

The Effect of Hydrostatic Weighting on the Vertical Temperature Structure of the Solar Corona

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ABSTRACT

We investigate the effect of hydrostatic scale heights $\lambda(T)$ in coronal loops on the determination of the vertical temperature structure $T(h)$ of the solar corona. Every method that determines an average temperature at a particular line-of-sight from optically thin emission (e.g. in EUV or soft X-ray wavelengths) of a multi-temperature plasma, is subject to the emission measure-weighted contributions $dEM(T)/dT$ from different temperatures. Because most of the coronal structures (along open or closed field lines) are close to hydrostatic equilibrium, the hydrostatic temperature scale height introduces a height-dependent weighting function that causes a systematic bias in the determination of the temperature structure $T(h)$ as function of altitude h . The net effect is that the averaged temperature seems to increase with altitude, $dT(h)/dh > 0$, even if every coronal loop (of a multi-temperature ensemble) is isothermal in itself. We simulate this effect with differential emission measure distributions observed by *SERTS* for an instrument with a broadband temperature filter such as *Yohkoh/SXT* and find that the apparent temperature increase due to hydrostatic weighting is of order $\Delta T \approx T_0 \times h/r_\odot$. We suggest that this effect largely explains the systematic temperature increase in the upper corona reported in recent studies (e.g. by Sturrock et al., Wheatland et al., or Priest et al.), rather than being an intrinsic signature of a coronal heating mechanism.

Subject headings: Sun: atmosphere — Sun: corona — Sun : X-Rays, gamma rays

1. INTRODUCTION

Attempts to solve the elusive *coronal heating problem* have been undertaken by determining the heating function $E_H(h)$ as function of height h , inferred from the vertical temperature structure $T_e(h)$ of the solar corona. In this context, a systematic temperature increase $T(h)$ with height h has been reported from numerous observations of the quiet diffuse corona, coronal arcades, or coronal loops (Mariska & Withbroe 1978; Kohl et al. 1980; Falconer 1994; Foley et al. 1996; Sturrock, Wheatland, and Acton 1996a; 1996b; Wheatland, Sturrock, & Acton 1997; Fludra et al. 1999; Priest et al. 1999; 2000). A common method that is chosen to infer the vertical temperature structure $T_e(h)$ is the extraction of soft X-ray fluxes in different wavelengths as function of height, say $F_1(h)$ and $F_2(h)$ from two different wavelengths 1 and 2, and then to use the filter-ratio method $Q(h) = F_2(h)/F_1(h)$ to determine the temperature as function of height, $T(h)$, by inverting the filter-ratio function $Q(T)$. The filter-ratio method has some obvious limitations, such as the limited range where the function $Q(T)$ is unique and thus permits only an inversion within this range, but the method has also some more subtle drawbacks in the case of a multi-temperature plasma, as it exists

in the solar corona. In principle, the filter-ratio method is only exact for an isothermal plasma, within the uniqueness range of $Q(T)$. The solar corona consists of myriads of open and closed field lines filled with plasmas of almost every temperature in the range of $10^4 \lesssim T \lesssim 10^7$ K, which is usually quantified with a differential emission measure distribution $dEM(T)/dT$. This multi-thermal nature can cause systematic errors in the determination of an average vertical temperature profile $T_e(h)$, due to a systematic weighting bias of the temperature-dependent pressure and density scale heights (Fig.1). The purpose of this Letter is to demonstrate this systematic error in the determination of the vertical temperature profile $T_e(h)$, for some typical observations of active regions and the quiet corona, using a broadband-filter instrument, such as the *Yohkoh Soft X-Ray Telescope (SXT)*.

2. MODEL

The soft X-ray flux measured along a given line-of-sight represents an integral over emission measure contributions from plasmas with different temperatures, which can be expressed by the differential emission measure distribution $dEM(T)/dT$, where the emission measure contribution at a given temperature $[T, T + dT]$ itself represents an integration along the line-of-sight z ,

$$\left(\frac{dEM(T)}{dT}\right)dT = \int n_e^2(T, z)dz . \quad (1)$$

The flux measured by a detector i is then given by the product of the differential emission measure function $dEM(T)/dT$ with the instrumental temperature response function $R_i(T)$,

