Massive warm dark matter

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Many independent high resolution simulations have indicated that the standard collisionless cold dark matter model does not reproduce the structure of observed present day galaxies well. Several possible solutions in the form of modifications to the physics of the dark matter particles have been proposed. One of the most promising is warm dark matter (WDM), particles with significant thermal motion in the early universe. It is usually assumed that such particles are relativistically decoupled particles with a mass of approximately 1 keV. However, here we have investigated the possibility that much more massive particles with highly non-thermal spectra could make up warm dark matter. Several possible production mechanisms are reviewed and the only one found to be viable is that the WDM is produced by the non-relativistic decay of some massive species in the early universe. Such very massive warm dark matter could possibly be detected in direct detection experiments, as opposed to standard thermal warm dark matter.

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I. INTRODUCTION

The standard Big Bang model has been very successful in explaining the large scale structure in the universe. An essential feature in the model is dark matter. By far the most successful candidate for dark matter is cold dark matter, collisionless particles which are so massive that they are very non-relativistic during the entire structure formation history [1,2].

However, in the past few years high resolution N-body simulations have shown that standard collisionless CDM apparently fails to reproduce observations on galactic or smaller scales. Observations indicate that the halo of a galaxy like the milky way has almost an order of magnitude fewer small satellite halos than is found in CDM simulations [3,4].

Also, the central cores of dark matter halos have singular density profiles, approaching almost $\rho \propto r^{-1.5}$ at small r [5–9]. This is in contrast to observations of low surface brightness galaxies, where the dark matter halos apparently have almost constant density.

This seems like a very serious problem for the CDM structure formation picture. However, at present it cannot be completely excluded that the discrepancy is due to the low quality of either simulations or observations. For instance very high resolution simulations which include baryons have yet to be carried out [10], and rotation curve measurements of LSB galaxies may have greater uncertainties than previously estimated [11].

However, the amount of observational data in conflict with the CDM model is quite large and it seems a very real possibility that we may have to abandon the simplest CDM collisionless models. The simplest possibility (other possibilities, such as self-interacting CDM have been proposed [12]) is that power is suppressed on small scales, either because of the initial power spectrum from inflation [13] or because of thermal motion of the dark matter particles. The latter possibility corresponds to warm [14,15] or hot dark matter and has received a lot of attention recently. WDM seems able to explain many of the problems which CDM suffers from, notably the substructure and the angular momentum problems [16–19]. However, it remains to be seen whether WDM can also prevent singular cores from forming [18,20].

Because WDM seems a quite promising candidate for dark matter it is definitely worthwhile to look closer at the possible particle physics mechanisms for producing it. The simplest possibility is that the warm dark matter is a relativistically decoupled species with a thermal distribution function. This possibility has been reviewed a number of times and we shall not discuss it further [14,15,21]. Rather we will investigate the intriguing possibility that warm dark matter could have been produced non-thermally. It is possible that dark matter particles with masses much higher than thermal warm dark matter could have been produced with enough thermal energy to make up warm dark matter. We shall discuss three distinct possibilities for this, namely production by decay of a heavy species [18], production by annihilation of a heavy species, and finally self heating by number changing self interactions of the warm dark matter particles [22–24,19]. We then go on to discuss any possible signatures that can distinguish massive warm dark matter from ordinary warm dark matter.

Note that there is another possibility for non-thermal warm dark matter, namely that the WDM particles are very light, but have been produced with very low average momentum. This could happen if the WDM particles are sterile neutrinos that have been produced via oscillations with an active species [25] or by relativistic decays of massive fermions into bosons [26,27].

II. PRODUCTION MECHANISMS FOR MASSIVE WARM DARK MATTER

A non-thermal warm dark matter candidate could have been produced in several different ways via interactions with other particles in the plasma. We here review three distinct possibilities: self-interactions, decays and pair-annihilations. In the following, ϕ is used to denote either the WDM field or the WDM particles, irrespective of whether they are bosons or fermions.

A. Production by self interactions

Carlson *et al.* [22,23] suggested an interesting way to heat cold dark matter relative to standard CDM, namely by number changing self-interactions. This was proposed as a means of reconciling the $\Omega_m = 1$ CDM model with large scale structure observations. However, subsequently it was shown by de Laix, Scherrer and Schaefer [24] that this model produced a very poor fit to observations.

