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Analytic Time Delays and H_0 Estimates for Gravitational Lenses

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ABSTRACT

We study gravitational lens time delays for a general family of lensing potentials, which includes the popular singular isothermal elliptical potential and singular isothermal elliptical density distribution but allows general angular structure. Using a novel approach, we show that the time delay can be cast in a very simple form, depending only on the observed image positions. Including an external shear changes the time delay proportional to the shear strength, and varying the radial profile of the potential changes the time delay approximately linearly. These analytic results can be used to obtain simple estimates of the time delay and the Hubble constant in observed gravitational lenses. The naive estimates for four of five time delay lenses show surprising agreement with each other and with local measurements of H_0; the complicated Q 0957+561 system is the only outlier. The agreement suggests that it is reasonable to use simple isothermal lens models to infer H_0, although it is still important to check this conclusion by examining detailed models and by measuring more lensing time delays.

1. Introduction

Refsdal (1964) first proposed that time delays between images in multiply-imaged gravitational lenses can be used to measure the Hubble constant H_0. This method is attractive because it is a single-step process and is based on the well-established theory of General Relativity. After a long ordeal, this method is finally beginning to bear fruit. In the last few years, the time delays in six gravitational lenses have been measured, which yield

$H_0 \approx 65 \pm 15\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (e.g., Koopmans & Fassnacht 1999; Browne 2000). The lensing measurement is an important way of confirming and extending local determinations of H_0 (see Freedman 1999 for a review), which are still subject to systematic uncertainties such as the LMC distance, metallicity effects, and photometric contamination (e.g., Stanek, Zaritsky & Harris 1998; Kennicutt et al. 1998; Mochejska et al. 1999). At present, the error budget in the lensing measurement is dominated by systematic uncertainties in the lens modeling (see Keeton et al. 2000a for a discussion). Ultimately the accuracy may be limited by the uncertainties induced by the large-scale structure along the line of sight at the few percent level (Seljak 1994; Barkana 1996; Schneider 1997), although if random these effects should shrink as $N^{-1/2}$ as the number N of lenses with measured time delays increases.

Lensing measurements of H_0 are typically derived from models based on isothermal galaxies, because such models are consistent with individual lenses, lens statistics, stellar dynamics, and X-ray galaxies (e.g., Fabbiano 1989; Maoz & Rix 1993; Kochanek 1995, 1996; Grogin & Narayan 1996; Rix et al. 1997). Modelers usually adopt a parametrized form, either an isothermal elliptical potential (e.g., Blandford & Kochanek 1987) or an isothermal elliptical density (e.g., Kassiola & Kovner 1993; Kormann, Schneider & Bartelmann 1994; Keeton & Kochanek 1998), and adjust the parameters to fit the data.[1] The purpose of this paper is to gain insights into the time delays in a more general family of lens models that includes these commonly used isothermal models and their variants.

The outline of this paper is as follows. In section 2, we show that the time delays for singular isothermal elliptical potential (SIEP) and singular isothermal elliptical density (SIED) distributions have a remarkably simple and elegant form, and that the result actually holds for a general family of potentials with the form $\phi = r\mathcal{F}(\theta)$. In section 3, we show how the time delay is affected when we incorporate an external shear and change the radial profile of the potential. In section 4, we combine our analytic results with data for the time delay lenses to estimate H_0. Finally in section 5, we offer a summary and discussion.

2. Time Delay for Generalized Isothermal Models

The time delay in gravitational lenses is given by (e.g., Schneider, Ehlers & Falco 1992)

$$\Delta t(x,y) = \frac{D}{2c}(1+z_{\rm d})\left[(x-\xi)^2 + (y-\eta)^2 - 2\phi(x,y)\right], \quad D \equiv \frac{D_{\rm d} D_{\rm s}}{D_{\rm ds}}, \tag{1}$$

[1] However, see Saha & Williams (1997) and Blandford, Surpi & Kundić (2000) for novel non-parametric methods.

where $z_{\rm d}$ is the redshift of the lens, (ξ,η) is the angular source position, (x,y) is the angular image position, $D_{\rm d}$ and $D_{\rm s}$ are angular diameter distances from the observer to the lens and source, respectively, and $D_{\rm ds}$ is the angular diameter distance from the lens to the source. The dimensionless potential ϕ satisfies the two-dimensional Poisson equation

$$\nabla^2\phi(x,y)=2\kappa, \quad \kappa=\frac{\Sigma(x,y)}{\Sigma_{\rm cr}}\ , \tag{2}$$

where Σ is the projected surface mass distribution of the lens and $\Sigma_{\rm cr}=c^2D_{\rm s}/(4\pi G D_{\rm d} D_{\rm ds})$ is the critical surface density for lensing.

