Scattering of energetic particles by anisotropic magnetohydrodynamic turbulence with a Goldreich-Sridhar power spectrum

Benjamin D. G. Chandran
Department of Physics & Astronomy, University of Iowa, Iowa City, IA 52242-1479
benjamin-chandran@uiowa.edu

— accepted for publication in *Physical Review Letters* —

Scattering rates for a Goldreich-Sridhar (GS) spectrum of anisotropic, incompressible, magnetohydrodynamic turbulence are calculated in the quasilinear approximation. Because the small-scale fluctuations are constrained to have wave vectors nearly perpendicular to the background magnetic field, scattering is too weak to provide either the mean free paths commonly used in Galactic cosmic-ray propagation models or the mean free paths required for acceleration of cosmic rays at quasi-parallel shocks. Where strong pitch-angle scattering occurs, it is due to fluctuations not described by the GS spectrum, such as fluctuations generated by streaming cosmic rays.

The scattering of energetic particles by turbulent magnetic and electric fields plays an important role in the acceleration and propagation of cosmic rays [1–7]. The turbulent fields responsible for cosmic-ray scattering can be excited by the cosmic rays themselves or by some mechanism that is independent of the cosmic rays. This paper focuses upon the latter case. In previous treatments of scattering, different turbulence models have been used, including fluctuations with wave vectors $\mathbf{k}$ parallel to the ambient large-scale magnetic field $\mathbf{B}_0$ (slab symmetry) or perpendicular to $\mathbf{B}_0$ (2D), or power spectra that are isotropic in k-space [7–10]. On the other hand, a number of studies suggest that in magnetohydrodynamic (MHD) turbulence excited by large-scale stirring, small-scale fluctuations have non-zero values of $k_\parallel$ that are $\ll k_\perp$, where $k_\perp$ and $k_\parallel$ are the components of $\mathbf{k}$ $\perp$ and $\parallel$ to $\mathbf{B}_0$ [12–14]. In this paper, the quasilinear approximation [11] is used to calculate general scattering rates for incompressible MHD turbulence and also shear-Alfvénic turbulence on the non-MHD scales shorter than the collisional mean free path of thermal particles [12]. These rates are then evaluated for the Goldreich-Sridhar power spectrum [12], which has significant power at small scales only for $k_\perp \gg k_\parallel$. The condition $k_\perp \gg k_\parallel$ is found to significantly decrease the efficiency of pitch-angle scattering relative to the slab-symmetric and isotropic cases. Astrophysical applications and limitations of quasilinear theory (QLT) are discussed.

It is assumed that there is an inertial-range spectrum of fluctuations extending from some large scale l to a much smaller scale d, with the fluctuations at scales $\sim l$ dominating the total magnetic energy. Only cosmic rays with gyroradii $\rho \ll l$ are considered. A scale l' is introduced, with $\rho \ll l' \ll l$. The energetically dominant fluctuations on scales $> l'$ are treated as a uniform field $\mathbf{B}_0$. The magnetic fluctuations on scales $< l'$, denoted $\mathbf{B}_1$, are small compared to $\mathbf{B}_0$ and are treated using QLT. It can be verified *a posteriori* that the QLT scattering rates are independent of l' to lowest order in ρ/l. In contrast to most previous treatments, the turbulence is treated as strong, in the sense that fluctuations decorrelate in one linear wave period.

In QLT, the turbulence causes the cosmic rays to diffuse in momentum space, with the diffusion coefficients determined by the statistical properties of the turbulence [15],

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \xi}\left(D_{\xi\xi}\frac{\partial f}{\partial \xi}\right) + \frac{\partial}{\partial \xi}\left(D_{\xi p}\frac{\partial f}{\partial p}\right) + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\left(D_{p\xi}\frac{\partial f}{\partial \xi} + D_{pp}\frac{\partial f}{\partial p}\right)\right], \tag{1}$$