At present the consensus is that the correct model for large scale structure is a flat ΛCDM model [1], and so there is no need to modify the physics of CDM to change the formation of large scale structure. Self-interacting dark matter could instead be invoked as a possible means of explaining the observed small scale structure. CDM with a large self-scattering cross section has been proposed [12], and subsequently warm dark matter with self-scatterings [28]. However, if the self-interactions are sufficiently strong there are very likely number changing reactions as well. This model is almost equivalent to that originally proposed by Carlson *et al.* [22,23]. If dark matter has such strong self-interactions that number changing reactions occur in equilibrium, the effective temperature of the species will drop much slower than for other species. In case of full thermal equilibrium maintained by self-interactions, the distribution function is (assuming Boltzmann statistics) $f = \exp(-E/T)$. Also, in thermal equilibrium the total entropy within a comoving volume is conserved. In this case, the effective temperature of the distribution is

$$\frac{T}{m} = (3\log(a) - K)^{-1}, \tag{1}$$

for $T \ll m$. a is the scale factor and K is a constant. This shows that the effective temperature of the species only drops logarithmically. Note that this behaviour is in stark contrast with that of the other species for which $T \propto a^{-1}$. Thus, the particle distribution is heated to higher temperature than the surrounding medium by transforming particles into kinetic energy. This leads to a much larger free streaming length for such particles than for thermally decoupled particles with the same mass. The observational bounds on the self-scattering cross section can be translated into a rough bound on the number changing reactions, depending on the specific type of WDM particle.

Pseudoscalars — A likely candidate in this category is a particle like the majoron (although there should be no coupling to standard model fields) with a simple $\alpha\phi^4$ self-interaction term. $4 \leftrightarrow 2$ number changing interactions take place via a ϕ^6 term in the effective Lagrangian [22]. Based on purely dimensional arguments the rate for this reaction should be [24]

$$\Gamma_{4\leftrightarrow 2} \simeq n^3\alpha^4/m^8, \tag{2}$$

where n denotes the number density. In all reasonable scenarios we should expect that the dimensionless coupling constant, $\alpha \lesssim 1$. However, we can also put another constraint on α from simple elastic scattering. The rate for elastic scattering of these particles is

$$\Gamma \simeq nv\alpha^2/m^2, \tag{3}$$

where v is their relative velocity. It has recently been shown in numerical simulations that dark matter with a large elastic cross section is not a good candidate because galaxies have very singular halos and are too spherical [12]. A safe upper bound on the 2-body elastic scattering cross section is [12]

$$\sigma \leq 10^{-22} m_{\rm GeV}\,{\rm cm}^2, \tag{4}$$

yielding an upper limit on α

$$\alpha^2 \lesssim 2.6 \times 10^{-22} m_{\rm eV}^3. \tag{5}$$

The quantity of interest here is the ratio of the number changing interaction rate and the Hubble parameter, Γ/H. For pseudo-scalars this can be written as

$$\frac{\Gamma}{H} \lesssim 1.1 \times 10^{30} (\Omega_\phi h^2)^3 \alpha^4 m_{\rm eV}^{-11} T_{\rm eV}^7 g_*^{-1/4}, \tag{6}$$

using the bound on α. Ω_ϕ is the present-day density contribution of the WDM and g_* is the number of relativistic degrees of freedom present. Freeze-out occurs at $\Gamma/H \simeq 1$, corresponding to the bound

$$m_{\rm eV}^{5/7} \lesssim 0.0017 (\Omega_\phi h^2)^{3/7} T_{\rm freeze,eV}. \tag{7}$$

As soon as the number changing reactions freeze out the particles become non-relativistic. Therefore, if this scenario is to work, we must demand that $T_{\rm freeze} \lesssim 100-200$ eV, so that the WDM has the correct amount of thermal motion. For pseudo-scalars this translates into

$$m_{\rm eV} \lesssim 0.22 (\Omega_\phi h^2)^{3/5} \ll T_{\rm freeze,eV}. \tag{8}$$

The conclusion is that massive WDM cannot be produced in this fashion unless special circumstances prevail. One way out is to tinker with the $4 \leftrightarrow 2$ reaction rate, increasing it by a large factor over the naive estimate. However, it seems safe to assume that this type of heating is an unlikely production mechanism for WDM.

