Quintessence-like Dark Matter in Spiral Galaxies

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Through the geodesic analysis of a static and axially symmetric space time, we present conditions on the state equation of an isotropic perfect fluid $p = \omega d$, when it is considered as dark matter in spiral galaxies. The main conclusion is that it can be an exotic fluid $(-1 < \omega < -1/3)$ as it is found for Quintessence at cosmological scale.

There is no doubt about the importance of the mystery concerning the nature of dark matter in the Universe and in particular in galaxies. The consequences of observations made on SNIa supernovae [1] have posed challenges to the available theoretical machinery, and certain models explaining such phenomena have arised which propose exotic types of matter and therefore unusual equations of state, such as a Cosmological Constant, Cold Dark Matter models, Dilaton Fields [2] and Quintessence [3]. However, at the galactic level there are no models consistent with the cosmological ones, and which give some light in the understanding of the nature of dark matter.

In order to be precise about the problem let us recall the situation of the galactic dark matter, for which we confine ourselves to the observations made by Rubin et al. [4] who found that for a few sample of spiral galaxies the interstellar gas and stars lying far away from the center (in the equatorial plane) of the corresponding galaxy behaves in a non Kepplerian way, but their circular velocity seems to be independent of the radius starting from a certain distance to the galactic center, i.e. the rotation curve profile of a spiral galaxy is flat outside a central galactic region. It was then inferred a distribution $\sim 1/r^2$ of non luminous matter (dark matter) which should contribute to the flatness of the rotation curves. There exists certain controversy about the flatness of such curves [5], but in general it is accepted that rotation curves are flat up to the precision of the meassurementes made by the astronomers and that this behavior is reproduced even for large samples of spiral galaxies [6].

The most accepted scenario for a spiral galaxy reads as follows: it is an object composed by a luminous disc whose density distributed in an exponentially decay which conspires with a dark halo whose density is distributed as $\sim 1/r^2$ [7]. In this way it is found an explanation for the kinetic behavior of gas and stars composing a spiral galaxy, but how was this mixture formed and what inspired nature to conspire in this way and not another one? If the dark matter is baryonic such as MACHOS for instance, why does its density have a non exponential distribution as luminous matter density does? If it is non baryonic, what is it made of, or at least which is its equation of state? This last question is the one that occupies ourselves in the present work.

In this letter we proceed in the following way: Assume that a spiral galaxy lies on a background axysimmetric static space time, which is characterized by the presence of a perfect fluid with an arbitrary equation of state, i.e. $p = \omega d$ being ω a free function, and then we find conditions over ω that permit flat rotation curves of test particles. Other types of candidates to dark matter are discussed in [8,9].

First of all it must be clear that our treatment is valid only in the dark matter dominated region, i.e. where the rotation curves are flat, and we do not consider the galactic core region. Observational data show that the galaxies must be composed by almost 90% of dark matter, distributed at the halo, in order to explain the observed dynamics of particles in the luminous sector of the galaxy. We can thus assume that luminous matter does not contribute in a very important way to the total energy density of the halo of the galaxy in the mentioned region. On the other hand, the exact symmetry of the halo is still unknown, but it is reasonable to suppose that the halo is symmetric with respect to the rotation axis of the galaxy, so we choose the space time to be axial symmetric. Furthermore, the

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rotation of the galaxy does not have a large effect on the motion of test particles around it; dragging effects in the halo of the galaxy can be considered too small to seriously affect the motion of tests particles (stars) traveling around the galaxy. The circular velocity of stars (like the Sun) is of the order of 230 Km/s, much less than the speed of light. Hence, in the region of interest we can suppose the space-time to be static as well.

Therefore, we start by considering the background described by the following line element:

$$ds^{2} = -e^{2\psi}dt^{2} + e^{-2\psi}[e^{2\gamma}(d\rho^{2} + dz^{2}) + \mu^{2}d\varphi^{2}],$$
(1)

which corresponds to an static axially symmetric space-time; the coordinates are the usual cylindrical ones.

