

Magnetic Flux Expulsion in the Powerful Superbubble Explosions and the α - Ω Dynamo.

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ABSTRACT

The possibility of the magnetic flux expulsion from the Galaxy in the superbubble (SB) explosions, important for the α - Ω dynamo, is considered. Special emphasis is put on the investigation of the downsliding of the matter from the top of the shell formed by the SB explosion which is able to influence the kinematics of the shell. It is shown that either Galactic gravity or the development of the Rayleigh-Taylor instabilities in the shell, limit the SB expansion, thus, making impossible magnetic flux expulsion. The effect of the cosmic rays in the shell on the sliding is considered and it is shown that it is negligible compared to Galactic gravity. Thus, the question of possible mechanism of flux expulsion in the α - Ω dynamo remains open.

Key words: galaxies: magnetic fields — ISM: magnetic fields — supernovae: general — MHD

1 INTRODUCTION

The magnetic field in our Galaxy and in other spiral galaxies is usually believed to have been amplified from a weak seed field by a hydromagnetic dynamo, which exists due to the presence of the large-scale differential rotation and small-scale cyclonic turbulence in the Galaxy (Parker 1970, 1971; Vainshtein & Ruzmaikin 1971, 1972; Moffat 1978). It has been suggested that any primordial magnetic field could be expelled from the Galaxy by the dynamic motions in less than a billion years (Parker 1971) so it would seem that some flux amplification is necessary to explain the Galactic field. The theory of such a dynamo has been formulated in a precise way through the mean field equations and the solutions to these equations indicate that the field would be amplified. The resulting field appears to correspond to the magnetic field patterns in our galaxy and others.

On the other hand, a number of criticisms of this theory have emerged. One of them concerns the intense development of the small scale fields, which could damp the turbulence and stop the dynamo action unless they saturate at levels which do not interfere with mean field dynamo (Kulsrud & Anderson 1992; Parker 1992; Vainshtein & Cattaneo 1992).

Another unresolved problem is the expulsion of flux from the galactic disc. A very important point is that the theory of the α - Ω dynamo predicts the amplification of some small preexisting magnetic field only if some magnetic diffusion is present in the Galaxy. However, too large a diffusion is destructive for the dynamo, because of its dissipative role in the process of generation and it seems attractive to

suppose that a dynamo without diffusion at all will be the most effective. But Ruzmaikin, Shukurov, & Sokoloff (1988) showed that this is not possible. The physical reason for this is rooted in the very strong flux freezing of the galactic plasma, because magnetic lines cannot break and the number of field lines in the disc can be increased only by toroidal stretching, which is accomplished by the dynamo action. Any stretching creates field of both signs, to conserve the total flux, and, for net amplification to occur, those portions of field lines which are of the wrong sign must be expelled from the Galaxy. In the standard α - Ω dynamo theory this is done by magnetic diffusion.

Ruzmaikin, Shukurov, & Sokoloff (1988) considered α - Ω -dynamo in the case of the thin disc and demonstrated that temporal evolution of the magnetic flux is governed by the following set of equations:

$$\frac{\partial}{\partial t} \int_0^1 B_r(t, z) dz = \beta \left. \frac{\partial B_r}{\partial z} \right|_0^1, \quad (1)$$

$$\frac{\partial}{\partial t} \int_0^1 B_\varphi(t, z) dz = \beta \left. \frac{\partial B_\varphi}{\partial z} \right|_0^1 + G \int_0^1 B_r dz, \quad (2)$$

where $G = r\partial\Omega/\partial r$ -measure of the differential rotation, β -magnetic diffusion, and 0 and 1 correspond to the center plane and boundary of the Galactic disc.

The β terms represent the expulsion of flux. It is clear, that setting $\beta = 0$ in (1) and (2) we immediately get that

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$$\int_0^1 B_r dz = \text{const}, \quad (3)$$

and that $\int_0^1 B_\phi dz$ may grow only linearly due to the stretching of lines by the galactic differential rotation. Thus, there is no exponential field growth, essential to the dynamo. One can easily see that for the dynamo to operate, there must be some nonzero flux escape through the upper boundary of the disc, that is $\beta \neq 0$.

This is consistent with the topological constraint that the total number of lines of force including those negative lines expelled from the disc must be constant. Indeed, a simple estimate of the expulsion terms making use of the numerical results in Ruzmaikin, Shukurov, & Sokoloff (1988) shows that the negative flux expelled during one growth cycle is comparable to the positive flux in the disc at the beginning of the e-folding.

Although the α - Ω dynamo theory is complicated, this physical intuition of flux expulsion can be considered in the absence of these complications. Further, the mechanism of expulsion need not be tied to the β diffusion inside the disc.

The main problem with the expulsion of flux is that this flux is loaded with matter so that it is related to the expulsion of matter against the strong gravity of the galactic disc. The most likely process to expel flux is the phenomenon of sequential supernova (SN) or superbubbles (SB), which sweep up matter into dense, radiatively cooled shells. Magnetic field, tied to the matter due to the strong flux freezing in the ISM, is also swept up and deposited in these shells. If some part of the shell leaves the Galaxy, it carries the frozen-in magnetic field with it, thus producing the flux expulsion.

But most of these superbubbles are not powerful enough to expel matter out of the gravitational well of the disc. The only possibility for flux expulsion seems to be: as the bubble expands, the field lines in the shells of SBs are not horizontal but form arcs, along which matter can slide down, lowering the amount of matter on the top of the lines and allowing some flux to escape.

There is further difficulty with the mechanism involving the SBs which is relevant to its application to the α - Ω -dynamo. The α - Ω -dynamo assumes *small-scale* turbulence while the cavities produced by SB explosions may reach ~ 500 pc or larger, which is greater than some of the length scales of the galactic disk. For the dynamo theory in its conventional form to be applicable it is important that turbulence be small scale, because it involves the expansion of the turbulent electromotive force \mathcal{E} , which describes the effect of turbulent motions on the mean (or ensemble-averaged) magnetic field, in terms of the mean magnetic field itself and its spatial derivatives:

$$\mathcal{E}_i = \alpha_{ij} < B_j > + \beta_{ijk} \frac{\partial < B_j >}{\partial x_k} \quad (4)$$

(Moffat 1978)

If the scale of the turbulence is too large, then the expansion is invalid and usual α - Ω -dynamo theory must be modified. That's why, for example, direct application of α and β tensors calculated by Ferrière (1995, 1998) for SBs

and SNs to the α - Ω -dynamo theory can lead to an overly optimistic estimates of the rate of flux escape.

Although α - Ω -dynamo theory is not strictly applicable to the case of SBs, the actual operation of them in amplifying the field is clear from the work of Ferrière (1991, 1995, 1998). It is also clear that the rapid escape of the flux from the disk is essential. In this paper we show that because of the deep gravitational well of the Galaxy it is difficult for the matter and field lines to escape, and consequently for the mean field to grow. In Ferrière's works she finds the lines of force rising with the SB but does not follow them long enough to see that they must fall back into the disk and inhibit the growth of the field.