where f is the cosmic-ray distribution function averaged over the small scales of the fluctuating fields, $\mathbf{p}$ is momentum, θ (the pitch angle) is the angle between $\mathbf{p}$ and $\mathbf{B}_0$, and $\xi = \cos\theta$. In equation (1) it has been assumed that the length scale characterizing variations in f is large compared to ρ, so that f can be taken to be independent of gyrophase. At each $\mathbf{k}$, $\mathbf{B}_1$ and the incompressible turbulent velocity $\mathbf{U}_1$ are decomposed into shear-Alfvén and pseudo-Alfvén components by projecting along the appropriate polarization vectors [12]. These components are denoted respectively by the superscripts s and p, so that

$$\mathbf{B}_1(\mathbf{k},t) = \mathbf{B}_1^{\rm s}(\mathbf{k},t) + \mathbf{B}_1^{\rm p}(\mathbf{k},t), \tag{2}$$

with an analogous equation for $\mathbf{U}_1(\mathbf{k},t)$. The electric field is given by Ohm's Law, $\mathbf{E}_1 = -(1/c)\mathbf{U}_1 \times \mathbf{B}_0$. The normalized power spectra of the shear-Alfvén modes are given by

$$M^{\rm s}(k_\perp, k_\|, \tau) = \langle \mathbf{B}_1^{\rm s}(\mathbf{k}, t) \cdot \mathbf{B}_1^{{\rm s}\,*}(\mathbf{k}, t+\tau)\rangle / B_0^2, \tag{3}$$

$$C^{\rm s}(k_\perp, k_\|, \tau) = \langle \mathbf{U}_1^{\rm s}(\mathbf{k}, t) \cdot \mathbf{B}_1^{{\rm s}\,*}(\mathbf{k}, t+\tau)\rangle / v_A B_0, \qquad \text{and} \tag{4}$$

$$K^{\rm s}(k_\perp, k_\|, \tau) = \langle \mathbf{U}_1^{\rm s}(\mathbf{k}, t) \cdot \mathbf{U}_1^{{\rm s}\,*}(\mathbf{k}, t+\tau)\rangle / v_A^2, \tag{5}$$

with analogous equations for the pseudo-Alfvén modes, where v_A is the Alfvén speed associated with $\mathbf{B}_0$, and $\langle \ldots \rangle$ denotes an ensemble average. It is assumed that the turbulence is homogeneous and stationary, that $\langle \mathbf{B}_1(\mathbf{x}, t)\mathbf{B}_1(\mathbf{x}+\mathbf{r}, t+\tau)\rangle = \langle \mathbf{B}_1(\mathbf{x}, t)\mathbf{B}_1(\mathbf{x}-\mathbf{r}, t+\tau)\rangle$ with analogous equations for $\langle \mathbf{U}_1\mathbf{U}_1\rangle$ and $\langle \mathbf{U}_1\mathbf{B}_1\rangle$ (no magnetic or kinetic helicity), that $\langle \mathbf{U}_1(\mathbf{x}, t)\mathbf{B}_1(\mathbf{x}+\mathbf{r}, t+\tau)\rangle = \langle \mathbf{U}_1(\mathbf{x}, t)\mathbf{B}_1(\mathbf{x}+\mathbf{r}, t-\tau)\rangle$ (which gives $D_{\xi p} = D_{p\xi}$), and that the shear-Alfvén and pseudo-Alfvén modes are statistically independent.

The contributions to the momentum diffusion coefficients from the shear-Alfvén modes and pseudo-Alfvén modes are, respectively,

$$\begin{pmatrix} D^{\rm s}_{\xi\xi} \\ D^{\rm s}_{\xi p} \\ D^{\rm s}_{pp} \end{pmatrix} = \lim_{L\to\infty} \Omega^2(1-\xi^2) \int \frac{d^3k}{L^3} \int_0^\infty d\tau \sum_{n=-\infty}^{\infty} e^{-i(k_\| v_\| + n\Omega)\tau} \frac{n^2 J_n^2(z)}{z^2} \begin{pmatrix} \delta^2\xi^2 K^{\rm s} + 2\delta\xi C^{\rm s} + M^{\rm s} \\ (\delta^2 \xi K^{\rm s} + \delta C^{\rm s})(-p) \\ p^2\delta^2 K^{\rm s} \end{pmatrix}, \quad \text{and} \tag{6}$$

