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Ferromagnetism of quark liquid and magnetars

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Spontaneous magnetization of quark liquid is examined on the analogy with that in electron gas. It is pointed out that quark liquid has potential to be ferromagnetic at rather low densities, around nuclear saturation density. Some comments are given as for implications on magnetars.

1 Introduction

Recently a new type of neutron stars with extraordinary magnetic field, usually called magnetars, has attracted much attention in connection with pulsars associated with soft-gamma-ray repeaters (SGR) and anomalous X-ray pulsars (AXP). There have been known several magnetar candidates such as SGR 1806-20 and SGR 1900+14 [1]. Various analysis including the $P - \dot{P}$ curve have indicated an intense magnetic field of $O(10^{14-15})$ G, while ordinary radio pulsars have a magnetic field of $O(10^{12-13})$G.

The origin of the strong magnetic field in compact stars, especially neutron stars, has been an open problem. Recent discovery of magnetars seems to renew this problem. Conservation of the magnetic flux during the collapse of a main sequence star has been a naive idea to understand the magnetic field in neutron stars [2]. Then the strength B should be proportional to R^{-2}, where R is a radius of a star; for example, the sun, a typical main sequence star, has a magnetic field of $O(10^3)$G with the radius $R \sim 10^{10-11}$cm. By decreasing the radius to 10^6cm for neutron stars $B = O(10^{12})$G, which is consistent with observations for radio pulsars. However, if this argument is extrapolated to explain the magnetic field for magnetars, we are lead to a contradiction: their radius should be $O(10^4)$cm to get an increase in B by a factor of $\sim 10^{12}$, which is much less than the Schwartzschild radius of neutron stars with the canonical mass $M = 1.4M_\odot$, $R_{Sch} = 2GM/c^2 = 4 \times 10^5$cm.

When we compare the energy scales for systems such as atomic system (e^-), nucleon system (p) and quark system (q), we can get a hint about the origin of the magnetic field. In Table 1. we list the interaction energy, $E_{int} = \mu_i B$, of the magnetic field $B = 10^{15}$G and each constituent with the Dirac magnetic moment, $\mu_i = e_i\hbar/2m_ic$. We also list a typical energy scale E_{typ} for each system. Then we can see that $E_{typ} \ll E_{int}$ for the electron system, while $E_{typ} > E_{int}$ for the nucleon and the quark systems; that is,

	e^-	p	q
m_i[MeV]	0.5	10^3	1- 100
E_{int}[MeV]	5 - 6	2.5×10^{-3}	2.5×10^{-2} - 2.5
E_{typ}	O(KeV)	$\geq O$(MeV)	$\geq O$(MeV)

Table 1:

the strength of $O(10^{15})$G is very large for the former system with the electromagnetic interaction, while it is not large for the latter systems with the strong interaction. Hence it may be conceivable that the strong interaction should easily produce the magnetic field of the above magnitude. Since there is a bulk hadronic matter beyond nuclear saturation density ($n_0 \sim 0.16\text{fm}^{-3}$) inside neutron stars, it should be interesting to consider the hadronic origin of the magnetic field; ferromagnetism or spin-polarization of hadronic matter may give such magnetic field.

In 70's, just after the first discovery of pulsars, there have been done many works about the possibility of the ferromagnetic transition in dense neutron matter, using $G-$matrix calculations or variational calculations with the realistic nuclear forces. Through these works there seems to be a consensus that ferromagnetic phase, if it exists, should be at very high densities, and there is no transition at rather low densities relevant to neutron stars [3].

We consider here the possibility of ferromagnetism of quark liquid interacting with the one-gluon-exchange (OGE) interaction [4]. One believes that there are deconfinement transition and chiral symmetry restoration at finite baryon density, while their critical densities have not been fixed yet. One interesting suggestion is that three-flavor symmetric quark matter (strange matter) may be the true ground state of QCD at finite baryon density [5,6]. If this is the case, quark stars (strange quark stars), can exist in a different branch from the neutron-star branch in the mass-radius plane [7]. Usually one implicitly assumes that the ground state of quark matter is unpolarized. We examine here the possibility of polarization of quark matter. We shall see our results should give an origin of the strong magnetic field for magnetars in the context of strange quark-star scenario.

2 Spontaneous magnetization of quark liquid

2.1 Relativistic formulation

Quark liquid should be totally color singlet (neutral), which means that only the exchange interaction between quarks is relevant there. This may remind us of electron system with the Coulomb force in a neutralizing positive charge background. In 1929 Bloch first suggested a possibility of ferromagnetism of electron system [8]. He has shown that there is a trade off between the kinetic and the exchange energies as a function of a polarization parameter, the latter of which favors the spin alignment due to a quantum effect; electrons with the same spin orientation can effectively avoid the Coulomb repulsion due to the Pauli exclusion principle. When the energy gain due to the spin alignment dominate over the increase in the kinetic energy at some density, the unpolarized electron gas suddenly turns into the completely polarized state.

