

COSMIC RAYS AND THE STRUCTURE OF SPACE-TIME

AURELIO F. GRILLO^{1,a} and ROBERTO ALOISIO^{a,b}

^a *Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali del Gran Sasso, 67010 Assergi (L'Aquila), (Italy).*

^b *Dipartimento di Fisica dell'Università di L'Aquila, 67100 L'Aquila, (Italy).*

Abstract

Even a fundamental symmetry like Lorentz Invariance is an experimental fact and must be experimentally verified. We show that the study of the interactions of Cosmic Rays with universal diffuse background radiation can provide very stringent tests of this symmetry. The interactions we consider are the ones characterized by well defined energy thresholds whose energy position can be predicted on the basis of special relativity. We argue that the experimental verification of these thresholds can address the physics of supra-Planckian scales.

1 Introduction

Symmetry principles have generally origin from experimental evidence and therefore their validity has to be verified to an ever increasing degree of precision, or falsified. Lorentz Invariance (LI) is no exception, being based on experimental facts (e.g. the impossibility of verifying the motion of the Earth from laboratory experiments) and having innumerable experimentally testable consequences (e.g. constancy of speed of light, equivalence of physics in different reference frames ...).

In a recent paper (Aloisio et al. 2000) we discussed the possibility of using Cosmic Ray (CR) experiments to put very stringent constraints on the validity of LI. That this is the case can be intuitively motivated in the following way. Consider the process giving

¹Talk presented by A.F. Grillo.

rise to the Greisen, Kuzmin, Zatsepin cut-off (Greisen 1966, Zatsepin and Kuzmin 1966), *i.e.* pion photoproduction. In the terrestrial laboratory the reaction is $\gamma p \rightarrow \pi N$ and has a threshold at $E_{th} \approx 100$ MeV for a (proton) target at rest. In the same frame the reaction initiated by UHE CRs on Cosmic Microwave Background (CMB) photons as a target ($p\gamma_{CMB} \rightarrow \pi N$) has a threshold of $\approx 5 \cdot 10^{19}$ eV. In this frame the two reactions appear very different: in particular a 10^{20} eV proton needs only an extremely tiny fraction of its energy ($\approx 10^{-23}$) to make a transition in the final state containing a pion. Obviously there is nothing mysterious in this, LI just implying that the two reactions are exactly the same taking place in two reference frames in motion with a relative Lorentz factor $\gamma \approx 10^{11}$. However it is clear that even very small deviations from strict relativistic invariance are likely to profoundly modify the value of the thresholds and of the associated absorption cut-offs, in a way in principle experimentally verifiable. A similar process, giving rise to an absorption threshold for VHE-UHE Cosmic γ rays, is $\gamma\gamma_{BCKG} \rightarrow e^+e^-$ where again γ_{BCKG} is a low energy background photon: IR (Stecker 1999), microwave (Nikishov 1962, Goldreich and Morrison 1964, Gould and Schreder 1966) or radio (Protheroe and Biermann, 1996, 1997).

On the other hand, it has been conjectured since the 50's (Wheeler 1957) that strict Lorentz invariance is likely to be profoundly modified at the scales at which quantum gravitational effects begin to be relevant: at Planck distances (times) the topology of space-time may become highly non trivial, making the definition of distance essentially impossible thus profoundly modifying our picture of the physical world.

In this talk I present arguments showing, on very general grounds and without referring to specific models, that in the processes which give rise to absorption thresholds for the propagation of cosmic rays in the Universe (due the onset of particle production on universal background radiation) it is possible to test the validity of LI to a very high degree of precision; in particular, if deviations from LI are ascribed to QG effects, they can be studied down to length scales orders of magnitude smaller than Planck length, even using beam particles with energy *far less* than the Planck mass!