$$F_i = \int \frac{dEM(T)}{dT} R_i(T) dT . \quad (2)$$

We characterize now the solar corona by a superposition of many different flux tubes (along open or closed magnetic field lines), each one having its own temperature and density function. For the purpose of this demonstration we make the simplest assumption that is compatible with observations, namely (1) that each flux tube is near-isothermal (as it has been established for many observed EUV loops in the temperature range of $T_e \approx 1.0 - 2.0$ MK, e.g. Neupert et al. 1998; Lenz et al. 1999; Aschwanden et al. 1999; 2000a; 2000b), and that each flux tube is in near-hydrostatic equilibrium (a condition that has been verified for EUV loops within factors of $\approx 1-3$, Schrijver et al. 1999; Aschwanden et al. 1999; 2000a; 2000b). Thus, the density structure of a (near-isothermal) fluxtube can be approximated by

$$n_e(h, T_e) = n_{e0} \exp\left[-\frac{h}{\lambda(T_e)}\right] . \quad (3)$$

where the density (or pressure) scale height $\lambda(T_e)$ in hydrostatic equilibrium is proportional to the temperature T_e ,

$$\lambda(T_e) = \frac{k_B T_e}{\mu m_p g_\odot} = \lambda_0 \left(\frac{T_e}{1 \text{ MK}}\right) \quad (4)$$

with $\lambda_0 = 47$ Mm for coronal conditions, with μm_p the average ion mass (i.e. $\mu \approx 1.4$ for H:He=10:1), and g_\odot the solar gravitation. The differential emission measure $dEM(T, h)/dT$ is proportional to $n_e(h)^2$, and thus has approximately an exponential height dependence with a scale height half of the density scale height. In first order, the height dependence of the line-of-sight integrated emission measure of plasma with a particular temperature T can then be characterized by the half density scale height, neglecting

the curvature of the corona. The fluxes $F_1(h)$ and $F_2(h)$ recorded in two detectors ($i=1,2$) with different wavelengths, characterized by the temperature response functions $R_1(T)$ and $R_2(T)$, is then

$$F_i(h) = \int \frac{dEM(T, h=0)}{dT} \exp\left[-\frac{2h}{\lambda_0 T}\right] R_i(T) dT, \quad (5)$$

When the filter-ratio method is applied, one takes the flux ratio of the two fluxes at every pixel (along a chosen altitude path h)

$$Q(h) = \frac{F_1(h)}{F_2(h)}, \quad (6)$$

which is now height-dependent, so that the resulting filter-ratio temperature $T(Q) = T(Q[h])$ yields the height dependence of the temperature, $T(h)$.

3. OBSERVATIONS AND SIMULATION

Some typical differential emission measure distributions $dEM(T)/dT$ have been determined with the NASA/GSFC *Solar EUV Rocket Telescope and Spectrograph (SERTS)*, using density-sensitive line ratios from 8 different ionization states of iron between Fe^{+9} (Fe X) and Fe^{+16} (Fe XVII), during two flights in 1991 and 1993 (Brosius et al. 1996). These line ratios provide density diagnostics between temperatures of $\log(T_e) = 5.0$ and $\log(T_e) = 6.7$ (i.e. $T_e \approx 0.1 - 5.0$ MK). Brosius et al. (1996, see their Fig.8 and 9) derived a differential emission measure curve $dEM(T)/dT$ in the temperature range of $\log(T_e) = 4.8 - 7.0$, which is reproduced in Fig.2 (top panel), for two observations of active regions (AR93, AR91) and two observations of Quiet Sun regions (QR93, QR91).

We consider now the instrumental response functions of *Yohkoh SXT*. For active regions, the two filters sensitive to the lowest temperatures are the thin aluminium (Al 1265 Å) and the Al/Mg/Mn composite filter (Tsuneta et al. 1991). The corresponding response functions $R_1(T)$ and $R_2(T)$ are shown in Fig.2 (second panel), and their filter ratio $Q(T) = R_2(T)/R_1(T)$ is given in Fig.2 (bottom panel). In order to understand the temperature contributions to the observed flux we show the differential soft X-ray flux $dF(T)/dT = [dEM(T)/dT] R(T)$ (Fig.1, third panel), for both filters and for all 4 regions. The differential soft X-ray flux exhibits a peak at a temperature of $T_e \approx 10^{6.65} = 4.5$ MK for the active regions, and at $T_e \approx 10^{6.3} = 2.0$ MK for the quiet Sun regions.