In this note we quantitatively examine the dynamics of the rising field lines carried by SBs and show that even with sliding the matter and flux are unlikely to escape. Thus, the requirement of the escape of flux provides a strong constraint for the α - Ω dynamo to overcome if it is to amplify the galactic magnetic field.

2 SLIDING OF MATTER FROM THE TOP OF SB; FORMULATION OF THE PROBLEM

Multiple supernovae from OB associations can carve out large cavities of hot gas, called superbubbles. McCray & Snow (1979) first described them. When the SB expands into surrounding medium it sweeps up interstellar matter, giving rise to a massive expanding shell. Inside the volume surrounded by this shell a hot rarefied low-density gas is contained which provides the pressure driving further expansion of the shell. The energetic source for sustaining this pressure is provided by continuous energy input from the SN explosions in the center of SB. Weaver et al. (1977) calculated the evolution of the bubble driven by the continuous wind from the central source and Mac Low & McCray (1988) applied this theory to the case of supernovae driven SBs. They show that the radius of such a SB, expanding in a uniform medium of density ρ_0 with continuous energy input in the center $L_{SN} = L_{38} 10^{38}$ ergs s^{-1} (the luminosity of SB conveniently expressed in units of 10^{38} ergs s^{-1}), is given by

$$R(t) = \left(\frac{125}{154\pi} \right)^{1/5} L_{SN}^{1/5} \rho_0^{-1/5} t^{3/5} = 267 \left(\frac{L_{38} t_7^3}{n_0} \right)^{1/5} \text{ pc}, \quad (5)$$

where $t_7 = t/10^7$ yr, with the velocity of the envelope changing as

$$u(t) = \dot{R}(t) \approx 15.7 \left(\frac{L_{38}}{n_0 t_7^2} \right)^{1/5} \text{ km s}^{-1}. \quad (6)$$

The inner pressure in the volume bounded by the shell varies as

$$P_{in}(t) = \frac{7}{(3850\pi)^{2/5}} L_{SN}^{2/5} \rho_0^{3/5} t^{-4/5} \\ = 4.1 \times 10^{-12} \left(\frac{L_{38}^2 n_0^3}{t_7^4} \right)^{1/5} \text{ ergs cm}^{-3}, \quad (7)$$

due to the work done on the expansion of the shell and the energy injection in the center of SB.

When the shell expands in the real Galactic environment, there is also a gravitational force which tends to slow down the vertical expansion. Also, the distribution of

medium is highly inhomogeneous with height z over the Galactic plane. Various components of ISM have different length scales and characteristic densities, but they all have exponential or Gaussian decreasing profiles in z and thus drop very rapidly with height. This effect is very noticeable for powerful superbubbles, for which the expansion radius may exceed the height scale of the matter distribution. As we will see this can lead to Rayleigh-Taylor instabilities.

At the final stage of the SB expansion, its velocity becomes comparable to the sound velocity of the ambient ISM and shock wave will no longer exist. However, this does not prevent the SB from further expansion, because there is still a lot of momentum in its massive shell of swept up material and the ram pressure at high z is negligible. This means that expansion actually continues until the velocity of the shell drops to zero under the action of Galactic gravity:

$$u = 0. \quad (8)$$

This stopping condition is different from the one which Ferrière used (1998) in her investigation of the role of SBs in the $\alpha - \Omega$ dynamo. Her condition (expansion stops when the shell velocity is of the order of speed of sound in the ambient ISM, in other words when the pressure inside the SB becomes comparable to the ram pressure) is applicable only for massless shells without any inertia. In reality shell is quite massive and the momentum stored in it drives further expansion of the bubble against the ram pressure and the galactic gravity.

Also when the SB expands in the presence of the gravity, the swept-up matter in its shell is likely to slide downward along the shell as mentioned before. This can influence the efficiency of flux removal by SBs. Obviously, in the absence of sliding, escaping matter takes with it all the frozen-in magnetic flux. Now, if we allow the matter in the shell to slide downward perpendicular to the field B in the shell it will take the magnetic field lines away from the top of the SB, thus reducing the magnetic flux to be removed. But it can also have a significant effect on the dynamics of the SB itself, because as the matter slides from the top in any direction, the upper part of the shell becomes lighter and acceleration due to the inner pressure of the hot gas will get larger in proportion to the surface density decrease, while the gravitational deceleration stays the same. This results in some additional acceleration of the top of the shell and, in principle, this can significantly change the dynamics of this part of SB if there is enough time before the shell stops. If this happens, and a substantial part of the mass slides down, the top of the shell may continue its expansion upward and drive the remaining matter and frozen-in flux to a further distance from the galactic plane and possibly expel the flux entirely from the Galaxy. For this reason it is very important to estimate the numerical value of this effect.

3 BASIC EQUATIONS

Giuliani (1982) gave a general formulation of the thin-shell approximation for hypersonic, hydromagnetic flows, axisymmetric about the z axis, including motions along the shell. We use his equations in our description of gravitational matter removal from the top of SB. All our further considera-

tion is restricted to the case when the angular distance of the shell element from the shell top, θ , is very small, $\theta \ll 1$.

We suppose the expansion of the SB is described by some law $R = R(t)$. We will also suppose for simplicity that the form of the shell near its top can be roughly approximated by a sphere. The validity of this assumption will be discussed later.

We made some further simplifications, one of which is the neglect of the pressure gradient along the shell. It is clear enough that any such pressure gradient would only reduce the downsliding of the matter. This means that our estimate is only an upper limit of the sliding and the real sliding will actually be smaller.

We also completely neglect the influence of the magnetic field on the dynamics of the sliding. This assumption is justified in the early periods of the Galaxy's life, when the magnetic field was weak. At the present time this is not completely valid, because the magnetic field is strong and magnetic tension may play some dynamical role in the expansion process. SBs with strong magnetic field were considered analytically by Ferrière (1991) and numerically by Tomisaka (1992).

Thus, the only external force in our analysis is the gravity due to the stars and ISM in the Galaxy. We take gravity as given, because the self-gravity of the bubble is negligible. With this in mind the system of the equations of Giuliani describing the gravitational fall of matter from the top of the shell reduce to

$$\frac{\partial}{\partial t}(R^2 \sin \theta \sigma) = \rho_0 u R^2 \sin \theta - \frac{\partial}{\partial \theta} (R(t) \sin \theta \sigma v_{\parallel}), \quad (9)$$

$$\frac{\partial v_{\parallel}}{\partial t} + \frac{\rho_0 u v_{\parallel}}{\sigma} + \frac{v_{\parallel}}{R} \left(u + \frac{\partial v_{\parallel}}{\partial \theta} \right) - g \sin \theta = 0. \quad (10)$$

Here v_{\parallel} is the tangential velocity of the matter along the shell arising from the presence of gravity, $u = \dot{R}(t)$ is the velocity of the shell expansion (dot means time derivative), σ is the surface density of the shell, ρ_0 is the density of the unperturbed gas in front of the shell, and g is the gravitational acceleration.