I want however to remark that we *do not* advocate violation of LI to explain the features of the spectrum of UHE Cosmic Rays (e.g. the apparent absence of the GZK cut-off). Our knowledge of the sources of the highest energy Cosmic and γ rays is rather poor, so that we prefer to stress the capability of CR experiments to test such a fundamental symmetry. However the situation is going to change drastically with the new generation of extremely large collecting area Cosmic Ray Experiments (Cronin 1992), or is already changing, for instance due to the high statistics studies of the VHE spectrum of extragalactic γ -sources like Mkn421 and 501 (see for instance Aharonian et al. 1999, Guy et al. 2000).

2 Absorption thresholds in non-LI world

As intuitively motivated above, the reactions that can lead to a verification of LI are those in which VHE and UHE Cosmic and γ rays interact with a universal diffuse (photon) background, that can be the very well known Cosmic Microwave Background Radiation (energy of maximum radiance $\epsilon \approx 10^{-3}$ eV), the less known Far Infrared

Radiation ($\epsilon \approx 10^{-2}$ eV) or the hypothetical Radio Background ($\epsilon \approx 10^{-7}$ eV). On this diffused radiation UHE protons and VHE γ 's can interact and produce secondary particles, mostly π in the case of primary protons, e^+e^- in the case of γ 's.

In all the cases the processes can be seen as Lorentz boosted from terrestrial laboratory with boost factors ranging from 10^7 to 10^{14} . The cross sections for these processes are large ($\approx 10^{-25}$ cm^2) so that the energetic particles are rapidly degraded in energy with an absorption length of the order of tens Kpc for γ absorption on CMBR to tens of Mpc for protons absorption in CMBR and γ absorption on Radio background. Both these facts are important quality factors for using CR' s to test LI. The first gives the range of parameters in which SR can be tested, while the second suggests that possible modifications of these reactions are expected to produce a (in principle easily) detectable signal.

To make the test quantitative one needs a parametrization of possible Lorentz violations ². The constancy of speed of light ($c(= 1)$ in the following) implies the existence of an invariant interval $ds^2 = dt^2 - dx^2$ ($=0$ for light signals), and that the norm of any four-vector is invariant: in particular for the Energy-Momentum four-vector of a particle of mass m one has the dispersion relation:

$$P_\mu P^\mu = m^2 = E^2 - |\vec{p}|^2 \quad (1)$$

This is the relation that we modify in order to parametrize violations of LI. We follow a phenomenological approach and we do not refer to any specific model (see however next section). Our guiding principles are:

- 1) Violations are universal, *i.e.* do not depend on particle type;
- 2) Preserve rotational invariance;
- 3) Violations are an high energy phenomenon, vanishing at low momenta.

Finally we only consider the $p \ll M_P$ range (relevant for the experiments we consider). We therefore modify the dispersion relation as ($p = |\vec{p}|$):

$$E^2 - p^2 = m^2 + p^2 f\left(\frac{p}{M}\right) + \dots \quad (2)$$

The dots stand for terms that are subleading in the regime we are considering; also in this regime the difference between E and p in the RHS of eq. 2 is a higher order correction. M is a mass parametrizing the violation of LI, which is expected to be of the order of the Planck mass M_P if violations originate from quantum gravity effects, and $f(0) = 0$. Under these conditions we can expand $f(p/M)$ and, reabsorbing numerical coefficients in M , we can write, for the leading correction:

$$E^2 - p^2 \approx m^2 \pm p^2 \left(\frac{p}{M}\right) \quad I_\pm \quad (3)$$

$$E^2 - p^2 \approx m^2 \pm p^2 \left(\frac{p^2}{M^2}\right) \quad II_\pm \quad (4)$$

being of first (second) order in the (small) parameter p/M .

²for earlier discussions of LI breaking see: Kirzhnits and Chechin 1971, Gonzalez-Mestres 1997, Coleman and Glashow 1997, Amelino-Camelia et al. 1997. A very similar approach is in Amelino-Camelia and Piran 2000.