$$\begin{pmatrix} D^{\rm p}_{\xi\xi} \\ D^{\rm p}_{\xi p} \\ D^{\rm p}_{pp} \end{pmatrix} = \lim_{L\to\infty} \Omega^2(1-\xi^2) \int \frac{d^3k}{L^3} \int_0^\infty d\tau \sum_{n=-\infty}^{\infty} e^{-i(k_\| v_\| + n\Omega)\tau} \frac{k_\|^2 J_n'^2(z)}{k^2} \begin{pmatrix} \delta^2\xi^2 K^{\rm p} + 2\delta\xi C^{\rm p} + M^{\rm p} \\ (\delta^2 \xi K^{\rm p} + \delta C^{\rm p})(-p) \\ p^2\delta^2 K^{\rm p} \end{pmatrix}, \tag{7}$$

where $\delta = v_A/v$, $z = k_\perp \rho$, $\rho = v_\perp/\Omega$, Ω is the cosmic-ray gyrofrequency, $v_\perp$ and $v_\|$ are the cosmic-ray velocity components $\perp$ and $\|$ to $\mathbf{B}_0$, L is the dimension of a window function that multiplies the variables before a Fourier transform is taken, and the arguments of K, C, and M are $(\mathbf{k}, \tau)$. Since the shear-Alfvén and pseudo-Alfvén modes are statistically independent, $D_{\xi\xi} = D^{\rm s}_{\xi\xi} + D^{\rm p}_{\xi\xi}$, etc. Equations (6) and (7) are derived using a standard method based on the linearized Vlasov equation [11], modified to treat strongly turbulent fluctuations instead of waves satisfying linear dispersion relations. Alternatively, they can be derived from equations (7a), (7b), and (7c) of [15], if one notes the typographical error on the eighth line of equation (7a), namely, that $Q_{R\|}$ should instead be $Q_{\|R}$.

A Goldreich-Sridhar spectrum of strong, anisotropic MHD turbulence [12] is now assumed, with

$$M^{\rm s}(k_\perp, k_\|, \tau) = \frac{L^3}{6\pi} k_\perp^{-10/3} l^{-1/3} g\left(\frac{k_\|}{k_\perp^{2/3} l^{-1/3}}\right) e^{-|\tau|/\tau_{\mathbf{k}}} \tag{8}$$

for $(l')^{-1} < k_\perp < d^{-1}$ with $d \to 0$, where $\tau_{\mathbf{k}} = (l/v_A)(k_\perp l)^{-2/3}$ is the Lagrangian correlation time appropriate for strong anisotropic incompressible MHD turbulence, and

$$g(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| \geq 1 \end{cases}. \tag{9}$$

The spectrum of equation (8) is also taken to describe the fluctuations on scales between l' and l, and the normalization has been chosen so that the total magnetic energy $\int_{l^{-1}}^\infty k_\perp dk_\perp \int_{-\infty}^\infty dk_\| M^{\rm s}(k_\perp, k_\|, 0) B_0^2/4 = L^3 B_0^2/8\pi$. At small scales in MHD turbulence, there is equipartition between magnetic and kinetic energies, so that $K^{\rm p} = M^{\rm p}$ and $K^{\rm s} = M^{\rm s}$. It is assumed that $M^{\rm p} = M^{\rm s}$ and $C^{\rm p} = C^{\rm s} = \sigma M^{\rm s}$, where the arguments of each of these spectra are $(k_\perp, k_\|, \tau)$, and where the fractional cross helicity $\sigma \in (-1, 1)$ is independent of $\mathbf{k}$.