Now, at an early stage of SB expansion, when its radial velocity is very high and the transverse velocity due to the galactic gravity is not very large, the effect of sliding is negligible, because the total velocity of the shell element is directed almost radially. Thus, it is reasonable to suppose that the most noticeable effect of sliding will occur only during the later stages of the shell evolution, when it sufficiently slows down in the radial direction. But by the onset of this later stage the SB has expanded in the direction perpendicular to the galactic plane to a distance larger than the scale height of matter distribution. This reasoning allows us to neglect the second term proportional the ambient density in equation (10). It makes equation (10) completely independent of equation (9) since it then contains no terms in σ .

However, in equation (9), we must keep terms in ρ_0 because σ still grows due to swept up matter.

Also we can neglect the term $\partial v_{\parallel} / \partial \theta$ in (10) compared to the radial velocity u . We discuss this omission later. Thus equations (9) and (10) reduce to the following simplified system of equations:

$$\frac{\partial}{\partial t}(R^2(t) \sin \theta \sigma) = \rho_0 u R^2(t) \sin \theta$$

$$-R(t)\sigma \frac{\partial}{\partial \theta} (\sin \theta v_{\parallel}) - R(t) \sin \theta v_{\parallel} \frac{\partial \sigma}{\partial \theta}, \quad (11)$$

$$\frac{\partial v_{\parallel}}{\partial t} + v_{\parallel} \frac{u}{R(t)} = g \sin \theta. \quad (12)$$

In general, we assume that gravitational acceleration, g , is a function of the z -coordinate in the Galaxy. Bearing in mind that $u = \dot{R}(t)$ and $v_{\parallel} = 0$ at $t = 0$, we can integrate equation (12) to get

$$\begin{aligned} v_{\parallel}(t) &= \frac{\sin \theta}{R(t)} \int_0^t R(t') g(t') dt' \\ &= \frac{\sin \theta}{R(t)} \int_0^t R(t') g(z_0 + R(t')) dt', \end{aligned} \quad (13)$$

where z_0 is the height in the Galactic plane where the explosion occurred.

We see that $v_{\parallel} \propto \sin \theta$. This means that in equation (11) we may neglect the last term near the top of SB since it is proportional to $\sin^2 \theta$. Then equation (11) further reduces to

$$\dot{\sigma} + \frac{1}{R} \left[2\dot{R} + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta v_{\parallel}) \right] \sigma = \rho_0 \dot{R}. \quad (14)$$

This equation can be integrated with the initial condition $\sigma = 0$ at $t = 0$:

$$\begin{aligned} \sigma(t) &= \frac{1}{R^2(t)} \exp \left(- \int_0^t \kappa(t') dt' \right) \\ &\times \int_0^t R^2 \dot{R} \rho(z_0 + R(t')) \exp \left(\int_0^{t'} \kappa(t'') dt'' \right) dt', \end{aligned} \quad (15)$$

where

$$\kappa = \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta v_{\parallel}). \quad (16)$$

Substitution of expression (13) into (16) gives

$$\begin{aligned} \kappa &= 2 \cos \theta \frac{1}{R^2(t)} \int_0^t R(t') g(t') dt' \\ &\approx \frac{2}{R^2(t)} \int_0^t R(t') g(z_0 + R(t')) dt', \end{aligned} \quad (17)$$

near the top of SB.

From the formulae (15) and (16) it is easy to see that the importance of sliding near the top is determined by the quantity

$$\zeta = e^{\kappa} = \exp \left(\frac{2}{R^2(t)} \int_0^t R(t') g(t') dt' \right), \quad (18)$$

which can be determined for any given $R(t)$.

If $\zeta \simeq 1$, we can safely neglect the sliding of matter but if $\zeta \gg 1$, sliding will play important role in the dynamics of late stages of SB expansion.

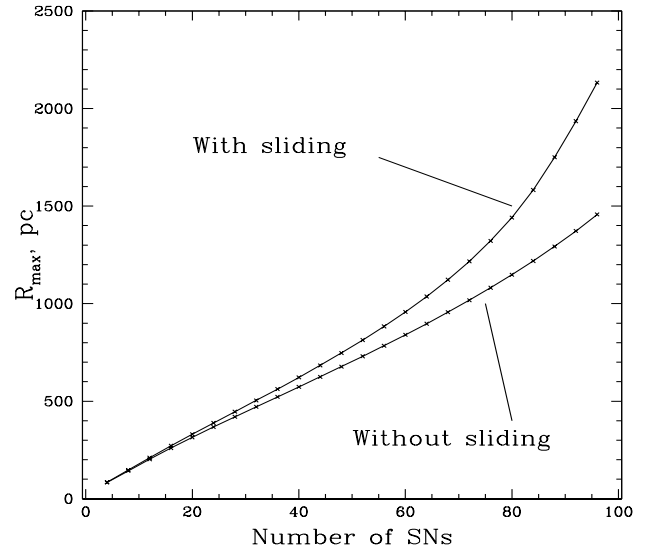
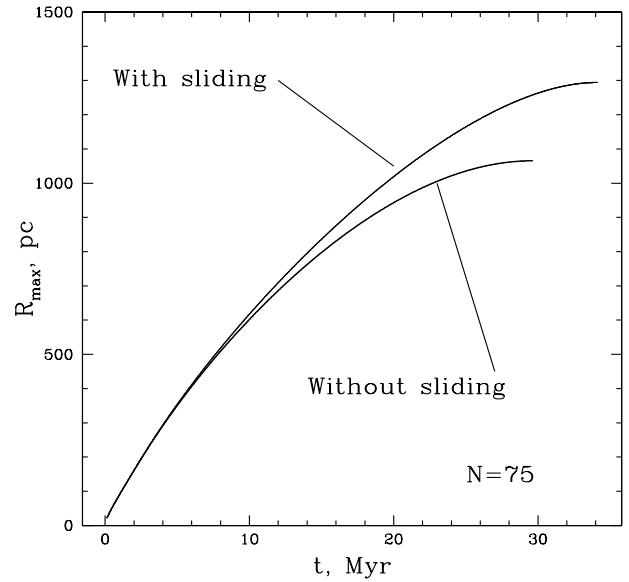


Figure 1. (a) Dependence of polar radius R (at $\theta = 0$) upon the expansion time t for the case of the sliding of matter due to the Galactic gravity (upper curve) and without sliding (lower curve). The case of 75 SNs in SB is considered, corresponding to the luminosity $L = 7.5 \times 10^{37}$ ergs s^{-1} . Note that SB with sliding has larger final size, than that without sliding. (b) Dependence of the maximum radii of the SB, R_{max} , on the luminosity (expressed through the number of SNs in the SB in equation (21)) with (upper curve) and without sliding (lower curve). The difference in final sizes of the SBs with and without this effect increases with the bubble's luminosity L .

4 INFLUENCE OF SLIDING ON THE SHELL EXPANSION

Let us examine the equation for $R(t)$ near the top of the bubble taking into account the effect of the unloading the matter from the shell's top on the radial expansion of the SB itself.

