

# Detectability of the Parallax-induced Deviations in the Astrometric Centroid Shift Trajectories of Gravitational Microlensing Events

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## ABSTRACT

The lens mass determined from the photometrically obtained Einstein time scale suffers from large uncertainty due to the lens parameter degeneracy. The uncertainty can be substantially reduced if the mass is determined from the lens proper motion obtained from astrometric measurements of the source image centroid shifts,  $\delta\theta_c$ , by using high precision interferometers from space-based platform such as the *Space Interferometry Mission* (SIM) and ground-based interferometers soon available on several 8 – 10 m class telescopes. However, for the complete resolution of the lens parameter degeneracy it is required to determine the lens parallax by measuring the parallax-induced deviations in the centroid shifts trajectory,  $\Delta\delta\theta_c$ .

In this paper, we investigate the detectabilities of  $\delta\theta_c$  and  $\Delta\delta\theta_c$  by determining the distributions of the maximum centroid shifts and the average maximum deviations expected for different types of Galactic microlensing events caused by various masses. From this investigation, we find that as long as source stars are bright enough for astrometric observations it is expected that  $\delta\theta_c$  for most events caused by lenses with masses greater than  $0.1 M_\odot$  regardless of the event types can be easily detected from observations by using not only the SIM (with a detection threshold  $\delta\theta_{\text{th}} \sim 3 \mu\text{as}$ ) but also the ground-based interferometers (with  $\delta\theta_{\text{th}} \sim 30 \mu\text{as}$ ). However, detection of  $\Delta\delta\theta_c$  from ground-based observations will be difficult for nearly all Galactic bulge self-lensing events, and will be restricted only for small fractions of disk-bulge and halo-LMC events, for which the deviations are relatively large. From observations by using the SIM, on the other hand, detecting  $\Delta\delta\theta_c$  will be possible for majority of disk-bulge and halo-LMC events and even for some fraction of bulge self-lensing events. For the complete resolution of the lens parameter degeneracy, therefore, SIM observations (or equivalent) will be essential.

**Key words:** gravitational lensing – astrometry

## 1 INTRODUCTION

The biggest difficulty in revealing the nature of Galactic dark matter by using microlensing is that one cannot determine the individual lens masses because the Einstein time scale, which is the only quantity related to the lens mass obtained from the photometrically measured event light curve, results from the combination of other lens parameters by

$$t_E = \left[ \frac{4GM}{c^2 v^2} \frac{D_{ol}(D_{os} - D_{ol})}{D_{os}} \right]^{1/2}, \quad (1)$$

where  $M$  is the lens mass,  $v$  is the lens-source transverse speed, and  $D_{ol}$  and  $D_{os}$  represent respectively the distances to the lens and the source from the observer. As a method to resolve the lens parameter degeneracy for general microlensing events, it was proposed to measure the shifts of the source star image centroid caused by lensing from follow-

up astrometric observations for events detected photometrically from the ground by using high precision interferometers from space-based platform such as the *Space Interferometry Mission* (SIM) and ground-based interferometers soon available on several 8 – 10 m class telescopes such as the Keck and the Very Large Telescope (Miyamoto & Yoshii 1995; Hø g, Novikov, & Polarev 1995; Walker 1995; Dominik & Sahu 2000).

When the lens-source separation is normalized by the angular Einstein ring radius  $\theta_E$ , the centroid shift vector with respect to the position of the un-lensed source for an event caused by a point-mass lens is represented by

$$\delta\theta_c = \frac{\mathbf{u}}{u^2 + 2}\theta_E; \quad (2)$$

$$u_x = \frac{t - t_0}{t_E}, \quad u_y = u_{\min}, \quad (3)$$

where  $\mathbf{u}$  is the lens-source separation normalized by  $\theta_E$ ,  $u_x$  and  $u_y$  represent the components of  $\mathbf{u}$  parallel and normal to the lens proper motion vector  $\boldsymbol{\mu}$ ,  $t_0$  is the time of the closest lens-source approach, and  $u_{\min}$  is the lens-source separation (also normalized by  $\theta_E$ ) at that moment (i.e. the impact parameter). The angular Einstein ring radius is related to the physical parameters of the lens by

$$\theta_E = \frac{vt_E}{D_{ol}} = \left( \frac{4GM}{c^2} \frac{D_{os} - D_{ol}}{D_{ol}D_{os}} \right)^{1/2}. \quad (4)$$