In many applications, there are two small parameters,

$$\epsilon = \frac{v}{l\Omega}, \qquad \text{and} \qquad \delta = \frac{v_A}{v}. \tag{10}$$

For $\sin\theta \gg \epsilon^{1/2}$, one finds from equations (6) and (7) and the assumed forms of the power spectra that to lowest order in ϵ and δ,

$$\begin{pmatrix} D^{\rm s}_{\xi\xi} \\ D^{\rm s}_{\xi p} \\ D^{\rm s}_{pp} \end{pmatrix} = \frac{v}{l}\left[\frac{2\epsilon^{3/2}|\cos\theta|^{11/2}}{13\sin\theta} \sum_{n=1}^{n=\infty} n^{-9/2} - \frac{\delta \ln\epsilon}{3}\sin^2\theta\right] \begin{pmatrix} 1 \\ -\sigma p\delta \\ p^2\delta^2 \end{pmatrix}, \text{ and} \tag{11}$$

$$\begin{pmatrix} D^{\rm p}_{\xi\xi} \\ D^{\rm p}_{\xi p} \\ D^{\rm p}_{pp} \end{pmatrix} = \frac{v}{l}\left\{\frac{2\epsilon^{3/2}|\cos\theta|^{7/2}\sin\theta}{13} \sum_{n=1}^{n=\infty} n^{-9/2} - \frac{\delta\ln\epsilon}{6}\frac{\sin^4\theta}{\cos^2\theta}\left[1 - \frac{v_A}{v_\|}\arctan\left(\frac{v_\|}{v_A}\right)\right]\right\} \begin{pmatrix} 1 \\ -\sigma p\delta \\ p^2\delta^2 \end{pmatrix}. \tag{12}$$

The terms on the right-hand sides of equations (11) and (12) proportional to $\epsilon^{3/2}$ correspond to fluctuations satisfying the magnetostatic gyroresonance condition $k_\parallel v_\parallel = n\Omega$, which states that the Doppler-shifted frequency of a static magnetic fluctuation in the reference frame of an energetic particle's motion along $\mathbf{B}_0$ is an integral multiple of the particle's gyrofrequency. A fluctuation is seen as static when a cosmic ray passes through one wavelength of the fluctuation in a time $(k_\parallel v_\parallel)^{-1} \ll \tau_{\mathbf{k}}$. The gyroresonant terms in equations (11) and (12) are much smaller than in the case of slab-symmetric or isotropic turbulence for $\sin\theta \gg \epsilon^{1/2}$ because equations (8) and (9) imply that $k_\perp > k_\parallel^{3/2} l^{1/2}$, so that fluctuations satisfying $k_\parallel v_\parallel = n\Omega$ also satisfy $k_\perp \rho > n^{3/2}\epsilon^{-1/2}\sin\theta\cos^{-3/2}\theta \gg 1$. The condition $k_\perp\rho \gg 1$ implies that a cosmic ray traverses many uncorrelated fluctuations of the required $k_\parallel$ during a single gyro orbit. The effects of these uncorrelated fluctuations tend to cancel. The weakening of gyroresonant scattering due to this gyro-orbit averaging would occur for any power spectrum in which all fluctuations on scales $\ll l$ satisfy $k_\perp \gg k_\parallel$. The terms on the right-hand sides of equations (11) and (12) proportional to $(-\ln\epsilon)\delta$ correspond to non-resonant interactions. In equation (12) the non-resonant term arises from the $n = 0$ term in equation (7), which represents the effects of the magnetic-mirror force of the pseudo-Alfvén modes (transit-time damping). This term becomes large as $\theta \to \pi/2$, since as $v_\parallel \to v_A$ particles can "surf" magnetic mirrors moving at speeds $\sim v_A$ more effectively.

For $\sin\theta \ll \epsilon^{1/2}$, scattering is dominated by magnetostatic gyroresonant interactions with shear-Alfvén modes, and to lowest order in ϵ and δ

$$\begin{pmatrix} D^{\rm s}_{\xi\xi} \\ D^{\rm s}_{\xi p} \\ D^{\rm s}_{pp} \end{pmatrix} = \left(\frac{v}{l}\right)\frac{\pi\theta^2}{8}\begin{pmatrix} 1 \\ -\sigma p\delta \\ p^2\delta^2 \end{pmatrix}. \tag{13}$$

Although $D_{\xi\xi}$ vanishes as $\theta \to 0$, the pitch-angle scattering frequency $\nu = 2D_{\xi\xi}/(1-\xi^2)$ (which unlike $D_{\xi\xi}$ is independent of θ for isotropic scattering) approaches $(\pi/4)(v/l)$ as $\theta \to 0$. Gyroresonant interactions are stronger for $\theta \lesssim \epsilon^{1/2}$ than for $\theta \gg \epsilon^{1/2}$, because when $\theta \lesssim \epsilon^{1/2}$ modes satisfying $k_\parallel v_\parallel = n\Omega$ also satisfy $k_\perp\rho \lesssim 1$, so that a cosmic ray doesn't traverse many uncorrelated resonant modes during a single gyro orbit.