To proceed further we adopt a specific potential or density form. Individual lenses and lens statistics are usually consistent with isothermal models (e.g., Maoz & Rix 1993; Kochanek 1995, 1996; Grogin & Narayan 1996), so it is common to adopt either the isothermal elliptical potential,

$$\phi=\frac{a_0}{2}(x^2+y^2/q^2)^{1/2}, \tag{3}$$

or the elliptical density distribution,

$$\nabla^2\phi=2\kappa, \quad \kappa=\frac{a_0}{2q}(x^2+y^2/q^2)^{-1/2}, \tag{4}$$

where a_0 specifies the overall angular size of the lens. For the SIED, the lens potential is given by, e.g., Kassiola & Kovner (1993), Kormann, Schneider & Bartelmann (1994), and Keeton & Kochanek (1998). Note that we have chosen a coordinate system that is centered on the lens galaxy and aligned the x-axis along the lens galaxy's major axis. For simplicity, we have also assumed that the isothermal potential and density distributions are singular at the origin, although we return to this issue briefly in §5.

In this paper, we study a more general family of lens models. The potential is assumed to obey the relation

$$\phi=x\phi_x+y\phi_y, \tag{5}$$

where ϕ_x and ϕ_y are the first derivatives of the potential, which are just the components of the deflection angle in the x and y directions. In polar coordinates (r,θ), eq. (5) can be written in the simple form

$$\phi=r\frac{\partial\phi}{\partial r}, \tag{6}$$

whose general solution is

$$\phi=r\mathcal{F}(\theta), \tag{7}$$

where $\mathcal{F}(\theta)$ is an arbitrary function of θ. This potential corresponds to a density of the form

$$\kappa=\frac{1}{2r}\mathcal{G}(\theta), \quad \mathcal{G}(\theta)=\mathcal{F}(\theta)+\frac{\partial^2\mathcal{F}}{\partial\theta^2}. \tag{8}$$

Thus eq. (5) describes a family of scale-free models that includes both the SIEP and SIED models but allows for general angular structure; in other words, these are generalized isothermal models. We now show that this family is extremely useful in deriving the time delays between images.

The lens equation is given by

$$\xi = x - \phi_x, \quad \eta = y - \phi_y. \tag{9}$$

Substituting eqs. (5) and (9) into the time delay expression in eq. (1), we obtain

$$\Delta t(x,y) = \frac{D}{2c}(1+z_d)\left[(x-\xi)^2 + (y-\eta)^2 - 2x(x-\xi) - 2y(y-\eta)\right], \tag{10}$$

which can be further simplified into

$$\Delta t(x,y) = \frac{D}{2c}(1+z_d)\,(\xi^2 + \eta^2 - x^2 - y^2). \tag{11}$$

Since only the relative time delay is observable, we compute the time delay between two images i and j, which is given by

$$\Delta t_{i,j} = \frac{D}{2c}(1+z_d)(r_j^2 - r_i^2), \tag{12}$$

where $r_i = (x_i^2 + y_i^2)^{1/2}$ is simply the distance of image i from the center of the galaxy. This surprisingly simple expression is valid for all lens potentials satisfying eq. (5), including both the SIEP and SIED models as well as more general angular structures.[2] Since r_i^2 and r_j^2 are rotationally invariant, eq. (12) is valid even after an arbitrary rotation. In other words, provided we know the center of the lens galaxy, the predicted time delay can be computed using eq. (12) irrespective of the lens orientation and without any need to search for the best fit model parameters. Since the predicted time delay scales as $\propto H_0^{-1}$, it can be compared with a measured time delay to determine H_0; we demonstrate this method in §4.

For completeness we note that the following relations hold for a potential which satisfies eq. (5):

$$x\phi_{xx} + y\phi_{xy} = 0,\ x\phi_{xy} + y\phi_{yy} = 0,\ \phi_{xx}\phi_{yy} - \phi_{xy}^2 = 0. \tag{13}$$

The last expression can be used to simplify the magnification of the images:

$$\mu^{-1} = (1-\phi_{xx})(1-\phi_{yy}) - \phi_{xy}^2 = 1 - \phi_{xx} - \phi_{yy} = 1 - 2\kappa, \tag{14}$$

revealing a simple relation between the magnification and the surface density in this class of potentials.