We calculate now the fluxes $F_1(h)$ and $F_2(h)$ in the two filters as function of the height h above the limb, using the hydrostatic distribution defined in Eqs.5-6, where each fluxtube with (different) temperature T has a (different) density scale height of $\lambda = \lambda_0 T$, while the total ensemble of fluxtubes is summed up by an integration over the entire temperature range (i.e. temperature integral in Eqs.5-6). The resulting SXR fluxes as function of height are shown in Fig.3 top, illustrating that the SXR flux drops exponentially with height. We derive now the filter ratio $Q(h) = F_2(h)/F_1(h)$, shown in Fig.3 (middle panel) for all 4 regions. The filter ratio $Q(h)$ clearly varies as function of height h , although each fluxtube is assumed to be isothermal.

We demonstrate now what effect this filter ratio variation $Q(h)$ has on the inference of a single-temperature model $T(h)$, as it is assumed in the classical filter-ratio method by definition. To invert the filter ratio $Q(T)$ as function of the temperature T , we find the following analytical approximation (accurate within $\lesssim 0.7\%$) in the temperature range of $T = 1.5 - 6.0$ MK (see fit in Fig.2 bottom),

$$Q(T) := \frac{R_2(T)}{R_1(T)} \approx 0.39 + 0.27[\log(T) - 6.18]^{1/2}. \quad (7)$$

This analytical approximation allows us conveniently to invert the filter-ratio temperature in the range of $Q = 0.4 - 0.6$, i.e.

$$\log(T[Q]) = 6.18 + \left(\frac{Q - 0.39}{0.27}\right)^2. \quad (8)$$

The inverted temperatures $T[Q(h)]$ are shown in Fig.3 bottom for all 4 regions. The filter ratio temperature $T(h)$ shows a height dependence from $T(h = 0) \approx 2.1$ MK to $T(h = 0.5r_{\odot}) \approx 3.1$ MK for the quiet regions, and from $T(h = 0) \approx 4.1 - 4.4$ MK to $T(h = 0.5r_{\odot}) \approx 5.4 - 6.3$ MK for the active regions. Thus, the weighting effect of temperature scale heights over the broadband response function introduces an apparent temperature gradient of $dT/dh \approx 0.003$ K m⁻¹ for the quiet corona regions, and about $dT/dh \approx 0.005$ K m⁻¹ for active regions. This corresponds about to a doubling of the apparent temperature over a distance of a solar radius r_{\odot} ,

$$\Delta T^{SXT} \approx T_0 \left(\frac{h}{r_{\odot}}\right). \quad (9)$$

4. DISCUSSION AND CONCLUSIONS

We have investigated the effect of hydrostatic density scale heights in coronal loops on the inference of a filter-ratio temperature from a broadband instrument, in particular for the two thinnest filters of *Yohkoh/SXT*, which are generally used to derive electron temperatures in active regions and in the quiet corona. The principal effect is that, with increasing altitude h (above the solar surface), the emission measure-weighted temperature T_e becomes systematically more weighted by the larger scale heights λ (Fig.1), which are associated with loops of higher temperature, and thus mimic an average temperature increase with height. We used differential emission measure distributions $dEM(T)/dT$ that have been observed in active regions and in quiet Sun regions and simulated the temperature bias on $T(h)$ for the instrumental response functions of *Yohkoh/SXT*. The resulting temperature bias can be quantified approximately as $\Delta T^{SXT} \approx T_0(h/r_{\odot})$. We discuss now the consequences of this result.