We recall the reader that observations are made upon objects lying in the galactic equatorial plane, thus the Lagrangian for a test particle travelling on such slide of the space time described by (1) is

$$2\mathcal{L} = -e^{2\psi}\dot{t}^2 + e^{-2\psi}[e^{2\gamma}\dot{\rho}^2 + \mu^2\dot{\varphi}^2], \qquad (2)$$

where dot means derivative with respect to the proper time τ of the test particle. The radial geodesic motion equation is then

$$\dot{\rho}^2 - e^{2(\psi - \gamma)} \left[E^2 e^{-2\psi} - L^2 \frac{e^{2\psi}}{\mu^2} - 1 \right] = 0.$$
(3)

where E, and L, are constants associated with this geodesic motion along the equatorial plane.

We are interested in circular and stable motion of test particles, therefore the following conditions must be satisfied

- i) $\dot{\rho} = 0$, circular trajectories
- ii) $\frac{\partial V(\rho)}{\partial \rho} = 0$, extreme ones
- iii) $\frac{\partial^2 V(\rho)}{\partial \rho^2}|_{extr} > 0$, and stable.

being
$$V(\rho) = -e^{2(\psi-\gamma)} \left[E^2 e^{-2\psi} - L^2 e^{2\psi} / \mu^2 - 1 \right].$$

Recalling that E and L are constants of motion for each circular orbit, it is straightforward to obtain expressions for the energy E, angular momentum L, angular velocity $\Omega = d\varphi/dt$ and the tangential velocity $v^{(\varphi)} = e^{-2\psi}\mu \Omega$ [10], corresponding to a circular, stable equatorial motion:

$$E = e^{\psi} \sqrt{\frac{\frac{\mu_{,\rho}}{\mu} - \psi_{,\rho}}{\frac{\mu_{,\rho}}{\mu} - 2\psi_{,\rho}}},\tag{4}$$

$$L = \mu e^{-\psi} \sqrt{\frac{\psi_{,\rho}}{\frac{\mu_{,\rho}}{\mu} - 2\psi_{,\rho}}},$$
(5)

$$\Omega = \frac{e^{2\psi}}{\mu} \sqrt{\frac{\psi_{,\rho}}{\frac{\mu_{,\rho}}{\mu} - \psi_{,\rho}}},\tag{6}$$

$$v^{(\varphi)} = \sqrt{\frac{\psi_{,\rho}}{\frac{\mu_{,\rho}}{\mu} - \psi_{,\rho}}},\tag{7}$$

and for the stability condition:

$$V_{,\rho\rho}|_{extr} = -\frac{2 e^{2(\psi-\gamma)}}{\frac{\mu,\rho}{\mu} - 2 \psi_{,\rho}} \left(\frac{\mu_{,\rho}}{\mu} \psi_{,\rho\rho} - \frac{\mu_{,\rho\rho}}{\mu} \psi_{,\rho} + 4 \psi_{,\rho}^{3} - 6 \frac{\mu_{,\rho}}{\mu} \psi_{,\rho}^{2} + 3 \left(\frac{\mu_{,\rho}}{\mu}\right)^{2} \psi_{,\rho} \right) > 0$$
(8)

where a coma stands for partial derivative.

Now, observe that if the functions ψ and μ are related by

$$e^{\psi} = \left(\frac{\mu}{\mu_0}\right)^l. \tag{9}$$

being l = const, we obtain a necessary and sufficient condition for the velocity $v_c^{(\varphi)}$ to be the same for two orbits at different radii, given $l = (v_c^{(\varphi)})^2 / (1 + (v_c^{(\varphi)})^2)$, and equation (8) tells us that this motion is stable. We call equation (9) together with such value of l the *flat curve condition*.

We now write the Einstein's equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ for an arbitrary energy momentum tensor for the line element (1):

$$\mu D^2 \psi + D\mu D\psi = 4\pi \,\mu \left[e^{-2(\psi - \gamma)} \left(e^{-2\psi} T_{tt} + \frac{e^{2\psi}}{\mu^2} T_{\varphi\varphi} \right) + T_{\rho\rho} + T_{zz} \right],\tag{10}$$

$$D^{2}\mu = 8\pi\,\mu\,[T_{\rho\rho} + T_{zz}] \tag{11}$$

$$\gamma_{\rho} \mu_{\rho} - \gamma_{z} \mu_{z} - \mu \left(\psi_{\rho}^{2} - \psi_{z}^{2}\right) + \mu_{zz} = 8 \pi \mu T_{\rho\rho}, \tag{12}$$