Scalars — If the particles are scalars there could be a $3 \leftrightarrow 2$ number changing term of the form $\Gamma \simeq n^2\alpha^3/m^5$ [22]. In this case Eq. (8) changes into

$$m_{\rm eV} \lesssim 4.1(\Omega_\phi h^2)^{2/5} \tag{9}$$

Again, the conclusion is that self-heating cannot be the production mechanism.

Fermions — For fermions the situation is quite different. The leading number changing reaction is $2f + 2\bar{f} \to f\bar{f}$. The fermion-fermion interaction necessitates some new force carrier, η. If it is a boson with mass smaller than the fermion then the reaction $f\bar{f} \to 2\eta$ will dominate completely and the fermion will quickly disappear by annihilation. Thus, the force carrier would have to be more massive than f. But in that case the reaction $2f + 2\bar{f} \to f\bar{f}$ will be suppressed by high orders of $T/m_\eta \ll 1$. Thus we can expect the number changing rate for fermions to be much smaller than for pseudoscalars and far too small to be of any practical interest.

Thus, no matter what nature the dark matter particles have, it is very unlikely that self-heating can increase the thermal energy sufficiently that WDM results.

B. Production by decay of a massive species

Another, and perhaps more obvious way is to produce the WDM particles, ϕ, via decays of strongly non-relativistic particles, X [18]. We can write the present day thermal velocity of the warm dark matter as

$$v = \frac{p}{m_\phi} \simeq \frac{T_{\gamma,0}}{m_\phi}\left(\frac{g_{*,0}}{g_{*,d}}\right)^{1/3}\left(\frac{m_X}{T_\gamma}\right)_{T=T_{\rm decay}} \tag{10}$$

We can use this relation to find how much energy density was in the X-particles when they decayed

$$\rho_{X,d} = \frac{m_X}{m_\phi}\rho_c\Omega_\phi\left(\frac{g_{*,d}}{g_{*,0}}\right)\left(\frac{T_{\gamma,d}}{T_{\gamma,0}}\right), \tag{11}$$

where Ω_ϕ is the present day contribution to Ω from the warm dark matter. From these relations we can calculate the ratio ρ_X/ρ_R at decay

$$\frac{\rho_X}{\rho_R} = 0.025\Omega_\phi h^2\left(\frac{v}{0.4{\rm km\,s^{-1}}}\right)\left(\frac{g_{*,d}}{g_{*,0}}\right)^{1/3}. \tag{12}$$

From this we can conclude that the X-particles never dominated the energy density of the universe prior to decay. However, the radiation produced at decay adds a significant contribution to ρ_R which could be detectable.

If the decay took place prior to BBN ($z \sim 10^{12}$) the ϕ particles were highly relativistic during BBN and their contribution to the radiation energy density is (in units of the energy density of a standard massless neutrino species)

$$\Delta N_\nu = 0.185\Omega_\phi h^2\left(\frac{v}{0.4{\rm km\,s^{-1}}}\right). \tag{13}$$

For $\Omega_\phi = 0.3$ and $h = 0.65$ this yields $\Delta N_\nu \simeq 0.023$, too small a perturbation to be detected. Even with high precision measurements of the primordial abundances this value seems out of reach [29]. If the decay is after BBN, the contribution of X during BBN is entirely negligible.

Next, one can ask whether this is detectable in the CMBR anisotropy. At recombination the ϕ-particles are already strongly non-relativistic. Their contribution to the radiation energy density can be estimated as

$$\rho_{\phi,R} \simeq \frac{1}{2}v^2\Omega_\phi h^2\left(\frac{T_\gamma}{T_{\gamma,0}}\right)^3. \tag{14}$$

Again, we can parameterize this, in units of neutrino species at recombination, to be

$$\Delta N_\nu \simeq 1.7\times10^{-8}\left(\frac{v}{0.4{\rm km\,s^{-1}}}\right)^2\left(\frac{T_\gamma}{T_{\gamma,0}}\right). \tag{15}$$

At recombination this gives $\Delta N_\nu \simeq 2\times10^{-5}$, which is too small to be detected, even with the upcoming high-precision experiments MAP and PLANCK [30].