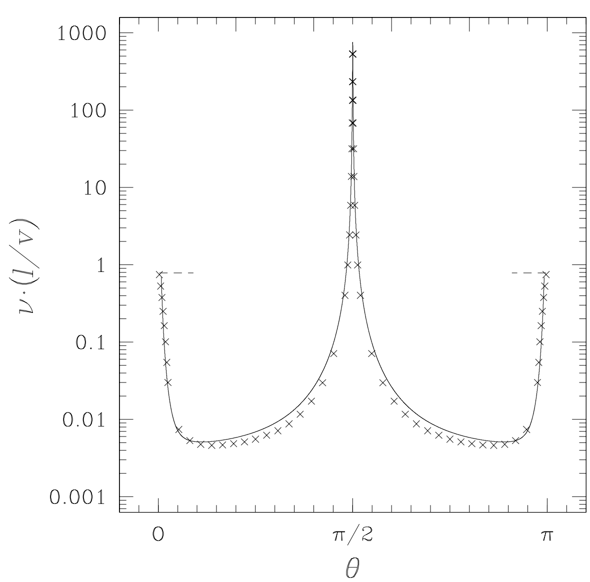

FIG. 1. The θ dependence of the pitch-angle scattering frequency $\nu = 2(D^{\rm s}_{\xi\xi} + D^{\rm p}_{\xi\xi})/(1-\xi^2)$ for $\epsilon = \delta = 10^{-3}$. The $\times$s indicate numerical evaluations of equations (6) and (7), the solid line gives the analytic results of equations (11) and (12), and the dashed lines give the limiting value from equation (13) of ν as $\sin\theta \to 0$.

In figure 1, the pitch-angle scattering frequency ν from equations (11) and (12) is plotted with the solid line, and the limiting value of ν as $\sin\theta \to 0$ from equation (13) is given by the dashed line. The $\times$s indicate numerical evaluations of ν from equations (6) and (7) for the assumed spectra in which only those terms in the infinite sum with $|n| \leq 10$ are kept and in which $l' = 0.1l$ (see introduction). The values $\epsilon = 10^{-3}$ and $\delta = 10^{-3}$ have been used. (σ, which only weakly affects ν, has been set to 0.) The characteristic values $\nu \sim v/l$ for $\sin\theta < \epsilon^{1/2}$, $\nu \sim \delta^{-1}v/l$ for $|\pi - \theta| < \delta$, and $\nu \sim [(-\ln\epsilon)\delta + \epsilon^{3/2}]v/l$ for moderate pitch angles can be extrapolated to all values of ϵ and δ much less than 1 in the quasilinear approximation.

When ν is sufficiently large, f_0 becomes nearly isotropic, and the pitch-angle-averaged distribution $\overline{f}$ can be treated in the diffusion approximation [7,16],

$$\frac{\partial \overline{f}}{\partial t} = \frac{\partial}{\partial l}\left(\kappa_\parallel \frac{\partial \overline{f}}{\partial l}\right) + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 \overline{D}_p \frac{\partial \overline{f}}{\partial p}\right) + \dots, \tag{14}$$

where l is distance along a field line, and the ellipsis indicates the omission of the advection and adiabatic-acceleration terms. The coefficient of spatial diffusion along $\mathbf{B}_0$ is given by [16]

$$\kappa_\parallel = \frac{v^2}{8}\int_{-1}^{1} d\xi \frac{(1-\xi^2)^2}{D_{\xi\xi}}. \tag{15}$$

To lowest order in ϵ and δ when $\epsilon^{3/2} \ll (-\ln\epsilon)\delta$, QLT gives

$$\kappa_\parallel^{\rm QLT} = vl(-\delta \ln \epsilon)^{-1}\left(\frac{5}{2} - \frac{3\pi}{4}\right). \tag{16}$$