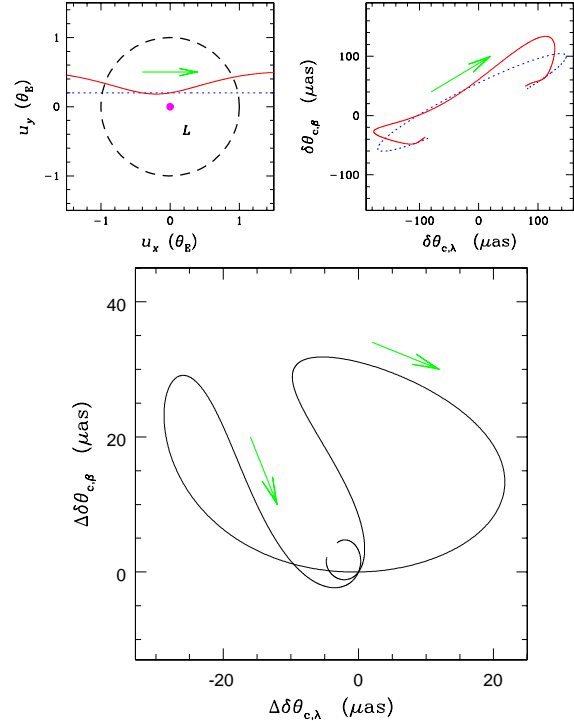
When  $u = \sqrt{2}$ , the amount of the centroid shift becomes maximum with

$$\delta\theta_{c,\max} = \frac{\theta_E}{\sqrt{8}}. \quad (5)$$

During the event, the trajectory of the centroid shifts traces an ellipse (astrometric ellipse). Once the trajectory is constructed from astrometric observations of the event, one can determine  $\theta_E$  because the size of the astrometric ellipse (semi-major axis) is directly proportional to  $\theta_E$ . Then one can determine the lens proper motion by  $\mu = \theta_E/t_E$  combined with the photometrically determined value of  $t_E$ . With the determined value of  $\mu$ , the uncertainty in the determined lens mass can be substantially reduced because  $\mu$  does not depend on  $v$ . However, the lens parameter degeneracy is not completely broken even with the determination of  $\mu$ , because it still results from the combination of the lens mass and the location.

The lens parameter degeneracy can be completely broken, and thus the lens mass can be uniquely determined if one determines the lens parallax by astrometrically measuring the deviations in the observed centroid shift trajectory,  $\Delta\delta\theta_c$ , caused by the change of the observer's location during the event due to the orbital motion of the Earth around the Sun: parallax effect.\* However,  $\Delta\delta\theta_c$ , in general, is much

\* The lens mass can also be determined by astrometrically monitoring nearby very high proper motion stars working as a lens instead of monitoring lensed source stars because the distance to the lens is known. This method was suggested by Miralda-Escudé (1996) and Paczyński (1998) and the specific candidate stars from the SIM monitoring were selected by Salim & Gould (2000). We note, however, that our analysis is focused on followup observations for events detected by the photometric lensing surveys from the ground.



**Figure 1.** The parallax effect on the astrometric behavior of an example halo-LMC event. The upper left panel shows the source star trajectories with respect to the lens ( $L$ ) with (solid curve) and without (dotted curve) the parallax effect. The dashed circle centered at  $L$  represents the Einstein ring. In the upper right panel, the trajectories of the centroid shifts  $\delta\theta_c$  resulting from the individual source trajectories in the upper left panel are marked by the same line types. In the lower panel, we present the trajectory of the deviation vector  $\Delta\delta\theta_c$ , which corresponds to the vector difference between the solid and dotted curve in the upper right panel. Both the trajectories of  $\delta\theta_c$  and  $\Delta\delta\theta_c$  are presented in the ecliptic coordinates. For the details about the lens system, see the text.

smaller than  $\delta\theta_c$ , and thus measurements of the parallax-induced deviations require higher precision observations. The possibility of astrometric parallax measurements was investigated by Paczyński (1998) and further analysis considering the actual performance of the specific instruments was done by Boden, Shao, & Van Buren (1998). However, their analyses were based on several example events.