In the following we discuss the possibility of ferromagnetism of quark liquid on the analogy with electron gas (Fig. 1).

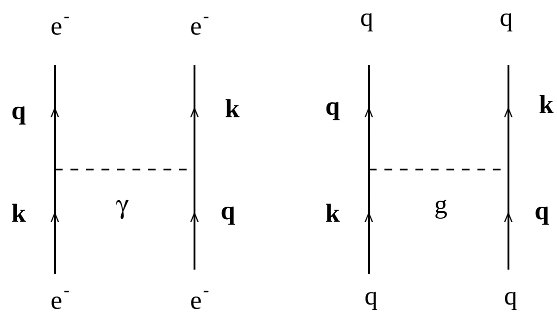

Figure 1: Exchange interactions for electrons with the Coulomb force (left) and quarks with OGE interaction (right).

It is to be noted that there is one big difference between them; quarks should be treated in a relativistic way. The concept of the spin orientation is not well defined in relativistic theories, while each quark has two polarization degrees of freedom. Here we define the spin-up and -down states in the rest frame of each quark. Then the projector onto states of definite polarization is given by $P(a) = (1 + \gamma_5 \not{a})/2$ with the 4-pseudovector a,

$$\mathbf{a} = \boldsymbol{\zeta} + \frac{\mathbf{k}(\boldsymbol{\zeta} \cdot \mathbf{k})}{m_q(E_k + m_q)}, \ a^0 = \frac{\boldsymbol{\zeta} \cdot \mathbf{k}}{m_q} \tag{1}$$

for a quark moving with the momentum $k = (E_k, \mathbf{k})$ [9]. The 4-pseudovector a is reduced into the axial vector $\boldsymbol{\zeta}$ ($|\boldsymbol{\zeta}| = 1$) in the rest frame, which is twice the mean spin vector in the rest frame. Hence a or $\boldsymbol{\zeta}$ can specify the polarized state.

The exchange interaction between two quarks with momenta $\mathbf{k}$ and $\mathbf{q}$ (Fig.

1) is written as

$$f_{\mathbf{k}\zeta,\mathbf{q}\zeta'} = \frac{2}{9}\frac{g^2}{m_q^2}\frac{m_q}{E_k}\cdot\frac{m_q}{E_q}[2m_q^2 - k\cdot q - m_q^2 a\cdot b]\frac{1}{(k-q)^2}, \tag{2}$$

where the 4-pseudovector b is given by the same form as in Eq. (1) for the momentum $\mathbf{q}$. The exchange energy is then given by the integration of the interaction (2) over the two Fermi seas [a] for the spin-up and -down states; eventually, it consists of two contributions,

$$\epsilon_{ex} = \epsilon_{ex}^{non-flip} + \epsilon_{ex}^{flip}. \tag{3}$$

The first one arises from the interaction between quarks with the same polarization, while the second one with the opposite polarization. The non-flip contribution is the similar one as in electron gas, while the flip contribution is a genuine relativistic effect and absent in electron gas. We shall see that this relativistic effect leads to a novel mechanism of ferromagnetism of quark liquid.

2.2 Symmetry consideration of ferromagnetic phase

Usual Heisenberg model describes the spin-spin interaction between adjacent spins localized at lattice points; that is, the Heisenberg ferromagnet is the spin alignment in coordinate space. On the other hand, the concept of spin alignment in quark liquid requires an extension to the phase space because of the coupling of spin with momentum. Since the spatial part of the quark wave functions take the plane wave, the spin orientation is obviously uniform in coordinate space, once $\boldsymbol{\zeta}$ is given. On the other hand, the spin does not necessarily take the same orientation in momentum space: generally $\boldsymbol{\zeta}$ should be momentum dependent (see Fig. 2).

The most favorite configuration in momentum space may be determined by an energetic consideration, while it seems to be a difficult task. We consider here only a naive case, where the spin orientation is uniform even in the phase space. This is a direct analog of the nonrelativistic version.

Anyway, the ferromagnetic phase is a spontaneously symmetry broken state with respect to the rotational symmetry in coordinate space: the order parameter is the mean value of $\boldsymbol{\zeta}$, $\langle\boldsymbol{\zeta}\rangle$, and symmetry is broken from $G = O(3)$ to $H = O(2)$ once $\langle\boldsymbol{\zeta}\rangle$ takes a special orientation.

[a] We, here, don't consider any deformation of Fermi spheres for simplicity, while they may be deformed in a realistic case due to the momentum dependent interaction.