To do this we consider a solid angle $d\Omega$ of the shell near its top. We can write the following equation for its motion:

$$\frac{d}{dt} (\sigma \dot{R} R^2 d\Omega) = (P_{in} - P_{out}) R^2 d\Omega - R^2 \sigma g d\Omega. \quad (19)$$

We can combine this equation with equation (15) to find $R(t)$ and $\sigma(t)$ near $\theta = 0$ as a functions of time t if we know how P_{in} behaves. All other quantities, namely P_{out} and g are given empirically, as a function of z , by the position of the shell's top.

Equations (19) and (14) can be combined to give

$$\ddot{R} + \dot{R}^2 \frac{\rho_0}{\sigma} - \frac{2\dot{R}}{R^2} \int_0^t R(t') g(t') dt' = \frac{P_{in} - P_{out}}{\sigma} - g \quad (20)$$

and together with equation (15) give $R(t)$.

These equations are solved numerically to get the behavior of R . First we supposed that the inner pressure is governed by equation (7). This underestimates the pressure and the height reached. But if there is no escape of flux under this assumption there is certainly no escape in real conditions. We have taken the model of ISM from Ferrière (1998), that is we supposed that density and pressure of ISM are contributed by 5 components, having different number densities, length scales and temperatures: neutral, cold, warm, ionized, and hot. We also use her approximation for the gravitational acceleration g . We suppose for simplicity that the luminosity of the SB is constant in time during 37 Myr, until the death of the $8M_{\odot}$ stars (Ferrière 1995), and equal to

$$L = 10^{36} \times N \text{ erg s}^{-1}, \quad (21)$$

where N is the number of SNs in the star cluster. For this calculation we suppose that the inner pressure P_{in} changes in accordance with equation (7) during the first 37 Myr of the SB expansion, with ρ_0 the mass density at the site of explosion. After 37 Myr the interior cools adiabatically because the inner pressure and inertia of the shell continue to drive the shell expansion so that the volume bounded by the shell increases. We also carry out calculations for the different pressure law. But we neglect radiative cooling of the hot gas in the interior of the bubble during the entire explosion, so that real SB expansion is always smaller than we obtain here.

In Figure 1a we show the dependence of R upon time t for the case of a SB with $N = 75$ SNs in it, going off near the Sun at an initial Galactic altitude 100 pc. For comparison we also depict the curve without sliding for the same SB, which is obtained by setting g in the integral in the left-hand side of equation (20) to zero (but not in the right-hand side!).

If the sliding is taken into account, the time when expansion stops is $t_s = 35$ Myr and $R_{max} = 1336$ pc; neglecting sliding we get $t_s = 30$ Myr and $R_{max} = 1094$ pc. It is obvious that effect of sliding should be more pronounced in more powerful bubbles because of their longer lifetimes and stronger gravity at the heights to which they bring the matter. To illustrate this we plot in Figure 1b the maximum radii of SBs of various luminosities with and without sliding for the same conditions as the Figure 1a. It is clearly seen that sliding plays important role for powerful SBs, making their final radii dozens of percent larger than that without sliding.

In Figure 2 we plot, for comparison, the expansion velocity of the $N = 75$ SB top and the velocity of sliding along the shell for the same SB, divided by $\sin \theta$ (which virtually

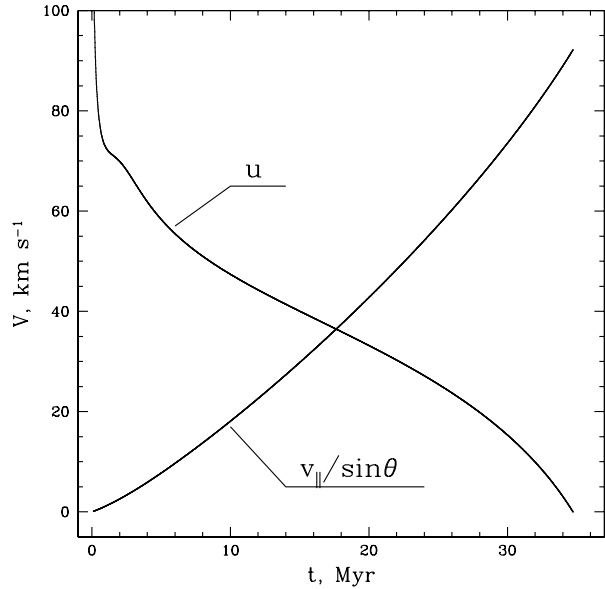


Figure 2. Plots of the expansion velocity of the SB top u (declining curve) and the velocity of the downsliding of the matter along the shell $v_{||}$ (growing curve), divided by $\sin \theta$. These plots show that at the early stages of the shell expansion we can neglect $\partial v_{||} / \partial \theta$ compared to u in equation (10).

equals to $\partial v_{||} / \partial \theta$ near the SB top). One can see that, up to the first 20 Myr, u is larger than $\partial v_{||} / \partial \theta$, which justifies our neglect of corresponding term in equation (10). By that time most of the SB expansion has already occurred, so the inclusion of the term with $\partial v_{||} / \partial \theta$ into (12) does not change the final results significantly. Moreover, if we do include it, it would only suppress sliding, as can be seen from (10), so that our results for sliding and shell expansion can be considered to give upper bounds for the final height of the shell.

5 RAYLEIGH-TAYLOR INSTABILITY

It is tempting to expect that for even more powerful SBs, than $N = 100$ SNs, the effect of sliding will be even more pronounced and the final size of the SB may reach scales comparable to the size of Galaxy, thus making possible the escape of the flux. But when the luminosity of the SB approaches 10^{38} ergs s^{-1} , corresponding to the number of SNs $N \approx 100$, another effect becomes important for the fate of SB. At such a large luminosity the shell starts accelerating some time before its expansion could be stopped by the gravity and it would accelerate, in principle, to a very high velocity if there is enough time for it.

However, it was first noticed by Mac Low & McCray (1988) and then proven numerically by them (Mac Low & McCray 1989) that as soon as the shell starts to accelerate it becomes Rayleigh-Taylor unstable and eventually breaks up. They called this process a “blowout” of the shell into the halo. Indeed, the effective gravity in the moving shell in the case of acceleration is directed towards the center of SB, that is the dense cold gas in the shell is pushed by rarefied hot gas of the interior and this leads to the instability. The shell fragments into blobs of cold, dense gas, which continue

moving with velocities they attained before fragmentation. There is no further significant acceleration of the shell, because the hot rarefied gas from the interior of the SB escapes into halo thus allowing the inner pressure to drop. After that time each blob will move ballistically, sweeping up some small mass, though this effect might be not so important at high altitudes, where the density of all the components of the ISM is very low. This means that if the speed of the blob at this time is less than the escape velocity from the Galaxy, matter can not leave but must return to the Galactic plane from halo and there is no contribution to the flux escape. For this instability the galactic gravity contributes to the effective gravity as well, thus making the shell unstable even when it is still decelerating.