It is worth noticing that we could have modified the Lorentz transformations of four-momenta

$$E' = \gamma MF\left(\frac{E + \beta p}{M}\right); \quad p' = \gamma MG\left(\frac{\beta E + p}{M}\right) \quad (F(0) = G(0) = 0) \quad (5)$$

which, with the appropriate choice of F and G can lead to (3) or (4). In this procedure there is however a large arbitrariness (eq. (5) is not even the most general) and we do not pursue it here ³.

We also *assume* Energy-Momentum conservation. This is not granted, if the violation of LI is associated to a violation of translational invariance, and has to be checked in specific models. However this assumption is legitimate if we want to put bounds on the violations of LI from the *observation* of the absorption thresholds.

We have now all the ingredients to compute the value of the threshold momenta in this new framework. It is important to notice that the computation must be performed in a specific frame, which we take the one in which the diffuse radiation is isotropic (neglecting the motion of the Earth), taking into account energy-momentum conservation and the modifications of the dispersion relations. This leads to algebraic equations (Aloisio et al. 2000)

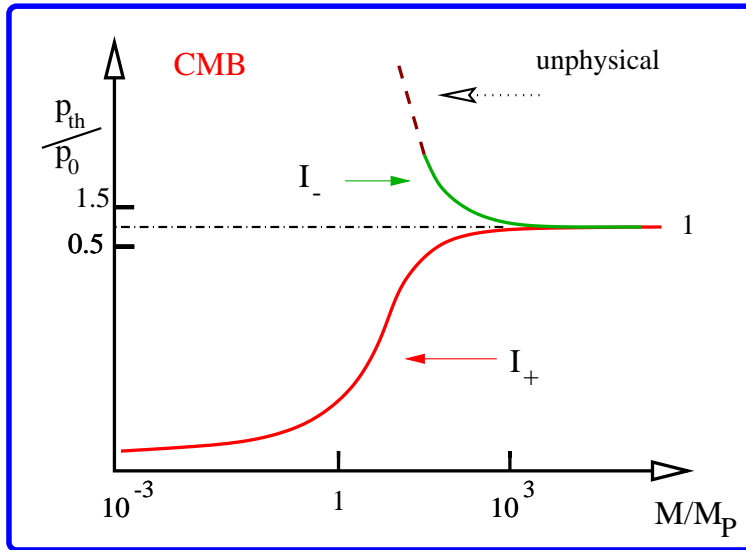


Figure 1: *Qualitative behaviour of the solutions, here for I-type modifications in the case of e^+e^- production on CMBR.*

$$\pm \alpha_I x^3 + x - 1 = 0 \quad (6)$$

$$\pm \alpha_{II} x^4 + x - 1 = 0 \quad (7)$$

³Notice that if $F(\infty), G(\infty) \neq \infty$ eq. (5) implies that there is a maximum value for energy (momentum) (Garay 1999).

where $x = p_{th}/p_0$ and p_0 are the thresholds as $M \rightarrow \infty$ ⁴, and (in parentheses the values for pion production)

$$\begin{aligned}\alpha_I &= \frac{p_0^3}{8m_e^2 M} && \left(\frac{p_0^3}{m_p^2 M}\right) \\ \alpha_{II} &= \frac{3p_0^4}{16m_e^2 M^2} && \left(\frac{3p_0^4}{m_p^2 M^2}\right)\end{aligned}\tag{8}$$

Notice that if we require that $p_{th} \approx p_0$ then we must have α_I (α_{II}) $\ll 1$, which, taking as example the pion production by protons, implies $M > 10^{14} M_P$ ($M > 10^3 M_P$) respectively. Namely, a verification of the LI threshold momentum value would put a limit on the violation parameter (much) larger than the Planck mass.

The qualitative behaviour of the solution is presented in Fig. 1. In case of positive modification the threshold moves towards lower values as M moves away from ∞ , while it increases, and becomes rapidly unphysical (*i.e.* negative or complex) for negative modifications; in this case the reaction becomes kinematically forbidden.