In this paper, we determine the distributions of the maximum centroid shifts,  $f(\delta\theta_{c,\max})$ , and the average maximum deviations,  $f(\langle\Delta\delta\theta_{c,\max}\rangle)$ , for different types of Galactic microlensing events caused by various masses. From the analysis of these distributions, we statistically investigate the detectabilities of  $\delta\theta_c$  and  $\Delta\delta\theta_c$  expected from ground- and space-based observations.

## 2 PARALLAX-INDUCED DEVIATIONS IN CENTROID SHIFT TRAJECTORIES

Due to the orbital motion of the Earth around the Sun, the lens location in the ecliptic coordinates changes by

$$\Delta\varphi_\lambda = \Pi \sin(\lambda_\odot - \lambda), \quad (6)$$

$$\Delta\varphi_\beta = -\Pi \sin\beta \cos(\lambda_\odot - \lambda), \quad (7)$$

where  $\Pi = 1 \text{ AU}/D_{ol}$  represents the lens parallax,  $(\lambda, \beta)$  is the ecliptic coordinates towards the direction where the event is occurred, and  $\lambda_\odot$  represents the ecliptic latitude of the Sun during the event. Then the change of the source location (normalized by  $\theta_E$ ) with respect to the lens becomes

$$\Delta u_x = -\left(\frac{\Delta\varphi_\lambda}{\theta_E} \cos\phi + \frac{\Delta\varphi_\beta}{\theta_E} \sin\phi\right) \left(\frac{D_{os} - D_{ol}}{D_{os}}\right), \quad (8)$$

$$\Delta u_y = -\left(-\frac{\Delta\varphi_\lambda}{\theta_E} \sin\phi + \frac{\Delta\varphi_\beta}{\theta_E} \cos\phi\right) \left(\frac{D_{os} - D_{ol}}{D_{os}}\right), \quad (9)$$

where  $\phi$  represents the angle between  $\boldsymbol{\mu}$  and the ecliptic plane. The factor  $(D_{os} - D_{ol})/D_{os}$  is included because all lengths are projected onto the source plane, and the minus sign is included because the source position changes towards the direction opposite to that of the lens position change. Then the parallax-induced deviation vector of the observed centroid shift from the centroid shift expected without the Earth's orbital motion is obtained by

$$\Delta\delta\theta_c = \delta\theta_c(\mathbf{u} + \Delta\mathbf{u}) - \delta\theta_c(\mathbf{u}), \quad (10)$$

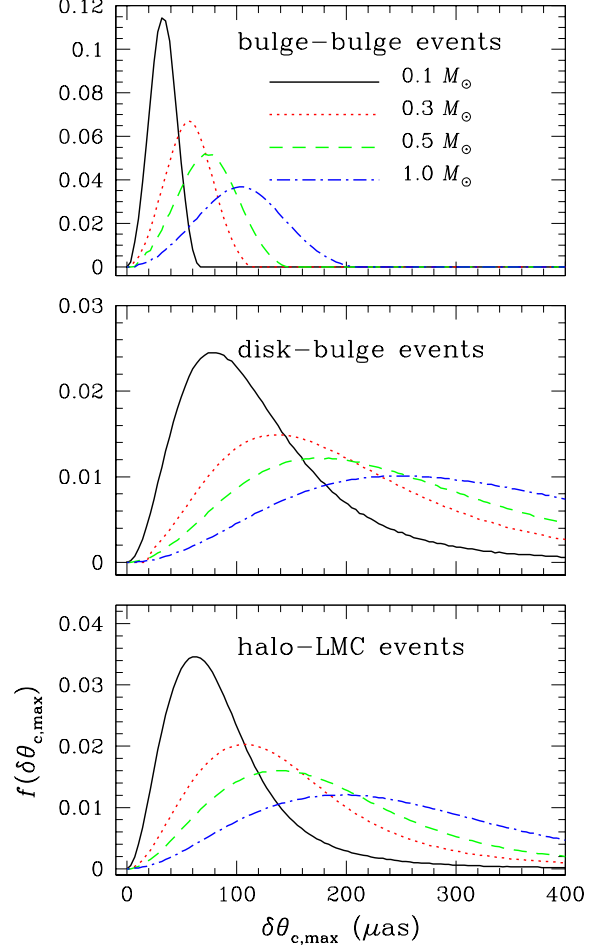
where  $\Delta\mathbf{u} = (\Delta u_x, \Delta u_y)$ .