[In this paper, from equation (11) on, $M^{\rm p} = M^{\rm s}$; however, if either $M^{\rm p}$ or $M^{\rm s}$ is set to zero, equation (16) becomes $\kappa_\parallel^{\rm QLT} = vl(-2\delta\ln\epsilon)^{-1}$.] To lowest order in ϵ and δ when $(-\ln\epsilon)\delta \ll \epsilon^{3/2}$ QLT gives

$$\kappa_\parallel^{\rm QLT} = vlc_1(-\delta\ln\epsilon)^{-5/11}\epsilon^{-9/11}, \qquad \text{where} \tag{17}$$

$c_1 = (\pi/22)\csc(5\pi/11)6^{5/11}[(2/13)\sum_{n=1}^{\infty} n^{-9/2}]^{-6/11} \simeq 0.88$. The pitch-angle-averaged momentum diffusion coefficient in equation (14) is given by [16]

$$\overline{D}_p = \frac{1}{2}\int_{-1}^{1} d\xi \left[D_{pp} - \frac{D_{\xi p}^2}{D_{\xi\xi}}\right]. \tag{18}$$

To lowest order in ϵ and δ, QLT gives

$$\overline{D}_p^{\rm QLT} = (\pi/24)(1-\sigma^2)(-\ln\epsilon)p^2 v_A^2/(vl). \tag{19}$$

When $\sigma^2 = 1$, $\overline{D}_p$ vanishes since the small-scale fluctuations all travel in a single direction along $\mathbf{B}_0$ at the speed v_A, and, in the reference frame that follows their motion, particle energies are conserved.

Although QLT is a useful and standard tool, it suffers from important inaccuracies. QLT assumes that during the time a particle is correlated with a turbulent fluctuation, the orbit of that particle is the same as in a uniform magnetic field. However, field-strength fluctuations $\triangle|B|$ with $\triangle|B|/|B| \equiv \alpha \ll 1$ magnetically trap cosmic rays with $|\xi| \lesssim \alpha^{1/2}$. (For incompressible turbulent fluctuations, which have phase velocities $\sim v_A$ along the magnetic field, and for cosmic rays with $v \gg v_A$, the trapping condition is essentially the same as if the fluctuations were stationary.) The trajectories of such trapped particles differ greatly from the trajectories of particles in a uniform field, violating the QLT assumptions. Because the integral in equation (18) is dominated by values of $|\xi| \lesssim \delta \ll 1$ for which trapping is important, the value of $\overline{D}_p$ in equation (19) is unreliable. Similarly, when $(-\ln\epsilon)\delta \ll \epsilon^{3/2}$, the integral in equation (15) is dominated by small $|\xi|$, and thus the value of $\kappa_\parallel$ in equation (17) is unreliable. In addition, assuming an unperturbed particle orbit in the presence of a slowly and non-periodically varying $\mathbf{E}_1$ or $\mathbf{B}_1$ leads to spurious changes in a particle's magnetic moment $\mu = mv_\perp^2/2B_0$, which, as an adiabatic invariant, should be virtually conserved when $\mathbf{E}_1$ and $\mathbf{B}_1$ vary on a time scale $\gg \Omega^{-1}$. The non-resonant terms in equations (11) and (12) arise from slowly varying modes and imply such spurious changes in μ, thereby significantly overestimating non-resonant pitch-angle scattering. Since $\kappa_\parallel$ in equation (16) is determined by this non-resonant pitch-angle scattering, equation (16) underestimates $\kappa_\parallel$.