In a more quantitative way, in the following table we present the values of p_{th}/p_0 for pair production, assuming $M = M_P$:

	Infrared	Microwave	Radio
I_+	≈ 1	0.06	$5 \cdot 10^{-7}$
I_-	<i>no solution</i>	<i>no solution</i>	<i>no solution</i>
II_+	≈ 1	1	$2 \cdot 10^{-3}$
II_-	≈ 1	≈ 1	<i>no solution</i>

Notice however that, for positive modifications the process $\gamma \rightarrow e^+e^-$ becomes allowed, giving rise to absorption even in absence of target (Coleman and Glashow 1997).

If we leave M as free parameter, *assuming* experimental verification of thresholds (with a 100% uncertainty in momentum determination) we have

	Infrared	Microwave	Radio
I_+	$\frac{M}{M_P} \geq 0.2$	$\frac{M}{M_P} \geq 800$	$\frac{M}{M_P} \geq 10^{18}$
I_-	$\frac{M}{M_P} \geq 6$	$\frac{M}{M_P} \geq 3 \cdot 10^4$	$\frac{M}{M_P} \geq 8 \cdot 10^{19}$
II_+	$\left(\frac{M}{M_P} \geq 3 \cdot 10^{-8}\right)$	$\left(\frac{M}{M_P} \geq 7 \cdot 10^{-6}\right)$	$\frac{M}{M_P} \geq 10^5$
II_-	$\left(\frac{M}{M_P} \geq 3 \cdot 10^{-7}\right)$	$\left(\frac{M}{M_P} \geq 10^{-4} M_P\right)$	$M \geq 10^6$

Analogously for pion production (the process giving rise to the GZK cut-off), for $M = M_P$

	GZK
I_+	$2 \cdot 10^{-5}$
I_-	<i>no solution</i>
II_+	0.02
II_-	<i>no solution</i>

⁴ $p_0 \approx (m_\pi m_p)/(2\omega)$ for pion production by protons, $p_0 \approx m_e^2/(2\omega)$ for pair production by γ . ω is the energy of background photons.

Also in this case for positive modifications the process $p \rightarrow \pi N$ becomes kinematically allowed. If we leave M as free parameter, assuming experimental verification of thresholds (again within a factor 2 in energy)

	GZK
I_+	$M \geq 3 \cdot 10^{13} M_P$
I_-	$M \geq 10^{15} M_P$
II_+	$M \geq 500 M_P$
II_-	$M \geq 6 \cdot 10^3 M_P$

It is important to notice that these solutions confirm the intuitive expectations described in the introduction: in fact, the values of threshold momenta are (in most cases) profoundly modified by the modifications of dispersion relations produced by violations of Lorentz Invariance.

3 Theoretical Motivation

The above analysis is quite general and does not refer to specific models. It is however important to notice that there are, in the literature, models which lead violations of LI in the form discussed above (see for instance Amelino-Camelia et al. 1997, Amelino-Camelia et al. 1998 and for a more complete list Aloisio et al. 2000).

More generally, *if* violations of LI are ascribed to Quantum Gravitational effects we can classify the violations in a general way. In fact QG effects imply that the metric of space-time is non trivial when examined near the Planck scale

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad (9)$$

where $\eta_{\mu\nu} = \text{diag}(1,-1,-1,-1)$ is the flat metrics and $h_{\mu\nu}$ is a term fluctuating in the vicinity of the Planck length ⁵.

