Can a Very Low-Luminosity and Cold White Dwarf be a Self-gravitating Bose Condensed System

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An entirely new model for the structure as well as for the cooling mechanism of white dwarfs has been proposed. We have argued that the massive part of the constituents of white dwarfs- the positively charged ions are boson and under the extreme physical condition (density and temperature) at the interior, it is possible to have condensation of charged bose gas. We have tried to establish that a cold white dwarf is a self gravitating charged bose condensed system.

Unlike the other branches of physical science, astrophysics deals with giant macroscopic objects and also tiny constituents of microscopic world. In the astrophysical calculations very often we use the numbers as large as Planck mass (M_p) and very small number like Planck's constant (h). The observational data from most of the giant macroscopic stellar objects, e.g., stars, white dwarfs, neutron stars, etc. are analyzed by various models based on micro-physical processes. To be more precise, the processes like stellar evolutions, supernova explosions, structure of white dwarfs and neutron stars etc. are explained by microscopic models based on nuclear physics, hydrodynamics, quantum mechanics, statistical mechanics, etc. Various astrophysical processes, e.g., emission of high energy radiations (in particular X-rays and γ-rays), neutrinos, many physical phenomena from pulsar observations, viz., pulsar glitches, existence of strong magnetic fields in neutron stars, etc. are also explained by various micro-physical processes. The theoretical analysis of thermal evolution and also the evolution of magnetic fields of neutron stars are based on the physics of superfluidity, superconductivity etc. of neutron matter [1]. Similarly, the theoretical studies on the structure and the thermal evolution (which is again related to luminosity function) of white dwarfs of different masses and temperatures, in which we are especially interested in this article are also based on some interesting micro physical models [2]. It is believed that the structure of white dwarfs is built on the following two assumptions. Electrons are all degenerate and they contribute to the total pressure of the system. On the other hand, ions are non-degenerate and almost the entire mass of the star comes from their rest masses. Based on these two assumptions and by employing the condition of hydrostatic equilibrium, the equation for the structure of white dwarfs can be obtained. Now the basic constituents of white dwarfs are mainly oxygen, carbon, helium with an envelope of hydrogen gas. It is however, sometime assumed that the whole star is made up of either carbon or helium or a composite structure of these three elements, with the heaviest at the core and lightest at the crust. The structure of a white dwarf is strictly determined by its mass. In some relatively massive white dwarfs, one can think of the presence of heavier elements like neon, magnesium or even iron within the stars. The class of white dwarfs which have originated as main sequence stars with masses around $5M_\odot$ or below are carbon-oxygen (CO) white dwarfs, the other category, which originated from main sequence stars with masses from $5M_\odot$ to about $12M_\odot$ are oxygen-neon-magnesium ($ONeMg$) white dwarfs. It is also possible to have white dwarfs with helium rich interiors. Stars of low enough mass are supposed to evolve, after its hydrogen burning phase, to a stage where degeneracy appears before helium burning.

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But such isolated stellar objects are not expected to have yet evolved beyond the main sequence. However, it is possible to have white dwarfs with He-core [3] from more massive progenitor in a time scale smaller than the age of the galaxy if they belong to a binary system which allows mass loss before He-burning. Now because of high density all these elements within the white dwarfs are in completely ionized condition. The stability of the star against gravitational collapse is supported by degenerate electron pressure. Now the conventional model of white dwarf cooling is based on the following three consecutive processes: (i) relaxation of lattice thermal vibration, (ii) crystallization of the ionic part which are in the gaseous/liquid phase and (iii) further release of energy by the separation of different masses during crystallization in presence of gravity [4,5] (see also [6–8]). The total available energy in these processes depends on the constituent mass or equivalently on the mass of the white dwarf and temperature of the system. The time scale for cooling process is $\sim$ 10Gyr [9]. In this article we shall try to establish an entirely new picture for both the structure and the cooling of white dwarfs based on the condensation of charged bose gas. Now it is well known from the nuclear structure studies, that the kind of isotopes of He, C, O, Mg, Ne, etc. present in white dwarfs are all bosons. The white dwarf at relatively high temperature is therefore a self gravitating (one-component or multi-component) positively charged giant macroscopic bose system with a back ground of electron gas (negatively charged degenerate Fermi system). Based on this fact, we have tried to establish from a very simple physical argument that within the white dwarf it is possible to achieve a temperature which leads to condensation of charged bose gas. We shall also try to explain the cooling of white dwarfs with this new physical picture by assuming the condensation of bose gas as a first order phase transition.