$$\gamma_{\rho}\,\mu_{z} + \gamma_{z}\,\mu_{\rho} - 2\,\mu\,\psi_{\rho}\,\psi_{z} - \mu_{\rho z} = 8\,\pi\,\mu\,T_{\rho z}.$$
(13)

where we have introduced the operator $D = (\partial_{\rho}, \partial_z)$, see reference [11]. In order to have flat tangential curve velocities, it is introduced the *flat curve condition* (9). This condition is valid on the equatorial plane. Nevertheless, the halo is expected to be almost spherically symmetric, that means that if we know the functional dependence of the gravitational potential on the equatorial plane, this dependence should be the same one in almost the rest of the halo. In that case it is reasonable to suppose that the *flat curve condition* (9) is valid in a region around the equatorial plane. Thus, in this region we substitute the relations (9) into the left hand side of equation (10) obtaining $\mu D^2 \psi + D\mu D \psi = l D^2 \mu$ and with (11) we get a constrain equation amount the components of the stress energy tensor:

$$-\left(\frac{1-(v_c{}^{(\varphi)})^2}{1+(v_c{}^{(\varphi)})^2}\right)(T_{\rho\rho}+T_{zz}) = e^{-2(\psi-\gamma)}\left(e^{-2\psi}T_{tt} + \frac{e^{2\psi}}{\mu^2}T_{\varphi\varphi}\right)$$
(14)

Notice that this relation must be satisfied by any stress energy tensor which, within the approximation made in the analysis, curves the space time in such a way that the motion of test particles corresponds to the observed one.

Let us consider the case of a stress energy tensor corresponding to a perfect fluid, $T_{\mu\nu} = (d+p) u_{\mu} u_{\nu} + g_{\mu\nu} p$, with d the density of the fluid and p its pressure. In this case we are thinking on a "dark fluid", which is not seen but it is thought that it could be there affecting the geometry in the way needed in order to have the observed behavior in the tangential velocities of the luminous matter, as just mentioned. Considering the dark fluid as static, the four velocity of such dark fluid is given by $u^{\alpha} = (u^0, 0, 0, 0)$ which, for the line element (1) reads: $u^0 = E e^{-2\psi}$, thus $u_0 = -E$ and from $u^{\mu}u_{\mu} = -1$, we obtain that $E = e^{2\psi}$. Therefore, the stress energy tensor has the form:

$$\Gamma_{tt} = e^{2\psi}d,\tag{15}$$

$$T_{\rho\rho} = T_{zz} = e^{-2(\psi - \gamma)}p,\tag{16}$$

$$T_{\varphi\varphi} = \mu^2 e^{-2\psi} p. \tag{17}$$

Substituting these expressions into (14), we obtain that in the equatorial plane, in order to satisfy the observed behavior on the tangential velocities, the "dark fluid" has to fulfill the relation:

$$-2\left(\frac{1-(v_c{}^{(\varphi)})^2}{1+(v_c{}^{(\varphi)})^2}\right)p = (d+p)$$
(18)

Let us see which are the permitted relations between pressure and density of the perfect fluid providing flat rotation curves, we thus obtain:

$$p = -\frac{1 + (v_c{}^{(\varphi)})^2}{3 - (v_c{}^{(\varphi)})^2}d$$
(19)

relation from which the *d* coefficient is identified with the square velocity dispersion of the dark particles, that appears to be negative. We are now in a convenient position to strict the state equation. As the velocities of the gas and stars rotating in the flat region must be within $0 < v_c^{(\varphi)^2} < 1$, (the observed ones are of the order of $v_c^{(\varphi)} \sim 10^{-3}$ [6]), relation (19) implies $-1 < \omega < -1/3$, being $p = \omega d$. This result coincides with the one obtained at cosmological scale for the respective equation of state in the Quintessence model [3,8].

Therefore the analysis presented in this letter, gives support to the hypothesis that a Quintessence-like equation of state could be the solution for the dark matter problem at galactic scale. In both cases it turns out the need of an exotic equation of state, with $\omega = -0.33$ at a galactic scale and $\omega = -0.64$ for the cosmos [2].

In any case, we have shown that galactic dark matter satisfying an exotic equation of state certainly can be used to explain the observed behavior on the rotational curves of spiral galaxies.

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