In our situation, when we have a semi-infinite hot rarefied medium and a dense slab of gas of finite thickness H up on it, the fastest growing mode of the instability has a scale of the order of H , so that the growth rate of the instability is given by

$$\gamma^2 = -(\ddot{R} + g) / H. \quad (22)$$

The increment of instability or the amount of e-foldings reduces to

$$\int \gamma dt = \int dt \sqrt{(g + \ddot{R}) / H}. \quad (23)$$

For the fragmentation to proceed effectively we require

$$\int \gamma dt > 1. \quad (24)$$

To get the shell thickness, let us note that the gas initially located between the heights $z_0 + z$ and $z_0 + z + dz$ is deposited into the shell between $h + dh$ and h from the shell outer surface. The conservation of the number of particles accounting for the sliding gives that number density of particles in the shell at local thickness h related to the number density of particles initially at the point $z_0 + z$ as

$$\frac{\sigma}{\sigma_0} n_0(z_0 + z) z^2 dz = n(h) R^2 dh, \quad (25)$$

where

$$\sigma_0(t) = \frac{1}{R^2(t)} \int_0^t R^2 \dot{R} \rho(t') dt', \quad (26)$$

is just σ without sliding. The number density n is given by $n = P_{sh}(h) / kT_{sh}$, where T_{sh} is the temperature in the shell, which we assume to be equal to 10^4 K, and P_{sh} is the pressure at the local thickness h in the shell. Here we have taken into account only the thermal pressure and neglected the cosmic ray pressure. This seems to be the reasonable assumption, because, as will be shown later, they have very high drift velocity and may easily escape from the compressed shell along the magnetic field lines deposited into it, since the lines themselves leave the shell. The pressure P_{sh} is comparable to the inner pressure P_{in} , because inside the shell it has to drop from P_{in} on the inner surface to $P_{out} \ll P_{in}$ outside the shell.

Combining all these considerations we obtain that

$$H \approx \frac{\sigma}{\sigma_0} \int_0^R \frac{n_0(z_0 + z) k T_{sh} z^2}{P_{in} R^2} dz. \quad (27)$$

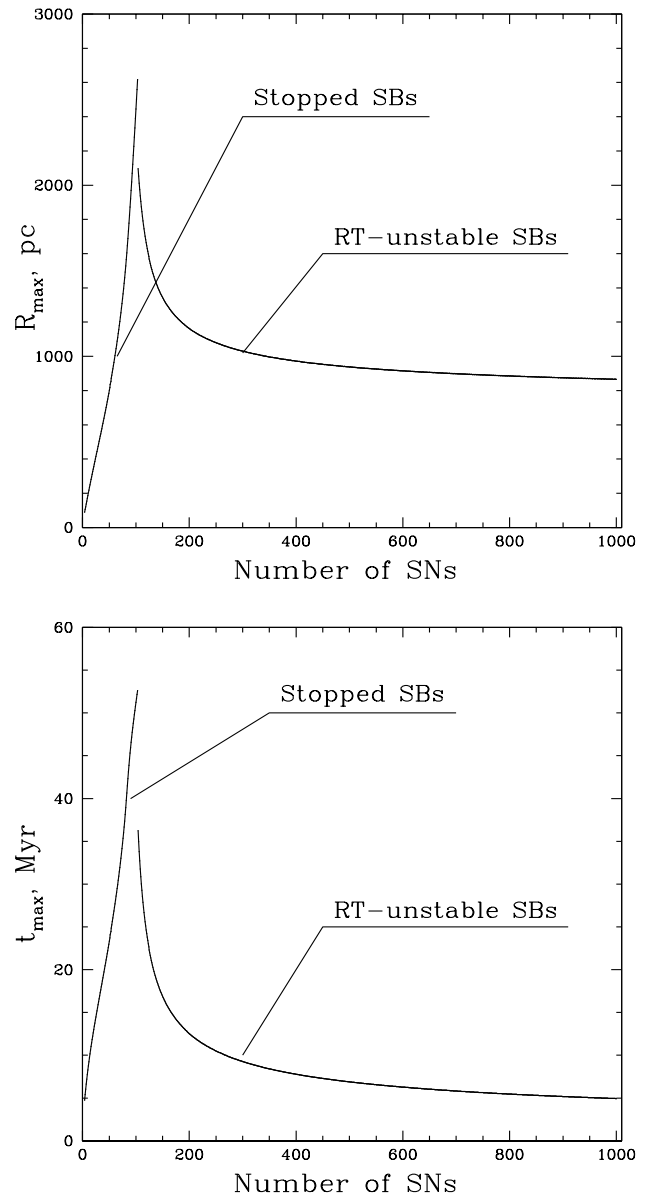


Figure 3. (a) Final size of the SB, R_{max} , as a function of N – the number of SNs producing different luminosities. Shells of the low luminosity bubbles are stopped by galactic gravity (left branch), while those of the more powerful bubbles (right branch) are disrupted by the Rayleigh-Taylor instability. (b) Distribution of times when the SB expansion stops, t_{max} , either due to the galactic gravity (left branch) or the Rayleigh-Taylor instability (right branch) for different number of SNs.

Numerical estimates show that during the first several e-foldings of the Rayleigh-Taylor instability, the shell thickness H changes very little, less than 10%. During this time the size of the shell changes less than $\sim 30\%$, so that the geometrical effect of the stretching the scale of the perturbation mode in the expanding shell is quite moderate. For this reason the modes which were unstable at the very onset of the instability stay unstable during several e-foldings thus developing the nonlinear stage of the instability and disrupting the shell.

During the development of the instability the velocity of

the shell does not change drastically, it is larger only (3–5)% than the velocity at the very onset of instability. Thus, we may safely assume that after the Rayleigh-Taylor instability is fully developed in the shell, we get no further acceleration of the shell fragments.

We consider the role of the Rayleigh-Taylor instability on the fate of SBs of various luminosities with inner pressure from (7) located at the altitude $z_0 = 100$ pc near the Sun. The results for maximum size of the shell and time when it either stops due to gravity or fragments because of Rayleigh-Taylor instability, are shown on Figure 3. Equation (24) is chosen to be the condition for Rayleigh-Taylor fragmentation of the shell, because it corresponds to the onset of the nonlinear stage of this instability when the shell is being disrupted. The two branches of Figure 3 correspond to the bubbles which were stopped (left branch) and to those which were disrupted (right branch). We see that for a chosen dependence of inner pressure upon time the transition to the Rayleigh-Taylor regime occurs for $N = 104$ supernovae in the SB. The maximum possible size and the greatest lifetime of the SB are achieved just before this N and are equal to $R_{max} = 2616$ pc and $t_{max} = 52.6$ Myr. After that size is reached the expansion drops rapidly due to the early onset of the Rayleigh-Taylor instability in the shell, and we see that for SBs with $N \sim 1000$ shell fragmentation occurs at a very early time $\sim 5 - 6$ Myr. It must be emphasized that at such an early stages the approximation of constant luminosity may not be justified and dynamics might be more complex. We believe that the result of the analysis is not very sensitive to the model assumptions.

In Figure 4 we plot the dependence of the blob velocity upon the luminosity of the SB. One can see that it is sufficiently less than ~ 430 km s $^{-1}$, the estimated lower bound on the Galactic escape velocity (Leonard & Tremaine 1990), so that we may conclude that shells formed by SBs with number of SNs in them $N \leq 10^3$ do not give rise to the mass and flux outflow from the Galaxy. Even if we go to a SB with a luminosity an order of magnitude larger ($N = 10^4$), the velocity at the moment of fragmentation is only $v_{max} \approx 336$ km s $^{-1}$, which is obviously not enough to leave the Galaxy.