Before we go into the detail discussion of bose condensation in a cold white dwarf, we give a very brief introduction on the condensation of ideal and weakly interacting hard-core bose gases. The condensation of weakly interacting bose gas- one of the oldest subjects has got a new dimension after the remarkable experimental achievement of bose condensation of alkali atoms in magnetic traps in the laboratory ([10], also see [11] for an overall knowledge in this subject). This discovery has given re-birth to this old subject and created an enormous interest in both theoretical and experimental studies of weakly interacting bose gas [11,12]. It is well known that in the case of an ideal bose gas, the condition determining the fugacity z in the normal phase is $\lambda^3 n = g_{3/2}(z)$, where n is the particle density [13],

$$\lambda = \left(\frac{2\pi\hbar^2}{mkT}\right)^{1/2} \tag{1}$$

is the thermal wavelength and $g_\nu(z) = \sum_{l=0}^{\infty} z^\nu/l^\nu$. Now the condition that the zero momentum state (i.e. the condensed state) is macroscopically occupied is $\lambda^3 n > \zeta(3/2)$, where $\zeta(3/2) = g_{3/2}(z=1) \simeq 2.612$, the standard zeta function. Hence the transition temperature for the condensation of ideal bose gas is given by,

$$T_0 = \frac{2\pi\hbar^2}{mk}\left[\zeta\left(\frac{3}{2}\right)\right]^{-\frac{2}{3}} n^{\frac{2}{3}} \tag{2}$$

On the other hand for an imperfect bose gas with non negligible hard core radius of the constituent, it was shown in several interesting theoretical calculations in the last few years, that the difference $\triangle T = T_c - T_0 \neq 0$, where T_c is the critical temperature in the case of imperfect bose gas. But there is no unique relation for the variation of $\triangle T$ with scattering length a. Not even there is consensus on the sign of $\triangle T$. Following the argument that a spatial repulsion of the constituents is equivalent to the momentum space attraction- which leads to condensation of hard core bose gas at relatively high temperature. Now for a system of identical bosons, assuming that the interaction potential acts locally (i.e. the range of interaction is small compared to the interparticle distance) and characterized entirely by s-wave scattering length a, the limit for which the quantum many body perturbation is valid is $a \ll \lambda$. With these assumptions it was shown [14,15] by Stoof that $\triangle T \sim a^{\frac{3}{2}}$, Bijlsma and Stoof showed that $\triangle T \sim a^{\frac{1}{2}}$. The Monte-Carlo simulation gives $\triangle T = T_0 C_0 (na^3)^\gamma$. Where $C_0 = 0.34 \pm 0.06$ and $\gamma = 0.34 \pm 0.003$. The mean field calculation shows $\triangle T = 0.7T_0(na^3)^{\frac{1}{3}}$. The simple calculation by Huang shows that $\triangle T = 3.527T_0(na^3)^{\frac{1}{6}}$. All these interesting results show that although there is no consensus on how $\triangle T$ depends on scattering length, however, there is no confusion that the condensation of imperfect bose gases occur at relatively higher temperature compared to that of an ideal bose gas.

We shall now go back to the discussion on the possibility of bose condensation in cold white dwarfs. The system we have considered contains massive ions carrying Z-unit positive charge (Z is fixed for one-component systems, on the other hand it is a variable in the case of multicomponent systems) which form a non-degenerate charged bose gas neutralized by a back ground of degenerate electron gas. Now in gases with short range forces the weak coupling limit corresponds to low density range, whereas the weak coupling limit in the case of charged bose gases interacting via coulomb force, this corresponds to the high density limit. Hence in the case of white dwarfs of mass density $\geq 10^6$gm/cc, and at temperature $\sim 10^6 - 10^8$K, the weak coupling limit is a good approximation. Therefore, if we simulate the screened coulomb potential of a positive ion by some equivalent hard-core potential with range $r_s \approx a$, the scattering length, we can apply the results of hard-core bose gas in this dense bosonic system. The scattering length a defined above indicates the beginning of the repulsive region of the ion.

To get an idea of the order of magnitude of transition temperature in such dense bosonic systems we equate the thermal de Broglie wavelength (eqn.(1)) with the inter-ionic spacing $\sim n^{-1/3}$. The transition temperature $T_c^{(1)}$ in ^oK obtained from this equality is displayed in Table I for three different elements (He, C and O). We have considered four possible mass densities. The transition temperatures so obtained for various physical conditions are found to be well within the limit of expected internal temperature of white dwarfs. Therefore the possibility of bose condensation inside cold white dwarfs can not be ruled out. The critical temperature $T_c^{(0)}$ obtained from eqn.(2), assuming ideal bose gas model is also shown in the table for the same set of elements and densities. We have also displayed the modified value of critical temperature $T_c^{(h)}$ from Huang's paper [14] (the values do not change significantly with other results as cited before). Now it is almost impossible to obtain the scattering length a within the white dwarf matter, we therefore treat it as a fixed parameter. Since we would like to use the results derived for hard-core bose gas obtained from perturbative calculation, keeping in mind the limit of its validity, we use the value $a = 0.01\lambda$. We believe that even under extreme condition ($\rho = 10^9$gm/cc), the scattering length can not go below this small value. The thermal wavelengths for two different cases: with and without hard-core radius are also presented in the table in $\mathring{A}$ unit. We have denoted these two quantities by $\lambda_T^{(h)}$ and $\lambda_T^{(0)}$ respectively. Assuming that the bose condensation is a first order phase transition, we have calculated the total amount of energy released in the process. The total available energy is given by $\triangle E = LM$, where $L \approx kT_c$, the latent heat of condensation and M is the total mass of the system. Assuming 5.0×10^3Km as the radius of a typical white dwarf which undergoes a bose condensation transition at a certain critical temperature (for hard-core gas), we have shown the amount of energy liberated during this process within the star for the same set of elements and mass densities as discussed above. The detail analysis of thermal evolution of white dwarfs taking bose condensation into account will be presented in some future publication.