The parallax effect on the astrometric behavior of a microlensing event is demonstrated in Figure 1. The upper left panel shows the source star trajectories with respect to the lens ( $L$ ) with (solid curve) and without (dotted curve) the parallax effect. In the upper right panel, the centroid shift trajectories resulting from the individual source trajectories in the upper left panel are marked by the same line types. In the lower panel, the trajectory of the deviation vector  $\Delta\delta\theta_c$  is presented, which corresponds to the vector difference between the solid and dotted curves in the upper right panel. The event is assumed to be caused by a lens located in the Galactic halo with  $D_{ol} = 10 \text{ kpc}$  and  $M = 0.5 M_\odot$  for a source star located in the Large Magellanic Cloud (LMC) with  $D_{os} = 50 \text{ kpc}$ . We also assume that the lensing parameters are  $u_{\min} = 0.2$  and  $t_E = 100$  days, and the closest lens-source approach occurs at the moment when the source is at the largest angular separation from the Sun (i.e.  $\lambda_\odot - \lambda = 180^\circ$ ). Observations are assumed to be carried out during  $-5t_E \leq t_{\text{obs}} \leq 5t_E$  with respect to the time of maximum amplification. The ecliptic latitude of a star in LMC is  $\beta \sim \sin^{-1}(-0.99)$ . Note that both the trajectories of  $\delta\theta_c$  and  $\Delta\delta\theta_c$  are presented in the ecliptic coordinates. From the figure, one finds that  $\Delta\delta\theta_c$  is much (nearly an order) smaller than  $\delta\theta_c$ .

### 3 DETECTION OF CENTROID SHIFTS

In this section, we show that as long as source stars are bright enough for astrometric observations<sup>†</sup> one can detect  $\delta\theta_c$  for most Galactic events caused by lenses with masses

<sup>†</sup> Note that astrometric observations of microlensing events are restricted by source brightness. With the SIM having a detection limit of  $V \sim 20$  mag, although most Galactic bulge events can be observed, a substantially fraction (nearly half) of the LMC events cannot be observed owing to the faintness of their source stars (Han & Chang 1999).



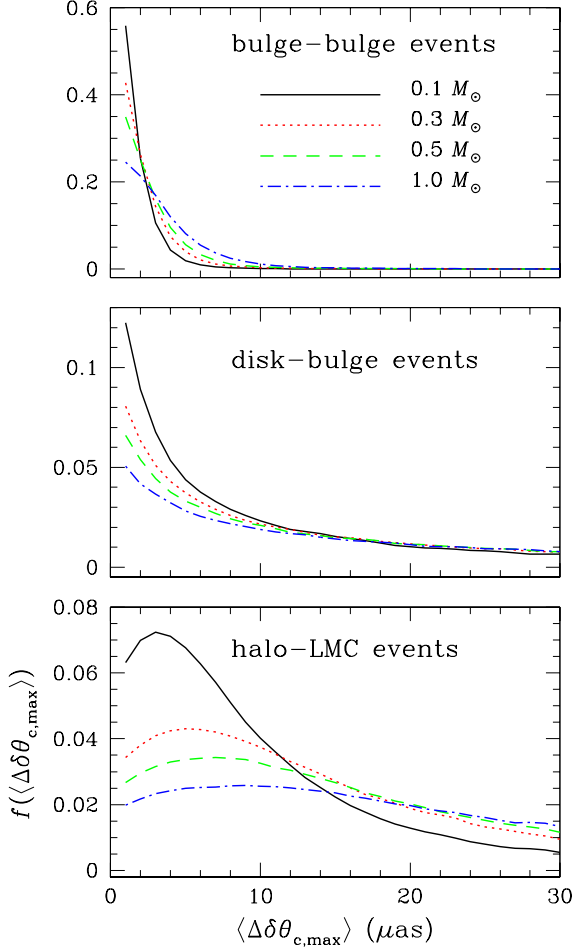
**Figure 2.** The distributions of the maximum astrometric centroid shifts expected from different types of Galactic events caused by various masses. Note that the distributions are arbitrarily normalized.

greater than  $0.1 M_\odot$  regardless of the event types from observations by using not only the SIM but also the ground interferometers.