6 IMPORTANCE OF THE SHAPE OF THE SB TOP

In our treatment of the SB expansion we have considered the shape of the shell near its top to be spherical. This enables us to use a simple fact that in this case the projection of Galactic gravitational acceleration along the shell is just $g_{\parallel} = g \sin \theta$ which sufficiently simplifies the problem.

In reality, of course, the surface density is not uniform on the top but depends upon the angle θ . The nonuniformity grows with growing θ . The inner pressure will accelerate parts of the shell closer to the top stronger than ones further from the top and it will distort the form of the shell. This distortion in its turn changes g_{\parallel} which influences the sliding of the matter and thus leads to further changes of the shell's form. The accurate treatment of the problem requires including this effect self-consistently in our calculations, but we can avoid this by noting that influence of a change of the shell's shape on the process of sliding can be attributed to

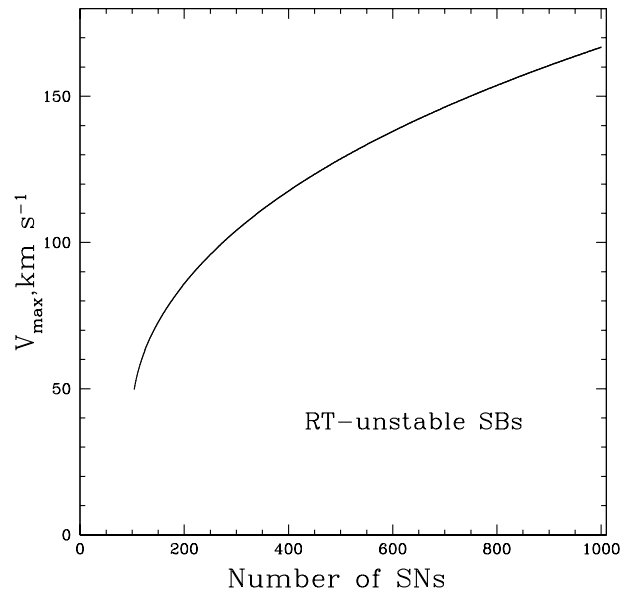


Figure 4. Velocity of blobs formed in the process of the fragmentation of SB shell, v_{max} , versus the luminosity of the SB (number of SNs in the initial star cluster). Though this velocity increases with the luminosity of the SB, it does not reach the escape velocity from the Galaxy.

the change in gravitational acceleration g , rather than the projection angle.

We carried out calculations identical to those with spherical top but have taken g in equation (17) to be 4 times larger than it is in reality. The result was that, as the luminosity of the SB was increased, they expanded faster, due to the more effective sliding of the matter from the top. But this in turn lead to a more rapid onset of Rayleigh-Taylor instability in the shell: it started to develop for SBs with $N > 69$. The bubble with $N = 69$ stops at a time $t_{RT} = 45.7$ Myr and size $R_{max} = 2398$ pc. The conclusion is obvious: the change of the shell shape might influence the sliding of the matter, but it leads to the development of the Rayleigh-Taylor instability for even smaller luminosities than in the case of the bubbles with the spherical top, making the impossibility of the expulsion of the flux from Galactic disc even more certain. That is why we think a more self-consistent approach to the problem of the shell shape will not change the general result.

7 DIFFERENT PRESSURE LAW

The pressure law (7) which we used in all our calculations was derived actually for the case of the SB expanding in a uniform medium and thus may be not a very good approximation for our purposes especially when the sliding of the matter influences the expansion of the shell and its size cannot be described by equation (5). For that reason we decided to use a different, more realistic pressure dependence to check if it makes a significant difference in our results.

Maciejewski & Cox (1999) proposed a simple, explicit, analytical approximation for the kinematics of the blast wave propagating in an exponentially stratified medium:

$$\rho_0 = \rho_* e^{-z/h}, \quad (28)$$

where ρ_* is the density at the explosion site and h is the stratification length scale. They considered an explosion in the framework of the Kompaneets approximation (Kompaneets 1960) with inner pressure constant throughout the volume engulfed by the shock. They showed that the form of the shell is very close to an ellipsoid with minor and major axes b and a related by

$$\tan \frac{b}{2h} = \sinh \frac{a}{2h} \quad (29)$$

and the distance from the explosion site to the ellipsoid center, s ,

$$\tan \frac{s}{2h} = \cosh \frac{a}{2h}. \quad (30)$$

At the same time, Weaver et al (1977) showed that internal energy of the SB interior is constant fraction of the total energy and for a spherical SB expanding in a homogeneous medium

$$E_{in} = \frac{5}{11} L_{SN} t. \quad (31)$$

We assume that this is also valid for the case of SB in a non-homogeneous density distribution, so that that inner pressure

$$P_{in} = \frac{E_{in}}{V} = \frac{5}{11} \frac{L_{SN} t}{\pi a b^2}. \quad (32)$$

We relate the semi-major axis a to the distance from the center of explosion to the top of the shell: $a + s = R$, by

$$\frac{a}{2h} + \log \left(\cosh \frac{a}{2h} \right) = \frac{R}{2h}. \quad (33)$$

Formulae (32),(33), and (29) give us P_{in} for a given R . The approach of Maciejewski & Cox (1999) includes neither galactic gravity nor the slippage from the top but it takes the inhomogeneity of the surrounding medium into account and enables us to test the stability of our results against different model assumptions.

The real ISM contains many components distributed with various length scales so we take rather arbitrarily $h = 200$ pc in our case. The particular choice of h turns out not to play a significant role. The results for superbubbles of various luminosities located at $z_0 = 200$ pc near the Sun are shown in Figures 5a and 5b.

We see that differences are only quantitative compared to the case of pressure law (7). Strong Rayleigh-Taylor instability starts to dominate the kinematics of the shell when the number of SNs in SB is larger than $N = 18$. This SB reaches the maximum size $R_{max} = 447$ pc at a time $t_{max} = 41$ Myr. Due to the specifics of the the chosen pressure law, SB with the higher luminosity, developing the Rayleigh-Taylor instability, reach somewhat larger size, maximum is $R_{RT} = 630$ pc. If we take, for example, $h = 100$ pc, then the shells start to be not stopped by the gravity but disrupted by Rayleigh-Taylor instability even for smaller N . Thus, again, for powerful SBs, expansion is limited by the shell fragmentation so that all major results of the consideration with the simplified pressure law remain valid.