From the tabulated results it may be inferred that the critical temperature for condensation increases with density. The transition process will therefore be favored in massive white dwarfs. It is also obvious that for a given density the lighter ions condense at relatively higher temperature. Now the ionic distribution in a multicomponent white dwarf is such that the matter at the core region is mainly dominated by massive ions, whereas the lighter ions built the crustal part. Therefore, the argument given by Lamb [16] long ago on the possibility of separation of normal and bose condensed phases in presence of gravity may be ruled out. However, the process strongly depends on so many factors, e.g., the mass, central density and temperature, the basic constituents and finally on the radial distribution of mass and temperature within the star. If there is a phase separation in presence of gravity, some extra energy will also be released in the process. A detail analysis will be presented in a future paper. Therefore the final conclusion is that the possibility of bose condensation can not be ruled out, particularly in massive white dwarfs of very low luminosity, at least at the central region. The structure of such massive white dwarfs could therefore be a bose condensed core with crystalline normal crustal matter. We can compare such structures with the typical structure of a neutron star with superfluid core and normal neutron matter crust. In a future publication, we shall discuss the most important issue- the controversy of white dwarf ages in globular clusters using this new ideas of white dwarf structure and cooling mechanism introduced in this letter.

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Element				*Helium*
ρ (gm/cc)	10^6	10^7	10^8	10^9
$T_c^{(1)o}$K	6.8×10^5	3.1×10^6	1.5×10^7	6.8×10^7
$T_c^{(0)o}$K	1.1×10^5	5.3×10^5	2.4×10^6	1.1×10^7
$T_c^{(h)o}$K	2.6×10^5	1.2×10^6	5.7×10^6	2.6×10^7
$\lambda_T^{(0)} \mathring{A}$	2.6×10^{-2}	1.2×10^{-2}	5.6×10^{-3}	2.6×10^{-3}
$\lambda_T^{(h)} \mathring{A}$	1.7×10^{-2}	7.9×10^{-3}	3.7×10^{-3}	1.7×10^{-3}
ΔE (ergs)	1.0×10^{46}	5.0×10^{47}	2.0×10^{49}	1.0×10^{51}
Element				*Carbon*
ρ (gm/cc)	10^6	10^7	10^8	10^9
$T_c^{(1)o}$K	1.1×10^5	5.0×10^5	2.3×10^6	1.1×10^7
$T_c^{(0)o}$K	1.8×10^4	8.5×10^4	3.9×10^5	1.8×10^6
$T_c^{(h)o}$K	4.2×10^4	2.0×10^5	9.1×10^5	4.2×10^6
$\lambda_T^{(0)} \mathring{A}$	3.7×10^{-2}	1.7×10^{-2}	8.0×10^{-3}	3.7×10^{-3}
$\lambda_T^{(h)} \mathring{A}$	2.5×10^{-2}	1.1×10^{-2}	5.3×10^{-3}	2.5×10^{-3}
ΔE (ergs)	6.0×10^{44}	2.8×10^{46}	1.7×10^{48}	6.1×10^{49}
Element				*Oxygen*
ρ (gm/cc)	10^6	10^7	10^8	10^9
$T_c^{(1)o}$K	6.7×10^4	3.1×10^5	1.4×10^6	6.7×10^6
$T_c^{(0)o}$K	1.1×10^4	5.2×10^4	2.4×10^5	1.1×10^6
$T_c^{(h)o}$K	2.6×10^4	1.2×10^5	5.6×10^5	2.6×10^6
$\lambda_T^{(0)} \mathring{A}$	4.1×10^{-2}	1.9×10^{-2}	8.9×10^{-3}	4.1×10^{-3}
$\lambda_T^{(h)} \mathring{A}$	2.7×10^{-2}	1.3×10^{-3}	5.8×10^{-3}	2.8×10^{-3}
ΔE (ergs)	3.0×10^{44}	1.3×10^{46}	6.0×10^{47}	2.8×10^{49}

TABLE I

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