The radial variation of temperature in the inner corona (out to 0.7 and 0.95 solar radii) has been examined for the diffuse corona from long-exposure *Yohkoh/SXT* images by Wheatland, Sturrock, & Acton (1997). These authors find a systematic temperature increase from $T_e \approx 1.6$ MK near the solar surface to $T_e \approx 2.4$ at a height of 0.5 solar radii for the 7-9 May 1992 active region, and from $T_e \approx 1.8$ MK to $T_e \approx 2 - 3$ MK at 1 solar radius for the 26 August 1992 region. This systematic temperature increase of the solar corona was interpreted in terms of a downward heat flux, leading to the conclusion of a heat deposition above the observed height. According to our model (Eq.9), we estimate fully consistent temperature increases [$T_e(h = 0) = 1.6$ MK $\mapsto T_e(h = 0.5r_{\odot}) = 2.4$ MK for the first case, and ($T_e(h = 0) = 1.8$ MK $\mapsto T_e(h = r_{\odot}) = 2.7$ MK for the second case] from the emission measure-weighted hydrostatic scale heights alone, even if all fluxtubes are isothermal. Therefore, if the hydrostatic weighting effect on the *Yohkoh/SXT* filter ratio method would be corrected, no net temperature increase would result, and thus no support for a heating function in the upper corona is warranted.

With the same measurement technique, Priest et al. (1999; 2000) analyzed large-scale arcades and loops and found a temperature increase from $T_e(h = 0) = 1.6$ MK to $T_e(h = 0.5r_{\odot}) = 2.2 - 2.3$ MK for a first loop observed on 1992 Oct 3, an increase from $T_e(h = 0) = 1.6$ MK to $T_e(h = 500$ Mm) = 2.4 - 2.6 MK in a second loop, and an increase from $T_e(h = 0) = 1.6$ MK to $T_e(h = 350$ Mm) ≈ 2.1 MK in a third loop. The authors fitted three heating models to these temperature increases $T_e(h)$ and found that a uniform heating function provides the best fit for all 3 cases, while heating functions localized at the loop

top was found to be less likely, and a heating function localized near the loop footpoints was rejected. From our model (Eq.9) we can reproduce the same temperature increases for these 3 cases, so that virtually no net temperature increase remains, if the *Yohkoh/SXT* filter ratios would be corrected for the hydrostatic emission measure weighting. Comparing the corrected temperature profiles $T_e(h) \approx const$ with the heating models shown of Figs.8 and 9 in Priest et al. (2000), one would conclude that the data are most consistent with the theoretical model of footpoint heating, a conclusion that would also be more in line with other recent observations from *TRACE* (Schrijver et al. 1999; Aschwanden et al. 2000b).

In summary, we like to point out that filter ratio temperatures from broadband instruments may lead to systematic errors in the determination of vertical temperature profiles $T_e(h)$, that can only be corrected properly by forward-fitting of models which contain both temperature $T_e(h)$ and density profiles $n_e(h)$. The systematic effects are larger for broadband filter ratios (e.g. *Yohkoh/SXT*) than for narrowband filters (e.g. *SoHO/EIT* or *TRACE*). Any detected temperature increase derived from an emission measure-weighted temperature definition is subject to the *hydrostatic weighting of a multi-temperature plasma*, and does not directly describe a variation (dT/dh) of the electron temperature along a magnetic field line.

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Figure Captions

Fig.1: This cartoon illustrates scale height-weighted contributions of hydrostatic loops or open fluxtubes to the emission measure observed along two line-of-sights above the solar limb. The left line-of-sight at a height of $h = 100$ Mm above the limb samples significant emission from the 3 loops with temperatures of 1.5-2.5 MK. The right line-of-sight at a height of $h = 200$ Mm above the limb samples significant emission only from the hottest loop with $T = 2.5$ MK.

Fig.2: The differential emission measure distribution $dEM(T)/dT$ of two active regions (AR93, AR91) and two quiet Sun regions (QR93, QR91) measured by Brosius et al. (1996) with SERTS data (top panel). The Yohkoh/SXT response function for the two thinnest filters (second panel). The differential SXR fluxes $dF(T)/dT = [dEM(T)/dT] * R(T)$ for the two SXT filters (thin and thick linestyles) for all 4 regions (third panel). The filter ratio $Q(T) = R_2(T)/R_1(T)$ for the two Yohkoh/SXT filters and an analytical approximation in the range of $T = 1.5 - 6.0$ MK (bottom panel).

Fig.3: The height dependence of the observed SXT fluxes $F(h)$ for the two filters (thin and thick linestyles) and all 4 regions (different linestyles) (top panel). The resulting filter ratio $Q(h)$ for all 4 regions (second panel), and the inferred filter-ratio temperatures (bottom panel). Note that the filter-ratio temperature $T(h)$ shows a systematic increase with height, although a model with isothermal loops was assumed.