Consider $g_{\mu\nu} P^\mu P^\nu$ in such a metric so that

$$g_{\mu\nu} P^\mu P^\nu = \eta_{\mu\nu} P^\mu P^\nu + h_{\mu\nu} P^\mu P^\nu \quad (10)$$

A particle traveling with energy E averages the fluctuations over a scale λ/l_P where λ is its de-Broglie wavelength ($\lambda/l_P \approx 10^8$ for a 10^{20} eV proton) so that:

$$\langle g_{\mu\nu} P^\mu P^\nu \rangle_{\frac{\lambda}{l_P}} = \eta_{\mu\nu} P^\mu P^\nu + \langle h_{\mu\nu} P^\mu P^\nu \rangle_{\frac{\lambda}{l_P}} = m^2 \approx E^2 - p^2 + \left(\frac{l_P}{\lambda}\right)^n \bar{h}_{\mu\nu} P^\mu P^\nu + \dots \quad (11)$$

where the two parameters $n, \bar{h}_{\mu\nu}$ describe the (possible) violations of LI due to QG effects; clearly in the spirit of the discussion of Sect. 2, if $n > 2$ then the effects are in any case likely to be negligible when $p \ll M_P$. On the other hand \bar{h} may be:

1. $\bar{h}_{\mu\nu} = 0$: the dispersion relation is not modified;
2. $\bar{h}_{\mu\nu} \propto \eta_{\mu\nu} \implies E^2 - p^2 \propto f(m^2), f(0) = 0$: this gives the mildest violation, that does not affect photon propagation.

⁵Notice that we consider (8) as a phenomenological description. In QG the concept of a background (flat) metric might be ill-posed.

3. $\bar{h}_{\mu\nu}$ non diagonal: $E^2 - p^2 \neq m^2$. For instance: $\bar{h}_{ij} = \mp\delta_{ij}$, $\bar{h}_{00} = \bar{h}_{0i} = 0$ is a possible (among many others) choice that leads to $E^2 - p^2 = m^2 \pm p^2 (p/M)^n + \dots$ and for $n = 1(2)$ generates the violations of type $I_{\pm}(II_{\pm})$.

Even in case 1, in general, one can have (Ford and Yu 1999)

$$\langle \bar{h}_{\mu\nu} \bar{h}_{\mu'\nu'} \rangle \neq 0 \implies \langle (E^2 - p^2 - m^2)^2 \rangle_{\frac{\lambda}{l_P}} \neq 0$$

with possibly observable effects.

The vacuum of Quantum Gravity is often described as filled by virtual, Planck mass Black Holes (Hawking 1995). Even if the full vacuum metric might be difficult to manage, it is interesting to notice that in the field of a single BH the dispersion relation becomes: $E^2 - p^2 = m^2 \pm \frac{l_P}{L} (E^2 + p^2)$ which holds when ($\frac{l_P}{L} \ll 1$), and we expect $L \approx \lambda$. The average over the ensemble of fluctuating BHs is non trivial, since it depends on the QG dynamics; it seems however natural to expect that $\langle (E^2 - p^2 - m^2)^2 \rangle^{\frac{1}{2}} \approx \mathcal{O}(\frac{p^3}{M_P})$

4 Conclusions

Cosmic Ray experiments already in operation have the capability to investigate the validity of Lorentz Invariance to a high degree of sensitivity. There are already a number of events above the GZK cut-off and the situation will improve dramatically in a few years with the beginning of data taking of the Auger experiment. And γ -telescopes are already in the position of studying the spectrum of a few (up to now) extragalactic sources up to tens of TeV.

These experiments can test LI down to length scales in principle much smaller than the Planck length. This is extremely important, since it means that we do not need Planck energies to study the Planck physics if we appropriately chose the processes to study.

The experimental situation is still rather unclear: no sign of the GZK cut-off has been seen up in the proton spectrum up to a few in 10^{20} eV (Takeda et al 1999, Abu-Zayyad et al. 1999) while the γ spectrum of Mkn501 although showing a bend, might be inconsistent with expectations based on a new estimate of IR background (Protheroe and Meyer 2000).