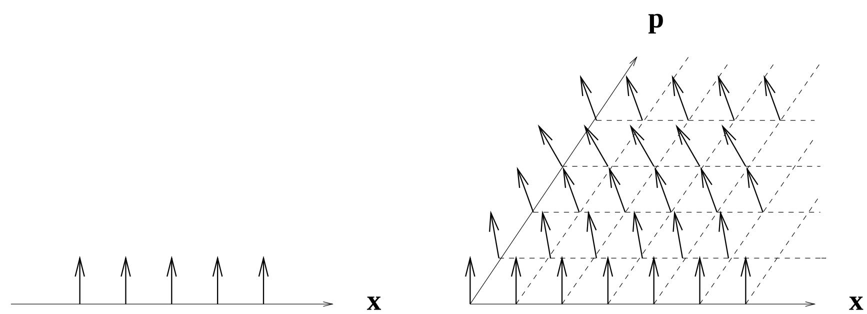

Figure 2: Heisenberg ferromagnet in the coordinate space (left) and quark ferromegnet in the phase space (right).

3 Examples

We show some results about the total energy of quark liquid, $\epsilon_{tot} = \epsilon_{kin} + \epsilon_{ex}$, by adding the kinetic term ϵ_{kin}. Since gluons have not the flavor quantum numbers, we can consider one flavor quark matter without loss of generality. Then quark number density directly corresponds to baryon number density, if we assume the three flavor symmetric quark matter as mentioned in §1.

There are two QCD parameters in our theory: the quark mass m_q and the quark-gluon coupling constant α_c. These values are not well determined so far. In particular, the concept of quark mass involves subtle issues; it depends on the current or constituent quark picture and may be also related to the existence of chiral phase transition [10]. Here we allow some range for these parameters and take, for example, a set, $m_q = 300$MeV for strange quark and $\alpha_c = 2.2$, given by the MIT bag model [11]. In Fig. 3 two results are presented as functions of the polarization parameter p defined by the difference of the number of the spin-up and -down quarks, $n_q^+ - n_q^- \equiv pn_q$. The results clearly show the first order phase transition, while it is of second order in the Heisenberg model. The critical density is around $n_q^c \simeq 0.16\text{fm}^{-3}$ in this case, which corresponds to n_0 for flavor symmetric quark matter. Note that there is a metastable ferromagnetic state (the local minimum) even above the critical density.

Magnetic properties of quark liquid are characterized by three quantities, $\delta\epsilon, \chi$ and η; $\delta\epsilon \equiv \epsilon_{tot}(p = 1) - \epsilon_{tot}(p = 0)$, which is a measure for ferromagnetism to appear in the ground state. For small $p \ll 1$,

$$\epsilon_{tot} - \epsilon_{tot}(p = 0) = \chi^{-1} p^2 + O(p^4). \tag{4}$$

χ is proportional to the magnetic susceptibility. In our case it is less relevant

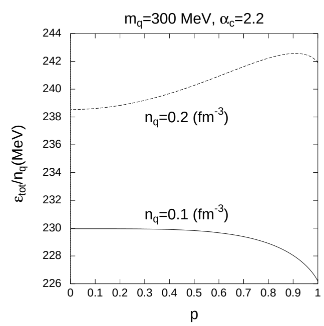

Figure 3: Total energy of quark liquid as a function of the polarization parameter for densities $n_q = 0.1, 0.2\mathrm{fm}^{-3}$.

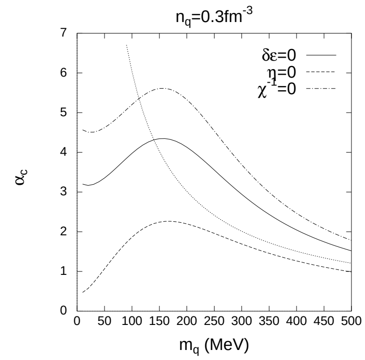

Figure 4: Phase diagram in the mass (m_q)- the coupling constant (α_c) plane. $\delta\epsilon$ in the nonrelativistic calculation is depicted for comparison (the dotted line) .

since the phase transition is of first order. Finally, $\eta \equiv \partial\epsilon_{tot}/\partial p\ |_{p=1}$, which is a measure for metastability to to exist.

In Fig. 4 we present a phase diagram in the $m_q{-}\alpha_c$ plane for $n_q = 0.3\mathrm{fm}^{-3}$, which corresponds to about twice n_0 for flavor symmetric quark matter. The region above the solid line shows the ferromagnetic phase and that bounded by the dashed and dash-dotted lines indicates the existence of the matastable state. For heavy quarks, which may correspond to the current s quarks or the constituent quarks before chiral symmetry restoration, the ferromagnetic state is favored for small coupling constant due to the same mechanism as in electron gas. The ferromagnetic state is favored again for light quarks, which may correspond to the current u, d quarks, while the nonrelativistic calculation never show such tendency. Hence this is due to a genuine relativistic effect, where the spin-flip interaction plays an essential role.

