in the K-band<sup>2</sup> and derive the flatness of disk ( $H_z/D_0$ ), which is  $0.010 \pm 26.4\%$ . Comparing this value with Peng’s (1988) ( $H_z/D_0 = 0.013 \pm 16.4\%$ ), we can find that the value of the flatness based on the K-band image is smaller. The reason is that, in the K-band image, the effect of dust can be minimized and the values of  $\rho_0$  and  $r_0$  may be reliably derived. We also emphasize that the images, which Ma et al. (1998) used, are from the Digitized Palomar Sky Survey, in which many images have burnt-out centers. There are some pictures that have burnt-out centers in Ma et al. (1998), but we can change the parameters of DISPLAY program to minimize the effect.

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<sup>2</sup> The image of M 51 in K-band is provided by Prof. Zaritsky.

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