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[2] After the completion of our paper, we learned that eq. (12) has been derived for particular cases by Koopmans, de Bruyn & Jackson (1998) and Zhao & Pronk (2000) using more complicated methods.

3. Time Delay in Other Potentials

3.1. Time delay in the presence of shear

In many gravitational lenses the images cannot be fit without the inclusion of a tidal perturbation from objects near the lens galaxy or along the line of sight (e.g., Keeton, Kochanek & Seljak 1997; Witt & Mao 1997). To lowest order, the perturbation can be modeled as an external shear with potential

$$\phi_\gamma = -\frac{1}{2}\gamma r^2 \cos 2(\theta - \theta_\gamma) = -\frac{1}{2}\left[\gamma_1(x^2 - y^2) + 2\gamma_2 xy\right], \tag{15}$$

where γ is the strength of the shear and θ_γ is its direction with respect to the lens galaxy's major axis, while $\gamma_1 = \gamma \cos 2\theta_\gamma$ and $\gamma_2 = \gamma \sin 2\theta_\gamma$. The total potential is then $\phi_{\rm tot} = \phi + \phi_\gamma$, and the lens equation can be written as

$$\xi = x - \phi_x + \gamma_1 x + \gamma_2 y, \quad \eta = y - \phi_y + \gamma_2 x - \gamma_1 y. \tag{16}$$

Following the same logic as in §2, we obtain the time delay

$$\Delta t(x,y) = \frac{D}{2c}(1+z_d)\left[\xi^2 + \eta^2 - r^2 - r^2\gamma\cos 2(\theta - \theta_\gamma)\right]. \tag{17}$$

The relative time delay between two images i and j in the presence of shear is then

$$\Delta t_{i,j} = \frac{D}{2c}(1+z_d)\left\{(r_j^2 - r_i^2) + \gamma\left[r_j^2\cos 2(\theta_j - \theta_\gamma) - r_i^2 \cos 2(\theta_i - \theta_\gamma)\right]\right\}. \tag{18}$$

This time delay depends on the shear amplitude and direction and therefore cannot be determined without detailed modeling. Nevertheless, we can make several remarks. The change in the time delay (relative to the no-shear case) is proportional to γ. For two-image lenses that have images at different distances ($r_i \neq r_j$) and a small shear, the shear should have a small effect on the time delay. However, when the images lie at approximately the same distance ($r_i \approx r_j$), such as in some four-image lenses, the shear term may be significant. In particular, perturbation theory reveals that in the presence of a shear the distance of an image from the critical curve scales $\propto \gamma$ (to lowest order; e.g., Kochanek 1991), so the two terms in eq. (18) can be comparable.

Although we cannot determine the time delay without knowing the shear amplitude and direction, we can put bounds on it. If the angle between the shear and the lens galaxy major axis is allowed to take on any value, then the time delay is bounded by

$$T_{i,j}^{(-)} \leq \Delta t_{i,j} \leq T_{i,j}^{(+)}, \tag{19}$$

where

$$T_{i,j}^{(\pm)} = \frac{D}{2c}(1+z_d)\left\{(r_j^2 - r_i^2) \pm \gamma\left[r_i^4 + r_j^4 - 2r_i^2 r_j^2 \cos 2(\theta_i - \theta_j)\right]^{1/2}\right\}. \tag{20}$$

If the images are directly opposite each other ($|\theta_i - \theta_j| = 180°$, as in a circular lens), the bounds are $T_{i,j}^{(\pm)} = (1 \pm \gamma)\Delta t_{i,j}^0$ where $\Delta t_{i,j}^0$ is the no-shear time delay from eq. (12). This means that, for example, a 10% shear ($\gamma = 0.1$) leads to a 10% uncertainty in the time delay (and hence H_0) if there is no information about the angle between the shear and the lens galaxy major axis. We apply eqs. (19) and (20) to obtain bounds for specific lenses in §4.