It is important however to use much caution in interpreting the experimental data: our knowledge of the possible sources of highest energy CRs is rather poor, and in general it is certainly premature to invoke violations of LI to explain experimental data. This might be true also in the case of VHE γ astronomy, although, at least in the case of Mkn501, also due to (almost) simultaneous multi-wavelength observation (Guy et al. 2000), the experimental data are constraining the source spectrum. Again, the statistics is at present low, and there is uncertainty on the IR flux, but the situation is likely to improve. And, γ -experiments performed in the PeV range (Catanese et al., Ghia et al.), where the knowledge of the background is extremely better, are possibly the best arena for testing Lorentz Invariance.

Due to the large cross-sections involved, this kind of experiments might not need to be only of cosmic nature. In fact a terrestrial photon target, containing 10^{21} infrared ($\epsilon \approx 0.01$ eV) photons/cm³ and 1 cm long, would have an efficiency of 10^{-4} to convert

TeV photons into $e^+ e^-$ pairs. This target does not seem unfeasible: a 1 W monochromatic source would produce as many IR photons in one second, and if TeV photons could be produced (at LHC?) with sufficient intensity, this device could test models of LI violation of type I up to the Planck scale.

5 Acknowledgements

While the content of section 3 is responsibility of the present authors, this talk uses much of the material of Aloisio et al. (2000). We thank Piera L. Ghia and Pasquale Blasi for continuous collaboration. We thank also Angelo Galante for collaboration and Giuseppe Di Carlo for discussions.

6 References

- Abu-Zayyad T. et al., 1999 Proc. 26th ICRC (Salt Lake City, USA), **3**, 264.
Aharonian F. A. et al., 1999, Astron. & Astroph. **349**, 11.
Aloisio, R., Blasi, P., Ghia, P.L. and Grillo, A.F., 2000, Phys. Rev. **D62**, 53010.
Amelino Camelia G., Ellis J., Mavromatos N.E. and Nanopoulos D.V., 1997, Int. J. Mod. Phys. **A12**, 2029.
Amelino Camelia G., Ellis J., Mavromatos N.E., Nanopoulos D.V. and Sarkar S., 1998, Nature **393**, 763.
Amelino Camelia G. and Piran T., 2000, preprint hep-ph/0006210.
Catanese M. et al., 1996, Astrophys. J. **469**, 572.
Coleman S. and Glashow S.L., 1997, Phys. Lett. **B405**, 249.
Cronin J. W., 1992, Nucl. Phys. B (Proc. Suppl.) **28B**, 213.
Hawking S.W., 1995 Phys. Rev. **D53**, 3099.
Ford L.H. and Yu H., 1999, preprint gr-qc/9907037.
Garay J.L., 1999, Int. J. Mod. Phys. **A14**, 4079.
Ghia P.L. et al., 1999, Nucl. Phys. B (Proc. Suppl.) **70**, 506.
Gonzalez-Mestres L., 1997, preprint physics/9702026.
Goldreich P. and Morrison P., 1964, Sov. Phys. - JETP **18**, 239.
Gould R.J. and Schreder G.P., 1966, Phys. Rev. Lett. **16**, 252.
Guy J., Renault C., Aharonian F.A., Rivoal M. and Tavernet J.P., 2000, preprint astro-ph 0004355
Greisen K., 1966, Phys. Rev. Lett. **16**, 748.
Kirzhnits D.A. and Chechin V.A., 1971, Sov. Jour. Nucl. Phys. **15**, 585.
Nikishov A.I., 1962, Sov. Phys. - JETP **14**, 393.
Protheroe R.J. and Biermann P.L., 1996, Astropart. Phys. **6**, 45 ; 1997, Erratum-ibid **7**, 181.
Protheroe R.J. and Meyer H., 2000, preprint astro-ph/0005349.
Stecker F.W., 1999, preprint astro-ph/9904416.
Takeda M. et al., 1999, Astrophys. J. **522**, 225.
Wheeler J.A., 1957, Ann. Phys. (N.Y.) **2**, 604.
Zatsepin G.T. and Kuzmin V.A., 1966, Pis'ma Zh. Eksp. Teor. Fiz. **4**, 114 [JETP Lett. **4**, (1966) 78].