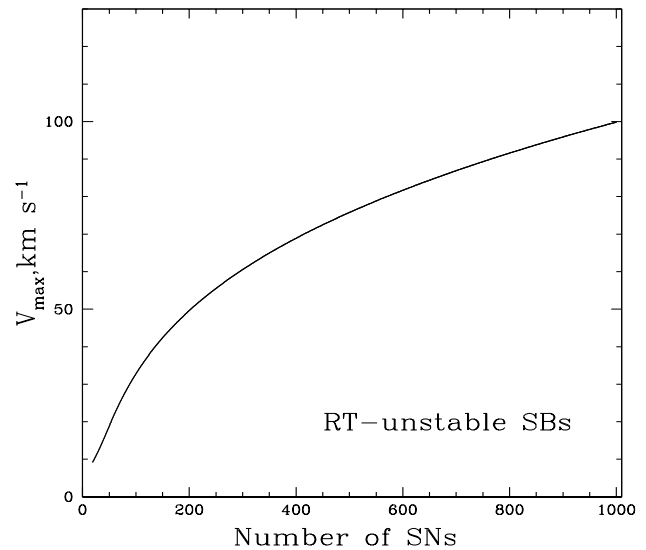
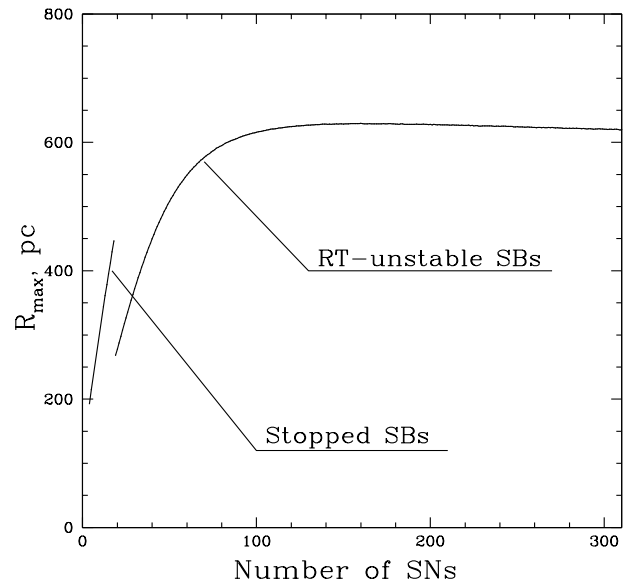


Figure 5. (a) Final size of the shell, R_{max} , and (b) velocity of the shell, v_{max} , at the moment of stopping or fragmentation versus the number of SNs in SB. Pressure law is different from the case represented on the Figure 3 and is given by equations (32),(33), and (29).

8 POSSIBLE IMPORTANCE OF THE CR PRESSURE IN THE SHELL

Kulsrud (1999) proposed that sliding of the matter from the top of the SB shell may be inhibited to some extent by cosmic ray (CR) pressure gradient, thus further supporting our conclusion about the impossibility of flux expulsion from the Galaxy. Now, on the basis of the better knowledge of the processes going on in SB shell, we can check this idea in more detail.

Let us consider the magnetic flux tube with the cross section constant along the tube in the shell which reached the size R and expands with velocity $u = \dot{R}$. This assumption is good for the cylindrical shell, with its axis lying in

the Galactic plane, but it will be clear further that this assumption is not important. The continuity equation for the CR along the flux tube reads:

$$\frac{\partial}{\partial t} (n_{CR}R) + \frac{\partial}{\partial \theta} [n_{CR} (v_{\parallel} + v_d)] = 0, \quad (34)$$

where n_{CR} is the number density of the CR, v_{\parallel} and v_d are the velocities of matter along the shell with respect to the rest system and of CR with respect to the matter correspondingly. The drift of the CR along the magnetic field is determined by the gradient of their number density and by the scattering of the CR by the Alfvénic turbulence. The scattering of CR by the self-generated Alfvén waves was first considered by Kulsrud & Pearce (1969) and Wentzel (1969) and they showed that CR cause the Alfvén waves to grow with a rate

$$\Gamma = C \frac{\pi}{4} \Omega_0 \frac{v_d - v_A}{v_A} \frac{n_{CR}(\epsilon)}{n}, \quad (35)$$

where Ω_0 is the nonrelativistic cyclotron frequency, $n_{CR}(\epsilon)$ is the number density of CR with energy greater than ϵ , n is the density of the ISM, $v_A = B/\sqrt{4\pi n m_H}$ is the Alfvén speed (m_H is the mass of hydrogen atom), and C is a constant of the order of unity whose value depends upon the energy spectrum of CR. These waves are also nonlinearly damped by the scattering of the beat waves on the ions:

$$\gamma_d = \sqrt{\frac{\pi}{8}} \frac{v_T}{c} \Omega \left(\frac{\delta B}{B} \right)^2, \quad (36)$$

(Lee & Völk 1973; Kulsrud 1978) with δB being the magnetic field perturbations due to the Alfvén waves, Ω – relativistic Larmor frequency, and v_T – thermal velocity of ions in ISM. Scattering of CR occurs at a rate

$$\nu = \Omega \left(\frac{\delta B}{B} \right)^2, \quad (37)$$

(Kulsrud 1995), so that the typical mean free path of the CR is

$$\lambda = \frac{c}{\nu} = \frac{c}{\Omega} \left(\frac{B}{\delta B} \right)^2. \quad (38)$$

On the other hand, the drift velocity is given by

$$n_{CR} v_d = c \frac{\lambda}{R} \frac{\partial n_{CR}}{\partial \theta}. \quad (39)$$

Assuming that $v_d \gg v_A$, that Alfvén waves are in equilibrium, $\Gamma = \gamma_d$, and combining equations (35), (36), (38), and (39) we get

$$\begin{aligned} |v_d| &= \left(\sqrt{\frac{2}{\pi}} \frac{c v_T n v_A}{C \Omega_0 n_{CR}^2} \left| \frac{1}{R} \frac{\partial n_{CR}}{\partial \theta} \right| \right)^{1/2} \\ &= \left(A \left| \frac{1}{R n_{CR}^2} \frac{\partial n_{CR}}{\partial \theta} \right| \right)^{1/2}. \end{aligned} \quad (40)$$

If we introduce the characteristic drift velocity

$$v_D = \sqrt{\frac{A}{n_{CR} R}}, \quad (41)$$

and suppose that

$$v_D \gg v_{\parallel}, \quad (42)$$

then it is easy to see from equation (34) that when $v_d \approx v_{\parallel}$ the variation of the CR density can be expressed as

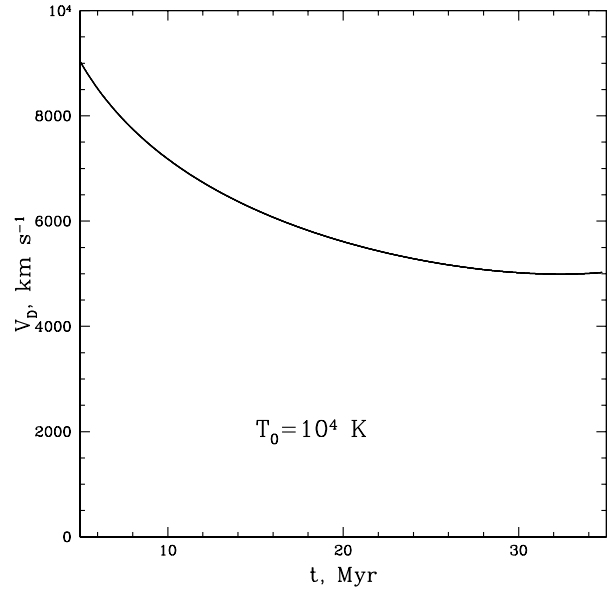


Figure 6. Characteristic drift velocity of CR, v_D , in the shell of SB due to the scattering of the CR by the self-generated Alfvén turbulence. Note the significant difference in the magnitudes of v_D and the sliding velocity v_{\parallel} in the shell (Figure 2).

$$n_{CR} R = n_{CR_0} R_0 \left(1 + \left(\frac{v_{\parallel}}{v_D} \right)^2 \chi_1(\theta) \right), \quad (43)$$

where $\chi_1(\theta)$ is some function of order of unity.