3.2. Time delay in non-isothermal models

Observed lenses seem to be consistent with isothermal galaxies (see introduction), but lenses with images at similar distances from the lens galaxy can often be modeled with other density profiles as well. Models of PG 1115+080 and B 1608+656 show that steeper density profiles lead to larger predicted time delays and hence larger values for H_0 (Keeton & Kochanek 1997; Impey et al. 1998; Koopmans & Fassnacht 1999). We analyze deviations from an isothermal profile by relaxing the condition in eq. (5) to

$$\beta\phi = x\phi_x + y\phi_y, \tag{21}$$

where β is a constant. (Isothermal models have $\beta = 1$.) This condition can be understood by finding the general solution for the potential,

$$\phi = r^\beta \mathcal{F}(\theta), \tag{22}$$

which corresponds to a density of the form

$$\kappa = \frac{1}{2} r^{\beta-2}\mathcal{G}(\theta), \quad \mathcal{G}(\theta) = \beta^2\mathcal{F}(\theta) + \frac{\partial^2\mathcal{F}}{\partial\theta^2}. \tag{23}$$

So this model has a radial power law for the potential and density, and β is the power law slope of the potential. Relaxing eq. (3) to fit eq. (21), an obvious (but not unique) family of solutions is given by $\phi \propto (x^n + y^n/q^n)^{\alpha/2}$, with $\beta = n\alpha/2 = \text{const}$. For $n > 2$, the iso-potential contours become boxier, while for $n < 2$, the contours become diskier. These contour shapes mimic those seen in the inner parts of elliptical galaxies (e.g., Bender et al. 1989).

More generally, for any potential satisfying eq. (21) the time delay is given by

$$\Delta t(x,y) = \frac{D}{2c}(1+z_d)\left[(\xi^2+\eta^2) + 2\frac{(1-\beta)}{\beta}(\xi x + \eta y) - \frac{(2-\beta)}{\beta}(x^2+y^2)\right]. \tag{24}$$

The time delay between two images i and j is then

$$\Delta t_{i,j} = \frac{D}{2c}(1+z_d)\left\{\frac{2-\beta}{\beta}\left(r_j^2 - r_i^2\right) + 2\frac{(1-\beta)}{\beta}\Big[\xi(x_i - x_j) + \eta(y_i - y_j)\Big]\right\}. \qquad (25)$$

Alternatively, if we substitute for the source position (ξ, η) using the lens equation we find

$$\Delta t_{i,j} = \frac{D}{2c}(1+z_d)\left[(r_j^2 - r_i^2) + 2(1-\beta)(\phi_j - \phi_i)\right], \qquad (26)$$

where $\phi_i = \phi(r_i, \theta_i)$. Eqs. (25) and (26) show that when the model is not isothermal ($\beta \neq 1$) we cannot eliminate the need for modeling to determine the source position and/or the potential at each image.

We computed time delays for non-isothermal elliptical potentials numerically, and we found that for small to moderate ellipticities the delays are well approximated by the isothermal time delay (eq. 12) modified by a multiplicative factor. For opposed images ($|\theta_i - \theta_j| \sim 180°$), the time delay is approximately $\Delta t_{i,j} \approx [2-\beta]\,\Delta t_{i,j}^{\rm iso}$, while for orthogonal images ($|\theta_i - \theta_j| \sim 90°$), the time delay is approximately $\Delta t_{i,j} \approx [(2-\beta)/\beta]\,\Delta t_{i,j}^{\rm iso}$. Similar scalings were found by Refsdal & Surdej (1994) and Witt, Mao & Schechter (1995, eq. 8). By contrast, the isothermal time delay is not a good approximation for close image pairs ($|\theta_i - \theta_j| \sim 0$), because the images are necessarily near the lensing critical curve and hence more sensitive to the particular lens model.

4. Implications for H_0

Lensing time delays are interesting because they offer a measurement of the Hubble constant independent of the distance ladder. Unfortunately, the method has proven to be somewhat more problematic than expected. Converting a time delay to H_0 requires a lens model, and it is often difficult to find a model that is both a good fit to the data and well constrained. At present we are in the paradoxical situation that better observational data seem to *increase* the uncertainties in H_0, because they require lens models that are more complex and thus harder to constrain (see, e.g., Bernstein & Fischer 1999; Keeton et al. 2000a).