To show this, we compute the distribution of the maximum centroid shifts by

$$\begin{aligned} f(\delta\theta_{c,\max}) &= \int_0^\infty dD_{os} \rho(D_{os}) \int_0^{D_{os}} dD_{ol} \rho(D_{ol}) \pi r_E^2 \\ &\times \int_0^\infty \int_0^\infty dv_x dv_y v f(v_x, v_y) \\ &\times \delta[\delta\theta_{c,\max} - \theta_E(D_{ol}, D_{os}, M)/\sqrt{8}], \end{aligned} \quad (11)$$

where  $r_E = D_{ol}\theta_E$  is the physical radius of the Einstein ring,  $\rho(D_{ol})$  and  $\rho(D_{os})$  represent the matter density distributions of the lenses and source stars along the line of sight,  $\delta$  represents the dirac delta function,  $v_x$  and  $v_y$  are the two components of  $\mathbf{v}$ , and  $f(v_x, v_y)$  is their distribution. For the physical and dynamical distributions of matter in the Galactic disk and bulge, we adopt the models of Han & Gould (1995). For the matter in the Galactic halo, we adopt an isothermal sphere model with a core radius of the form



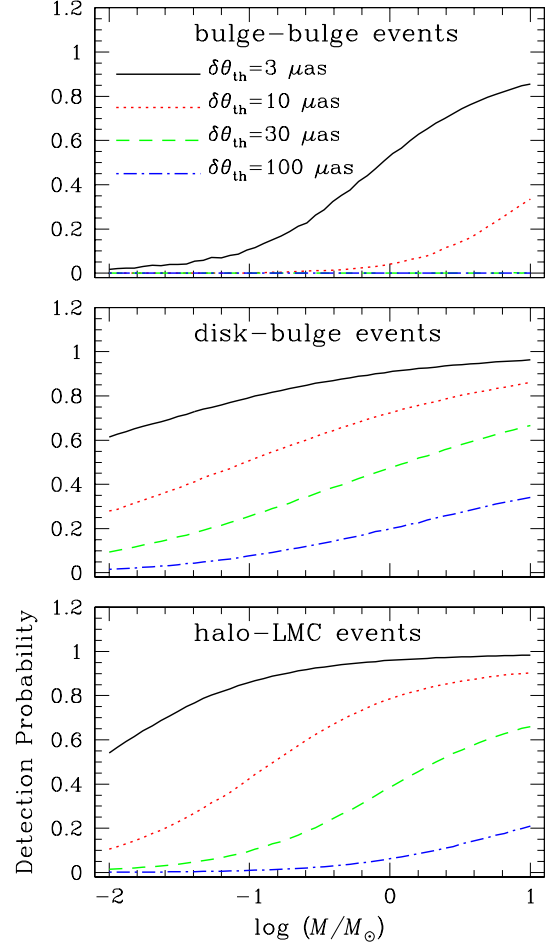
**Figure 3.** The distributions of the average maximum deviations in the centroid shift trajectory for different types of Galactic events caused by various masses. Note that the distributions are arbitrarily normalized.

$$\rho(r) = \rho_0 \frac{r_c^2 + R_0^2}{r_c^2 + r^2}, \quad (12)$$

where  $r$  is the distance measured from the Galactic center,  $R_0 = 8$  kpc is the adopted Galacto-centric distance of the Sun, and  $\rho_0 = 7.9 \times 10^{-3} M_{\odot} \text{pc}^{-3}$  is the halo mass density in the solar neighborhood. The adopted velocity dispersion and the core radius are  $\sigma = 250/\sqrt{2} \text{ km s}^{-1}$  and  $r_c = 2$  kpc. The factors  $\pi r_E^2$  and  $v$  in equation (11) are included to weight the event rate by the cross-section (i.e. Einstein ring area) and the transverse speed.