4 Strange quark star as magnetar

We have seen that quark matter has a potential to be ferromagnetic at rather low densities. Here we consider some implications on astrophysics.

Since the idea that nucleons are made of quarks has been confirmed, one has expected the existence of quark stars as a third branch of compact stars next to the neutron-star branch; when pressure or density is increased enough, there should occur the deconfinement transition and matter consists of quarks rather than nucleons. This naive expectation has been shown to be wrong; if the deconfinement transition occurs and quarks are liberated beyond the maximum central density of neutron stars (several times of n_0), they should behave like relativistic and almost free particles due to the asymptotic freedom of QCD. Thereby the adiabatic index (γ_{ad}) of quark matter becomes around 4/3. On the other hand, the criteria for the gravitationally stable stars reads $\gamma_{ad} > 4/3 + \kappa GM/R$, where the second term means the general relativistic correction, ~ 0.4 for $M \simeq 1.4M_\odot$ and $R \simeq 10$Km. Hence, the quark-star branch subsequent to that of neutron stars is impossible. If quark matter exists, it might occupy only the small portion of the core of neutron stars.

However, there is an alternative idea about quark matter and quark stars. As first indicated by Chin and Kerman [5], a large contamination of strange quarks are favorable for quark matter at low baryon density around 0.26fm^{-3}, which is about 1.5 n_0. Their calculation shows that the energy per baryon of quark matter is larger than that of nucleon, while less than Λ particle. Subsequently, Witten and Farhi and Jaffe [6] have pointed out the possibility that the almost flavor symmetric quark matter (strange matter) is the ground state of QCD at finite density within the reasonable range of QCD parameters.

Using the idea of strange matter some people suggested that quark stars with strange matter (strange quark stars) may be possible [7]. Since the EOS for strange matter shows the saturation property around n_0, strange quark stars can have any small radius and mass. Thereby, the quark-star branch can be clearly distinguished from that of neutron stars.

If a ferromagnetic quark liquid exists stably or metastably around or above nuclear saturation density, it has some implications on the properties of strange quark stars and strange quark nuggets: they should be magnetized in a macroscopic scale. For quark stars with the quark core of r_q, simply assuming the dipolar magnetic field, we can estimate its strength at the surface $R \simeq 10$Km,

$$B_{max} = \frac{8\pi}{3}\left(\frac{r_q}{R}\right)^3 \mu_q n_q, \tag{5}$$

with the quark magnetic moment μ_q. It amounts to order of $O(10^{15-17})$G for $r_q \sim O(R)$ and $n_q = O(0.1)$fm^{-3} , which should be large enough for magnetars. A sketch of a strange quark star is presented in Fig 5.

5 Summary and Concluding remarks

We have seen that the ferromagnetic phase is realized at low densities and the metastable state is possible up to rather high densities for a reasonable range of the QCD parameters.

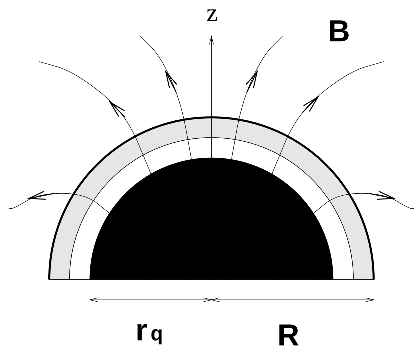

Figure 5: A model of strange quark star with $M \sim 1.4 M_{\odot}$ and $R \sim 10$Km. Almost all the portion is occupied by strange matter and a small vacuum gap may separate the quark core from the outer crust, which is composed of usual solid below the neutron-drip density.

We have found that ferromagnetic instability is feasible not only in the massive quark system but also in the light quark system: the spin-nonflip contribution is dominant in the nonrelativistic case as in electron gas, while a novel mechanism appears as a result of the large spin-flip contribution in the relativistic case.

If a ferromagnetic quark liquid exists stably or metastably around or above nuclear saturation density, strange stars may have a strong magnetic field, which strength is estimated to be strong enough for magnetars. Thereby it might be interesting to model SGR or AXP using our idea.

Our calculation is basically a perturbative one and the Fermi sea remains in a spherical shape. However, if we want to get more insight about the ferromagnetic phase, we must solve the Hartree-Fock equation and thereby derive a self-consistent mean-field for quark liquid. Moreover, we need to examine the long range correlation among quarks by looking into the ring diagrams, which has been known to be important in the calculation of the susceptibility of electron gas.

Recently, there have been done many works about the color superconductivity of quark matter. The order of the energy gap amounts to $O(100)$MeV, while the energy gain per particle is rather small and several MeV around n_0 , which should be the same order of magnitude as that for the ferromagnetism [12]. Hence it may be interesting to explore the phase diagram for ferromagnetic phase and superconducting phase.

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