As a result, some have advocated applying a simple and physically motivated class of models (such as isothermal ellipsoids) to an ensemble of time delay lenses and using the scatter in H_0 estimates to evaluate whether the result is robust (e.g., Koopmans & Fassnacht 1999; Browne 2000; E. Turner, private communication). The crucial assumption is that simple but statistically poor fits may be close enough to the truth to give reliable H_0

estimates – or if not, the discrepancy will show up as a large scatter in H_0 estimates. This approach yields a tantalizing concordance of lensing estimates at $H_0 \approx 65 \pm 15\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (Koopmans & Fassnacht 1999; Browne 2000), a value that is consistent with values from other methods. However, this value is based on a compilation of models from the literature that are very heterogeneous and include numerous explicit and implicit biases from the choice and quality of the data and the choice of classes of lens models.

Our analytic time delays now offer the ability to estimate H_0 for the time delay lenses with a method that is not only extremely simple but also uniform, i.e. derived using a uniform set of modeling assumptions and using the same type of data (image positions) for all lenses. We use the data for five of the time delay lenses (summarized in Table 1) to compare the observed and predicted time delays and determine H_0.[3] Figure 1 shows the H_0 estimates for the generalized isothermal models discussed in §2, assuming no external shear. Eq. (12) makes it essentially trivial to compute H_0 from easily measured quantities without any modeling. Moreover, the estimates depend only on the assumption of an isothermal profile for the lensing potential (and not on its angular structure). Finally, the H_0 estimates depend only weakly on the adopted cosmology.

With the exception of Q 0957+561, the naive H_0 estimates are consistent with each other and with values from other methods. Q 0957+561 stands out because the lens is complicated: the lens galaxy is embedded in a $\sigma \sim 700\,\mathrm{km\,s^{-1}}$ cluster and appears not to have a simple isothermal profile, two features that invalidate the class of potentials assumed in eq. (5) (see Bernstein & Fischer 1999; Keeton et al. 2000a; and references therein). Still, the agreement among the remaining four systems is surprising given that we have done no modeling. On the one hand, the agreement is somewhat misleading because two of the lenses have large systematic uncertainties in the lens galaxy position that lead to enormous H_0 errorbars (B 0218+357 and PKS 1830−211, which are discussed in detail by Lehár et al. 1999). On the other hand, the agreement between PG 1115+080 and B 1600+434 is intriguing because one has an elliptical lens galaxy and the other has an edge-on spiral lens galaxy. Moreover, the rough agreement between the H_0 estimates from the two time delays in PG 1115+080 provides a useful consistency check on the model assumptions.

Figure 1 suggests three conclusions about how to strengthen lensing's ability to provide an independent H_0 estimate. First, the dependence of the analytic time delays on the image positions emphasizes the importance of precise astrometry of not only the lensed images but also the lens galaxy. At present, the large H_0 uncertainties in B 0218+357 and

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[3]We exclude B 1608+656 because its two lens galaxies imply a potential that does not obey eq. (5); see Koopmans & Fassnacht (1999).

PKS 1830−211 are dominated by the 60–80 mas uncertainties in the lens galaxy positions. Second, the agreement between the naive H_0 estimates for four of the five systems suggests that the lens galaxies have a common density profile; furthermore, the agreement of the lens results with results from other methods suggests that assuming the generalized isothermal model is not unreasonable. (The same conclusion was reached by Koopmans & Fassnacht 1999.) A good way to test these conclusions is to increase the number of lenses with measured time delays and see whether the naive H_0 estimates continue to agree.

Third, there are still systematic uncertainties that can only be addressed with detailed modeling. Such modeling is clearly required for Q 0957+561. More generally, the modeling is required to fully understand the systematic uncertainties in the models and their effect on H_0. Models of PG 1115+080 reveal the importance of a group of galaxies surrounding the lens galaxy, whose effects cannot be modeled as a simple external shear; the models yield $H_0 = 44 \pm 4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (assuming an SIED galaxy; Impey et al. 1998), which is somewhat smaller than the naive H_0 estimate due in part to the gravitational focusing (or convergence) provided by the group. Although increasingly detailed models may be subject to degeneracies, the recent detection of the host galaxies of several lensed quasars (see Rix et al. 2000 for a summary) offers many new constraints that should break the model degeneracies and yield robust lens models (see Keeton et al. 2000ab; Blandford et al. 2000). Even if we can never break the degeneracies in all time delay lenses, comparing detailed models with our naive analytic H_0 estimates in a few systems will test whether the estimates are close enough to the truth to be useful.