In Figure 2, we present the computed distributions  $f(\delta\theta_{c, \max})$  for different types of Galactic events.<sup>‡</sup> In each

<sup>‡</sup> We note that both the distributions  $f(\delta\theta_{c, \max})$  and  $f(\langle \Delta \delta \theta_{c, \max} \rangle)$  are determined for all events with  $u_{\min} \leq 1.0$ , not considering the dependence of the detection efficiency on the event duration and the amplification. As a result, they are different from the distributions for events actually detected by the previous and the current lensing surveys (Alcock et al. 1993; Aubourg et al. 1993; Udalski et al. 1993; Alard & Guibert 1995; Abe et al. 1997). We determine the distributions in this way because the detection efficiency is greatly dependent on the adopted observa-



**Figure 4.** The distributions of the probability of detecting the parallax-induced deviations in the centroid shift trajectories for given values of detection threshold,  $\delta\theta_{\text{th}}$ , as a function of (logarithmic) lens mass.

panel, we mark the type of events according to the order of the lens and source locations. For example, ‘disk-bulge events’ refer to those caused by lenses located in the Galactic disk for source stars located in the bulge. From the obtained distributions, one finds that for a given lens mass bulge self-lensing (bulge-bulge) events are most likely to have the smallest value of  $\delta\theta_{c, \max}$ . One also finds that the expected most frequent value of  $\delta\theta_{c, \max}$  even for this type of events caused by small mass lenses with  $M \sim 0.1 M_{\odot}$  exceeds  $\sim 30 \mu\text{as}$ . Then, considering the detection threshold is  $\delta\theta_{\text{th}} \sim 3 \mu\text{as}$  for the SIM (Unwin, Boden, & Shao 1997) and  $\delta\theta_{\text{th}} \sim 30 \mu\text{as}$  for the ground interferometers (Colavita et al. 1998), detection of  $\delta\theta_c$  will be possible for most of events regardless of the event types from observations by using not only the SIM but also the ground interferometers.

tional strategy, the instruments, and the data analysis technique, which are evolving rapidly, and thus the analysis based on the observation condition and strategy of a specific lensing survey will have little meaning.

#### 4 DETECTION OF PARALLAX-INDUCED DEVIATIONS

In the previous section, we show that the amount of the maximum centroid shifts for most Galactic microlensing events are large enough to be detected both from space and ground-based astrometric observations. However, as shown in the example of Figure 1, the amount of the parallax-induced deviations will be much smaller than the shifts caused by the lensing itself. Then a naturally arising question is that for what fraction of events one can determine lens parallaxes by detecting  $\Delta\delta\theta_c$ . In this section, we answer this question.

To determine the fraction of events with detectable  $\Delta\delta\theta_c$ , we compute the distribution of the average maximum deviations by

$$\begin{aligned}
 f(\langle\Delta\delta\theta_{c,\max}\rangle) &= \int_0^\infty dD_{os}\rho(D_{os}) \int_0^{D_{os}} dD_{ol}\rho(D_{ol})\pi r_E^2 \\
 &\times \int_0^\infty \int_0^\infty dv_x dv_y v f(v_x, v_y) \\
 &\times \delta[\langle\Delta\delta\theta_{c,\max}\rangle - \langle\Delta\delta\theta_{c,\max}\rangle'(\theta_E, \Pi; D_{ol}, D_{os}, M)].
 \end{aligned}
 \quad (13)$$