To see if the condition (42) is fulfilled, we calculated v_D given by (41) and (40) at various time moments for the SB with $N = 75$ SNs, taking into account the compression of the matter and CR in the shell, which we calculate in the manner similar to the calculation of the thickness of the shell, and supposing that the temperature in the shell after cooling is $T_s = 10^4$ K. Compression of matter in the shell (and, consequently, of the CR) depends upon the temperature of ambient ISM T_0 , so that v_D scales as $v_D \propto T_0^{-3/4}$. In Figure 6 we plot v_D , defined by (41) for $T_0 = 10^4$ K.

One can see from comparison of Figures 6 and 2, that condition (42) is always fulfilled for $T_0 = 10^4$ K. It might be violated if $T_0 \sim 10^6 - 10^7$ K, that is if the SB expands into the predominantly hot ISM component, but near the top the condition (42) is always fulfilled even for these high temperatures, because $v_{\parallel} \propto \sin^2 \theta$.

In the approximation given by (42) the spatial CR density perturbations are small and the first term in equation (34) is negligible. This means that the drift velocity is almost equal to the velocity of matter v_{\parallel} but opposite to it in the direction, so that CR slide through the matter to maintain constant spatial density and the time variations of their density are only due to the shell expansion. Then the equation of continuity reduces to

$$\frac{\partial}{\partial \theta} \left(n_{CR} v_{\parallel} - \sqrt{\frac{A}{n_{CR}}} \frac{\partial n_{CR}}{\partial \theta} \right) = 0. \quad (44)$$

This equation can be integrated with initial condition $\partial n_{CR} / \partial \theta = 0$ at $\theta = 0$, where $v_{\parallel} = 0$, to give

$$\frac{\partial n_{CR}}{\partial \theta} = n_{CR} v_{\parallel}^2 \left(\frac{A}{n_{CR} R} \right)^{-1}. \quad (45)$$

Then the ratio of CR pressure term in the equation (10) to the gravitational acceleration along the shell is

$$\frac{\nabla p_{CR}}{\rho_s g_{\parallel}} = \frac{\partial p_{CR}}{\partial n_{CR}} \frac{\partial n_{CR}}{\partial \theta} \frac{1}{Rg \sin \theta \rho_s} \\ \sim \frac{p_{CR}}{\rho_s g R} \left(\frac{v_{\parallel}}{\sin \theta v_D} \right)^2 \sin \theta \sim 10^{-4} \sin \theta, \quad (46)$$

for $T_0 = 10^4$ K, or maybe $\sim 0.1 \sin \theta$ at $T_0 \sim 10^7$ K, so that the effect the CR pressure gradient has on the sliding of the matter from the shell's top is negligible for all interesting cases, and, actually, not only at the shell's top, but also for $\theta \sim 1$.

Obviously, the assumption that the flux tube has constant cross section and the cylindrical geometry implied here are not important, because the uniformity of the CR pressure is achieved due to the very high characteristic drift velocity v_D of the CR compared to the sliding velocity, which perturbs the uniformity, and not due to any peculiarities of the flux tube geometry.

9 SUMMARY

In this paper we consider the effect of the downsliding of the matter which takes place in the expanding superbubbles for application to the expulsion of the magnetic flux from the Galaxy. This expulsion is an important ingredient of the α - Ω dynamo theory. However, it is shown that even the inclusion of the sliding into the calculations of the kinematics of the superbubbles, does not enable matter and frozen in flux to leave the Galaxy in SBs. One must note, that the impossibility of the flux escape from the Galaxy weakens significantly the Parker's (1971) argument against the primordial magnetic field, because there is actually no mechanism to expel it.

Some authors (Korpi et al 1999a, Korpi et al 1999b) have considered dynamics of the superbubbles in the gravitational field of the Galaxy in greater detail and they also find that the matter does not reach terminal velocities larger than the escape velocity from the Galaxy. In their simulations they do observe the development of the SB and its blowout from the disk, but at the height of several kiloparsecs the velocity of the matter is too small for the matter and the field to leave the Galaxy, which agrees with our conclusions. However, they do not comment on its relation to the dynamo.

Other authors (e.g. Hanasz & Lesch 1998, Moss et al 1999), more directly concerned with dynamos, do not investigate the dynamics of escape but merely assume that once the magnetic field lines reach the boundary of the disk, they are advected away by some mechanism leading to a vacuum boundary conditions. They do mention magnetic buoyancy as a possible escape mechanism but buoyancy is unlikely to be important outside the disk and clearly plays no role in the SB escape mechanism, especially when the magnetic fields are first amplified from weak seed fields.

In this paper an analytical formalism for the consideration of sliding was built on reasonable assumptions, which enabled to include the back-reaction of lowering the density of the matter on the top of the SB, on the expansion of the shell in the radial direction. All the SB, depending on their luminosity and conditions in the surrounding ISM

were demonstrated to fall into two classes: low luminosity SBs which are stopped by the Galactic gravitational field and fall back, and powerful SBs, which are possibly able to reach low density regions at high altitude. Even without sliding these powerful SBs might be able to expel matter from the Galaxy. However, before this happens the shells of these high luminosity SBs fragment into separate blobs due to the Rayleigh-Taylor instability, which develop when the shells start accelerating at the high altitudes, where the density of the matter and the outer pressure are very small. These separate fragments of the shells continue to move ballistically with velocities too small to leave the Galaxy.

It was shown that our conclusion about the impossibility of flux to escape in SB explosions does not depend essentially upon the details of the inner pressure behavior in the SB or the shape of the SB top. The inclusion of the CR pressure gradient in the shell also does not influence the downsliding because of the high uniformity of CR density in the shell caused by the very large diffusion velocity of CR in the shell.

It should be remarked that throughout the paper we have assumed that magnetic field lines are tied to the gas. If the ionization is the shell is low, then ambipolar diffusion (under the influence of the CR pressure gradient perpendicular to the shell) could allow some small slippage of the field lines through the neutral component and, thus, some small amount of escape. We do not discuss this possibility in this paper.

All this lead us to the conclusion that if there does exist some mechanism responsible for the flux expulsions from the Galaxy needed by the α - Ω dynamo this mechanism is not superbubbles. The question of whether such a mechanism exist remain an open one.

10 ACKNOWLEDGEMENTS

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