One important systematic effect not included in Figure 1 is external shear, but our analytic results even permit an estimate of its effects (using eqs. 19 and 20). Figure 2 shows the bounds on H_0 for various values of the shear strength γ for the five lenses, assuming no knowledge of the angle between the shear and the lens galaxy major axis. The bounds would be narrower given constraints on this angle. An important qualitative result is that the H_0 estimate from the four-image lens PG 1115+080 is much more strongly affected by shear than those from two-image lenses, as expected from the fact that in four-image lenses the images tend to lie at approximately the same distance from the lens galaxy (see §3.1).

5. Conclusions

We have shown that for generalized isothermal models (with a potential of the form $\phi = r\mathcal{F}(\theta)$), the time delay between images can be expressed in a common and surprisingly simple form involving only the observed image positions (eq. 12). An external shear changes the time delay by an amount proportional to the shear strength, and affects 4-image lenses more than 2-image lenses. Changing the radial profile of the lens galaxy changes the time delay approximately by a multiplicative factor. In this paper, we have only considered singular potential and density distributions. This is justified because so far no convincing faint central image has been observed, and this sets stringent limits on the central core radius (Wallington & Narayan 1993; Kochanek 1996). Numerically we find that such small core radius changes the time delays negligibly ($\lesssim 1\%$).

Our simple expression for the time delay can be combined with directly observable quantities to give an estimate for the Hubble constant H_0 without the need for any modeling. The analytic H_0 estimates for four of five time delay lenses agree surprisingly well with each other and with distance ladder measurements of H_0. The outlier, Q 0957+561, is known to be a complicated system where the lensing potential is inconsistent with our assumptions of a pure isothermal model. Thus it appears that simple isothermal lens models are often reasonable for obtaining lensing estimates of H_0 (see also Koopmans & Fassnacht 1999).

Still, it is crucial to test this conclusion more rigorously, and there are three ways to do so. First, the astrometry for two of the time delay lenses (B 0218+357 and PKS 1830−211) must be significantly improved to reduce the uncertainties and verify the H_0 concordance. Second, more time delays should be measured (and good astrometric precision obtained for those lenses) to test whether all of the H_0 estimates derived from isothermal lens models are mutually consistent. Increasing the sample size will also reduce the random uncertainties due to large-scale structure. Third, more individual lenses should undergo detailed modeling to check the H_0 estimates obtained from isothermal models and understand the systematic uncertainties. These detailed models are most instructive when they are constrained by observations of the host galaxies of the lenses quasars, because requiring models to reproduce the distortion of the host galaxy significantly improves the ability to find a robust and well-constrained model (see Keeton et al. 2000ab; Blandford et al. 2000).

Finally, we recall that the dependence of the time delay on the density profile of the lens galaxy affects the H_0 estimates (also see Keeton & Kochanek 1997; Koopmans & Fassnacht 1999). If lens galaxies tend to have profiles steeper than isothermal ($\phi(r) \propto r^\beta$ with $\beta < 1$), then by insisting on using isothermal models we tend to underestimate H_0 (or vice versa). There are two possible situations:

- *Galaxies have a common density profile (little or no scatter in β).* If so, the H_0 estimates obtained by applying isothermal models to numerous lenses would show little scatter. If these estimates were to agree with distance ladder measurements of H_0, it would suggest that galaxies generally have isothermal profiles.

- *Galaxies have a scatter in β.* If so, isothermal lens H_0 estimates would show a significant scatter. It is possible that lenses are biased tracers of β: on the one hand, lenses with larger values of β tend to produce higher magnifications and hence have a larger magnification bias, while on the other hand they have smaller cross sections for lensing (e.g., Kochanek 1991; Wambsganss & Paczyński 1994; Witt et al. 1995; Witt & Mao 2000). However, when lenses are normalized to reproduce the image separations in observed lenses, these two effects essentially cancel (see Kochanek 1996). Thus it appears that lenses are not strongly biased with regard to β, so the scatter in lensing H_0 estimates would be similar to the scatter in β.

The present data suggest the first situation, namely that lenses are generally consistent with isothermal galaxies. By collecting more and better data, however, we will be able to test this conclusion more rigorously and obtain a robust and independent measurement of H_0.