For the computation of  $\langle\Delta\delta\theta_{c,\max}\rangle$ , we first compute  $\Delta\delta\theta_c$  of each event for given values of the lens parameters ( $M$ ,  $D_{ol}$ , and  $D_{os}$ ) and the parameters of the relative observer-lens-source positions ( $\lambda_\odot$  and  $\phi$ ) by using the equations in § 2. We then obtain the maximum value which is expected from the assumed duration of observation. Since astrometric observations will be carried out for events detected from photometric monitoring, we assume observation duration to be  $-1t_E \leq t_{\text{obs}} \leq 10t_E$ . Since the lens proper motion vector  $\boldsymbol{\mu}$  has no preferred orientation, the average value of the maximum centroid shifts is obtained by assuming that  $\phi$  is randomly distributed in the range  $0 \leq \phi \leq 2\pi$ . In addition, since we are interested in all lensing events not in the events detectable by a specific lensing survey, the impact parameters of events have random distribution in the range  $0 \leq u_{\min} \leq 1.0$ . The ecliptic latitude of a star towards the Baade's window is  $\beta \sim -6^\circ.5$ . For the physical and dynamical distributions of the lens and source, we assume the same distributions used for the computation of  $f(\delta\theta_{c,\max})$ .

Figure 3 shows the obtained distributions  $f(\langle\Delta\delta\theta_{c,\max}\rangle)$  for various types of Galactic lensing events. In Figure 4, we also present the distributions of the probability of detecting  $\Delta\delta\theta_c$  for given values of detection threshold,  $\delta\theta_{\text{th}}$ , as a function of (logarithmic) lens mass. From the figures, one finds the following facts. First, if observations are performed with the ground interferometers, it will be difficult to detect  $\Delta\delta\theta_c$  for nearly all bulge self-lensing events since the detection probability is less than 1% even for events caused by lenses with  $M = 1.0 M_\odot$ . In addition, detection will be limited for only small fractions of disk-bulge and halo-LMC events. For example, for these types of events caused by  $M = 0.3 M_\odot$ , the probabilities are 36% and 21%, respectively. Second, if events are observed by using the SIM, on the other hand, it is expected to detect the deviations for majority of disk-bulge and halo-LMC events and even for some fraction of bulge self-lensing events. For example, the detection probabilities for events with  $M = 0.3 M_\odot$  are 86% and 93% for disk-bulge and halo-LMC events and 27% for bulge self-lensing events. In Table 1, we summarize the

event type	lens mass ( $M_\odot$ )	detection probability	
		SIM	ground
bulge-bulge	0.1	0.11	0.00
	0.3	0.27	0.00
	0.5	0.37	0.00
	1.0	0.53	0.00
disk-bulge	0.1	0.79	0.26
	0.3	0.86	0.36
	0.5	0.88	0.41
	1.0	0.91	0.47
halo-LMC	0.1	0.86	0.10
	0.3	0.93	0.21
	0.5	0.94	0.28
	1.0	0.96	0.39

**Table 1.** The detection probabilities of the parallax-induced deviations in centroid shift trajectories which are expected from the astrometric observations by using the SIM and ground interferometers. The assumed detection thresholds are  $\delta\theta_{\text{th}} = 3 \mu\text{as}$  for the SIM and  $\delta\theta_{\text{th}} = 30 \mu\text{as}$  for the ground interferometers.

detection probabilities which are expected from the observations by using the SIM and ground interferometers.

#### 5 SUMMARY AND CONCLUSION

We compute the distributions of the maximum astrometric source star image centroid shifts,  $\delta\theta_{c,\max}$ , and the average maximum deviations in the centroid shift trajectories,  $\langle\Delta\delta\theta_{c,\max}\rangle$ , expected for different types of Galactic events caused by various masses. The findings from the analysis of these distributions are summarized as follows.

- (i) As long as source stars are bright enough for astrometric observations, one can detect  $\delta\theta_c$  for most events caused by lenses with masses greater than  $0.1 M_\odot$  regardless of the event types from observations by using not only the SIM but also the ground interferometers.
- (ii) If observations are performed with the ground interferometers, detecting  $\Delta\delta\theta_c$  will be possible for nearly none of bulge self-lensing events and for only a small fractions of disk-bulge and halo-LMC events.
- (iii) However, if events are observed by using the SIM, detection of  $\Delta\delta\theta_c$  will be possible for most disk-bulge and halo-LMC events and even for some fraction of bulge self-lensing events.

Therefore, for the complete resolution of the lens degeneracy and accurate lens mass determination, SIM observations (or equivalent) will be essential.

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