Note that thermally decoupled WDM gives the same contribution to the cosmic radiation density as does the decay-produced WDM. Thus, this effect would in no case allow us to distinguish the two different types of dark matter (unless the decay is after BBN, in which case the decay-produced WDM would not contribute to the radiation during BBN).

C. Production by annihilation of a massive species

The final production mechanism we shall discuss is via the pair-annihilation of some massive fermion species. Here, the ϕ-particles are produced via $X\bar{X} \to 2\phi$. ϕ can be either a boson or a fermion. The prime example would be a heavy neutrino species annihilating into massless fermions. Annihilations will in general not produce ϕ with much greater than thermal energies. The reason is that Γ/H is always an increasing function of T for non-relativistic particles, so that very few high energy annihilation products are produced (unless one invokes a specific physical mechanism to prevent annihilations at high temperature [31]).

In any case, if warm dark matter is produced by the annihilation of a massive species, the annihilation rate must be high enough that practically all heavy particles disappear before the annihilation freezes out. Otherwise the remaining heavy particles will quickly begin to dominate the energy density, resulting in a standard CDM scenario. But in the limit of strong interactions, the specifics of the annihilation reaction does not matter, the decay proceeds in equilibrium. The final energy density of the annihilation products can then be found by solving the simple differential equations (assuming $2 \leftrightarrow 2$ annihilation and Boltzmann statistics of the involved species)

$$\frac{dn_{\rm TOT}}{dt} = \frac{dn_{\rm X}}{dt} + \frac{dn_\phi}{dt} = -3Hn_{\rm TOT} \qquad (16)$$

$$\frac{d\rho_{\rm TOT}}{dt} = \frac{d\rho_{\rm X}}{dt} + \frac{d\rho_\phi}{dt} = -3H(\rho_{\rm TOT} + P_{\rm TOT}). \qquad (17)$$

$$f_{{\rm X},\phi} = \exp(-(\sqrt{m_{{\rm X},\phi}^2 + p^2} - \mu)/T), \qquad (18)$$

where μ and T denote the common pseudo-chemical potential and temperature of the two species.

Fig. 1 shows the outcome of the annihilation process. The average energy of the light particles is slightly higher than the expected $3T$, namely $\langle E_\phi \rangle = 3.78T$. However, this clearly shows that any annihilation-product cannot act as strongly non-thermal warm dark matter.

After reviewing these three possible production scenarios for non-thermal WDM we conclude that the only plausible possibility is that the WDM particles have been produced by the decay of some massive species.

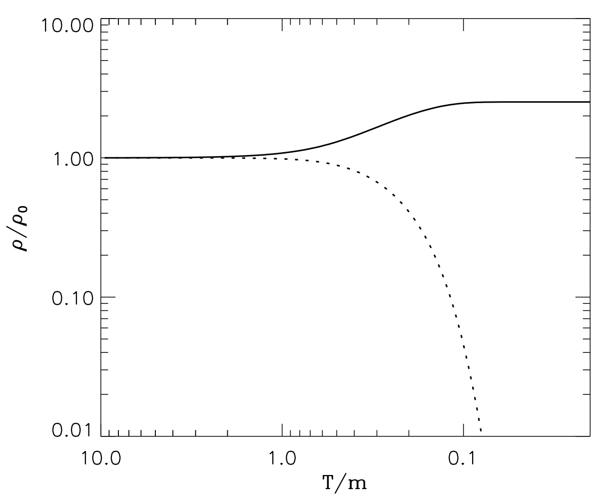

FIG. 1. The evolution of the energy density of parent (dashed line) and daughter (full line) for annihilation in equilibrium, parametrized in units of the initial high temperature density for both species (which is the same since they are in thermal equilibrium). Note that the density is multiplied with a^4 so that it is constant for a decoupled relativistic species.