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TABLE 1
OBSERVATIONAL DATA FOR TIME DELAY LENSES

Lens / Components	z_d	z_s	$\Delta t_{i,j}$ (days)	r_j ($''$)	r_i ($''$)	$\lvert\theta_i - \theta_j\rvert$ (°)	Ref
B 0218+357 / B−A	0.96	0.68	10.5 ± 0.2	0.24 ± 0.06	0.10 ± 0.06	176.4	3,8,10
Q 0957+561 / B−A	1.41	0.36	417 ± 3	5.2275 ± 0.0035	1.0340 ± 0.0035	154.2	2,7
PG 1115+080 / A−B	1.72	0.31	11.7 ± 1.2	1.147 ± 0.025	0.950 ± 0.004	115.5	1,5
C−B	1.72	0.31	25.0 ± 1.6	1.397 ± 0.004	0.950 ± 0.004	114.6	1,5
B 1600+434 / B−A	1.59	0.42	47 ± 6	1.14 ± 0.05	0.25 ± 0.05	179.4	4,6
PKS 1830−211 / B−A	2.51	0.89	26 ± 5	0.67 ± 0.08	0.32 ± 0.08	160.5	8,9

NOTE.— Observational data for five of the six time delay lenses. The remaining time delay system, B 1608+656, is excluded because the presence of two lens galaxies clearly rules out the simple potential assumed in the text (Koopmans & Fassnacht 1999). For B 0218+357, the position errorbars include the systematic uncertainty in the lens galaxy position (see Lehár et al. 1999). For PG 1115+080, the time delay has been measured between B and the combined A_1+A_2 components; the quoted position uncertainty includes the difference between A_1 and A_2. We do not give measurement uncertainties on the angle $|\theta_i - \theta_j|$ because they do not enter our calculations. References: (1) Barkana (1997); (2) Barkana et al. (1999); (3) Biggs et al. (1999); (4) Hjorth et al. (2000); (5) Impey et al. (1998); (6) Koopmans et al. (1998); (7) Kundić et al. (1997); (8) Lehár et al. (1999); (9) Lovell et al. (1998); (10) Patnaik et al. (1995).