Although the QLT results for the key particle-transport coefficients are inaccurate, QLT does show that resonant scattering by MHD turbulence with $k_\perp \gg k_\parallel$ is much weaker than resonant scattering by slab-symmetric or isotropic fluctuations. Moreover, equation (16) as a lower bound on $\kappa_\parallel$ has important implications. If $B_0 = 5\mu$G, $l = 100$ pc, and $v_A = 10^6$ cm/s [parameters characteristic of the interstellar medium (ISM)], then $\epsilon = 2.2 \times 10^{-9} E_{\rm GeV}$ for a relativistic proton, where $E_{\rm GeV}$ is the proton's energy in GeV, and $\delta = 3.3 \times 10^{-5}$. If $E_{\rm GeV} \ll 10^6$, then $\epsilon^{3/2} \ll (-\ln\epsilon)\delta$, and equation (16) gives a lower limit to the scattering mean free path $\kappa_\parallel/v$ of 430 kpc $\times$ $(20 - \ln E_{\rm GeV})^{-1}$. This value is so large that if the power-law spectrum of interstellar turbulence inferred from observations [17] is described by equation (8), then some mechanism besides such turbulence must be invoked to explain the confinement of cosmic rays to the Galaxy [6]. At energies $\lesssim 10^2 - 10^3$ GeV, such a mechanism is provided by resonant MHD waves that cosmic rays themselves excite, but at higher energies it is believed that self-confinement does not work [5,6]. For $E_{\rm GeV} > 10^2 - 10^3$, cosmic-ray confinement and isotropization can be explained even if turbulent scattering is weak if one takes into account molecular-cloud magnetic mirrors [18]. If the interstellar turbulence generated by supernovae and stellar winds is described by equation (8), the inefficient scattering associated with such turbulence may indicate that quasi-parallel shocks are unable to accelerate cosmic rays up to the $\sim 10^6$ GeV energies at the "knee" of the galactic cosmic-ray energy spectrum [2,19], although this suggestion is controversial [20]. Quasi-perpendicular shocks, however, may be able to accelerate cosmic rays to the knee and beyond [21,22].

I thank Steve Spangler, Eliot Quataert, Torsten Ensslin, Randy Jokipii, Steve Cowley, Jon Arons, Peter Goldreich, and Russell Kulsrud for valuable discussions.

---

[1] A. Bell, Mon. Not. R. Astr. Soc., 182, 147 (1978)
[2] R. Blandford, J. Ostriker, Astrophys. J., 221, L29 (1978)
[3] L. Drury, Rep. Prog. Phys., 46, 973 (1983)
[4] R. Kulsrud, W. Pearce, Astrophys. J., 156, 445 (1969)
[5] C. Cesarsky, Annu. Rev. Astr. Astrophys., 18, 289 (1980)
[6] V. Berezinskii, V. Bulanov, S. Dogiel, V. Ginzburg, and V. Ptuskin, *Astrophysics of Cosmic Rays* (New York, North-Holland, 1990)
[7] J. R. Jokipii, 1966, Astrophys. J., 146, 480 (1966)
[8] J. Bieber, W. Matthaeus, C. Smith, W. Wanner, M. Kallenrode, and G. Wibberenz, Astrophys. J., 420, 294 (1994)
[9] R. Schlickeiser, J. Miller, Astrophys. J., 492, 352 (1998)
[10] J. Giacalone, and J. R. Jokipii, Astrophys. J., 520, 204 (1999)
[11] T. Stix, *Waves in Plasmas* (Springer-Verlag, New York, 1992)
[12] P. Goldreich, and S. Sridhar, Astrophys. J., 438, 763 (1995)
[13] J. Shebalin, W. Matthaeus, and D. Montgomery, J. Plasma Phys., 29, 525 (1983)
[14] C. Ng, and A. Bhattacharjee, submitted, Astrophys. J.
[15] R. Schlickeiser, and U. Achatz, U., J. Plasma Phys., 49, 63 (1993)
[16] R. Schlickeiser, Astrophys. J., 336, 243 (1989)
[17] S. Spangler, and J. Cordes, Astrophys. J., 505 766, 1998
[18] B. Chandran, Astrophys. J., 529, 513 (2000)
[19] P. Lagage, and C. Cesarsky, Astron. Astrophys., 125 249 (1983)
[20] D. Ellison, P. Goret, M. Baring, in *Proc.* 26$^{\rm th}$ *Int. Cosmic Ray Conf., Salt Lake City* (Melville, AIP, 1999), 3, 468
[21] J. R. Jokipii, Astrophys. J., 313, 842 (1987)
[22] J. Kirk, P. Duffy, and Y. Gallant, Astron. Astrophys., 314, 1010 (1996)