III. COUPLING TO STANDARD MODEL FIELDS

Warm dark matter could in principle be coupled to standard model fields. Thermal dark matter is much lighter than the Z-mass and therefore any coupling to the standard model fields would likely show up as a branching to ϕ in the Z-decay. However, thermal WDM cannot be detected in standard dark matter search experiments that rely on nuclear recoil effects because the mass is much below detection threshold [32]. That would not need be the case for massive warm dark matter candidates. In principle the WDM particles could scatter on nuclei and be observed in direct detection experiments. However, the WDM should never come into equilibrium with the standard model particles in the early universe. Otherwise a purely thermal distribution results, and the end product is cold dark matter. This puts a strict upper limit on the coupling between ϕ and standard model fields.

As an example we calculate the energy equilibration rate for a Dirac neutrino-like particle, ϕ, in the early universe. The squared matrix element for ϕ scattering on a massless standard model fermion is roughly

$$\sum |M|^2 \simeq \eta G_F^2 (p_\phi \cdot p_f)(p_\phi^{'} \cdot p_f^{'}), \qquad (19)$$

for energy transfers below m_Z. The dimensionless parameter η denotes the effective coupling strength of ϕ to Z. From the above matrix element we estimate the scattering rate for a ϕ-particle to be

$$\Gamma_s = n_f \sigma \simeq \eta G_F^2 \langle E_\phi \rangle \langle E_f \rangle T_f^3. \qquad (20)$$

The average energy of ϕ is much bigger than that of f. Thus, on average the momentum transfer in each scattering is equivalent to $\langle E_f \rangle$. The energy equilibration rate can then be estimated as

$$\Gamma_E \simeq \eta G_F^2 \langle E_f \rangle^2 T_f^3 \simeq \eta G_F^2 T_f^5. \qquad (21)$$

If the WDM particles are to stay out of thermal equilibrium then

$$\frac{\Gamma}{H} < 1 \qquad (22)$$

must be fulfilled at all times after ϕ is produced. This above equation can be transformed into a bound on η

$$\eta < 2.8 g_*^{1/2} T_{\rm MeV}^{-3}. \qquad (23)$$

Thus, the bound strengthens with increasing temperature and is strongest at T_d, the decay temperature of the parent.

A direct detection experiment would typically use WIMP-nucleon elastic scattering recoil. The elastic scattering cross section is given approximately by [32]

$$\sigma \simeq \eta G_F^2 \mu^2, \qquad (24)$$

where $\mu = (m_N m_\phi)/(m_N + m_\phi)$ is the reduced mass of the system.

The present generation of experiments have reached a limit of $\sigma \simeq 1$ pb for masses of ϕ in the GeV range [33]. In that case $\mu/m_N \simeq 1$ and the detection limit corresponds roughly to

$$\eta \simeq 20. \qquad (25)$$

Using Eq. (23) we find that

$$T_{\rm d,MeV} \lesssim 0.5 \qquad (26)$$

if the ϕ-particles were to have been detected in the present experiments. The next generation of dark matter experiments will probably achieve about a factor 10^3 better sensitivity, yielding $T_{\rm d,MeV} \lesssim 5$. In any case, if this type of warm dark matter is to be detected in direct detection experiments it must have been produced at the epoch of BBN or later, otherwise the upper bound on the coupling to standard model fields becomes so tight that no direct detection experiment will be able to see it.

IV. DISCUSSION

We have discussed various possible means of producing warm dark matter, with specific focus on the possibility of very heavy warm dark matter. Dark matter self-heating was found to be excluded by present observational bounds. Also, production by pair-annihilation of a massive species cannot produce strongly non-thermal dark matter. The only viable production mechanism was found to be production by the non-relativistic decay of a massive relic.

From a structure formation point of view there is very little difference between massive warm dark matter and standard thermally decoupled warm dark matter for the same average free streaming length. Interestingly , as opposed to standard warm dark matter it could be possible to detect massive warm dark matter in a direct detection experiment, because the particle mass can easily be in the GeV range or above. However, it was also found that such WDM would have to have been produced at relatively low temperatures, otherwise the WDM distribution would have been brought into thermal equilibrium by interactions with the standard model fields.

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