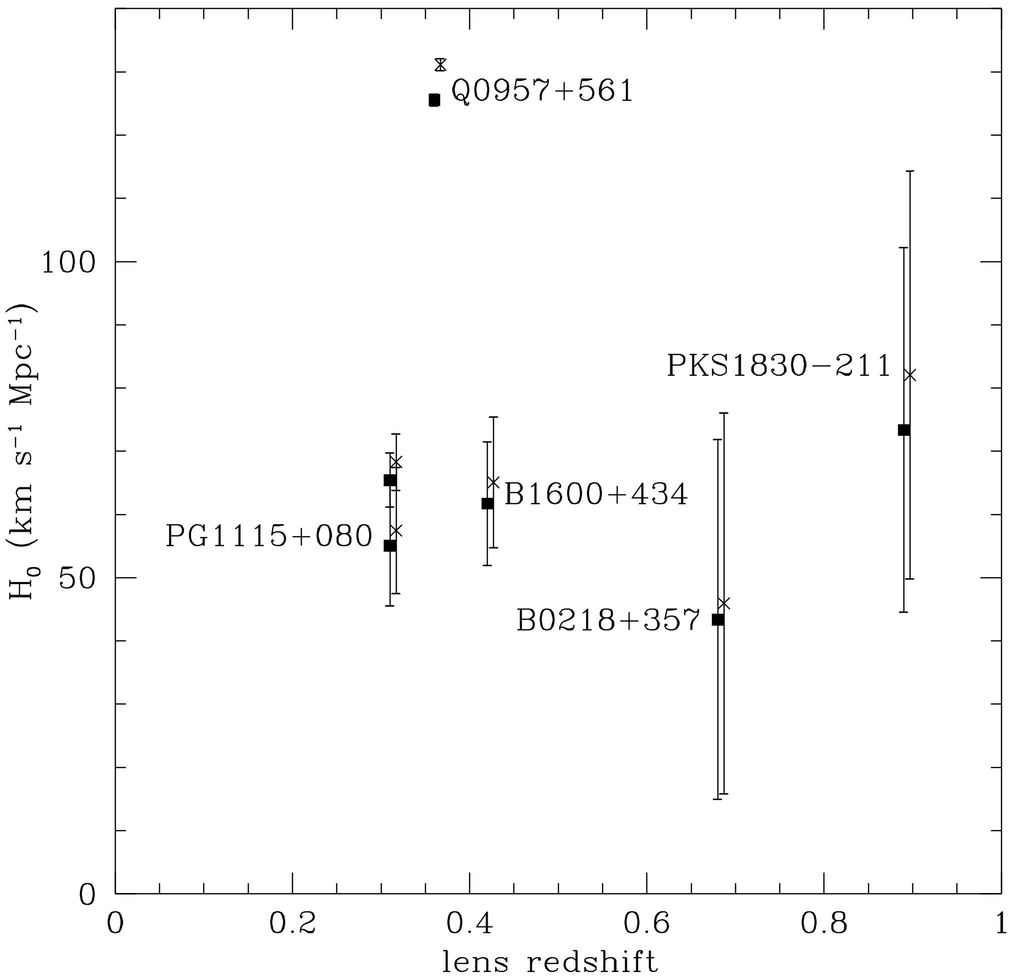

Fig. 1.— Naive H_0 estimates for the five time delay lenses listed in Table 1. The estimates are computed using eq. (12), i.e. under the assumption that the lensing potential obeys eq. (5) and there is no external shear. The errorbars indicate uncertainties due to measurement errors in the time delays and image positions. The filled squares are computed assuming a cosmology with $\Omega_M = 1$ and $\Omega_\Lambda = 0$, while the crosses (offset slightly in redshift) are computed assuming $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$. For PG 1115+080, there are two H_0 estimates because there are two measured time delays among the four images.

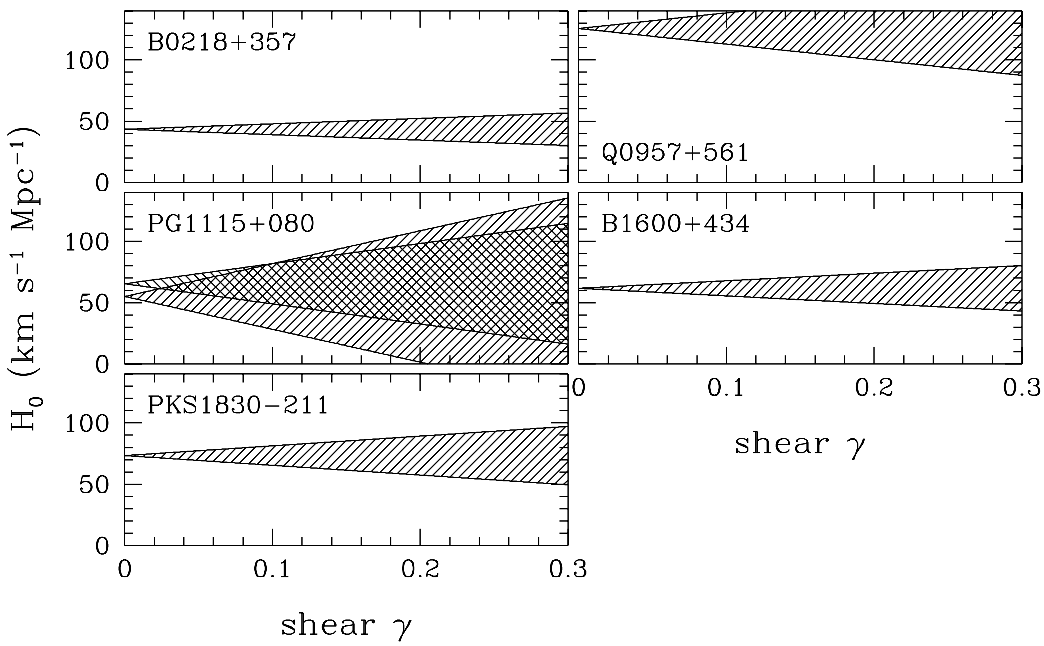

Fig. 2.— Effects of external shear on H_0 estimates. Each panel shows the bounds on H_0 as a function of the shear amplitude for the specified lens. (This figure does not include statistical uncertainties due to observational errors.) The bounds are computed using eqs. (19) and (20), assuming no knowledge of the angle between the shear and the lens galaxy major axis. The bounds would be narrower if this angle were constrained. For PG 1115+080, the two sets of bounds refer to the two different time delays. All results are computed for a cosmology with $\Omega_M = 1$ and $\Omega_\Lambda = 0$ but are nearly the same for other cosmologies (see Figure 1).