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Julian Schwinger

Dynamics of low-energy nuclear forces and Solar Neutrino Problems in the Nambu–Jona–Lasinio model of light nuclei

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January 1, 2019

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Abstract

The Solar Neutrino Problems (SNP's) are analysed in the Nambu–Jona–Lasinio model of light nuclei. In this model a possible clue to the solution of the SNP's is in the reduction of the solar neutrino fluxes relative to the predicted by the Standard Solar Model through the decrease of the solar core temperature. The former can be realized through the enhancement of the astrophysical factor for the solar proton burning. The enhancement the upper bound of which is restricted by the helioseismological data goes dynamically via the contribution of the nucleon tensor current coupled to the deuteron. The agreement of the reduced solar neutrino fluxes with the experimental data can be reached within a scenario of vacuum two-flavour neutrino oscillations without a fine tuning of the neutrino–flavour oscillation parameters. In the Nambu–Jona–Lasinio model of light nuclei an enhancement of the astrophysical factor for the solar proton burning entails a change of the cross sections for neutrino and anti-neutrino disintegration of the deuteron at low energies. This provides a theoretical foundation for a new check of a value of the astrophysical factor in terrestrial laboratories.

PACS: 11.10.Ef, 13.75.Cs, 14.20.Dh, 21.30.Fe, 26.65.+t
 Keywords: deuteron, proton burning, solar neutrino fluxes

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1 Introduction

The weak nuclear reaction $p + p \rightarrow D + e^+ + \nu_e$, the solar proton burning, plays an important role in Astrophysics [1,2]. It gives start for the p–p chain of nucleosynthesis in the Sun and main–sequence stars [1,2]. In the Standard Solar Model (SSM) [3] the total (or bolometric) luminosity of the Sun $L_\odot = (3.846 \pm 0.008) \times 10^{26}$ W is normalized to the astrophysical factor $S_{pp}(0)$ for the solar proton burning. The recommended value $S_{pp}^{SSM}(0) = 4.00 \times 10^{-25}$ MeVb [4] has been found by averaging over the results obtained in the Potential model approach (PMA) [5,6] and the Effective Field Theory (EFT) approach [7,8]. As has been shown recently [9] *the inverse and forward helioseismological approach indicate that higher values of $S_{pp}(0)$ seem more favoured*. However, as has been stated by Degl’Innocenti, Fiorentini and Ricci [10] the helioseismological data restrict the value of the astrophysical factor $S_{pp}(0)$ and predict $0.94 \leq S_{pp}(0)/S_{pp}^{SSM}(0) \leq 1.18$.

The value of the astrophysical factor $S_{pp}(0)$ for the solar proton burning is very closely connected with the Solar Neutrino Problem formulated by Bahcall [1] as a discrepancy between theoretical and experimental values for the solar neutrino fluxes of high energy neutrinos. Recently [11] following the contemporary experimental data GALLEX[12], SAGE [13], HOMESTAKE [14], KAMIOKANDE [15] and SUPERKAMIOKANDE [16] Bahcall formulated three Solar Neutrino Problems (SNP’s), *three different discrepancies between the calculations and the observations* of solar neutrino fluxes. In Table 1 we adduce the experimental data on the solar neutrino fluxes, whereas Table 2 contains the theoretical values of the solar neutrino fluxes calculated within the SSM [3] and normalized to the recommended value for the astrophysical factor $S_{pp}^{SSM}(0)$ [4]. According to Bahcall’s classification [11]:

The *first* SNP is the disagreement between the calculations and observations for the chlorine experiment [14]. The measured rate is 2.56 ± 0.23 SNU¹, whereas the theoretical prediction is about 3 times larger $7.7_{-1.1}^{+1.2}$ SNU. As has been emphasized by Bahcall [11] *most of the predicted rate in the chlorine experiment is from the rare, high energy ^8B neutrinos, although the ^7Be neutrinos are expected to contribute significantly*.

The *second* SNP results from a comparison of the measured event rates in the chlorine experiment and in the Japanese purewater experiments, these are KAMIOKANDE [15] and SUPERKAMIOKANDE [16]. According to the SSM [3] the main contribution to neutrino fluxes measured by KAMIOKANDE and SUPERKAMIOKANDE Collaborations comes from the β decay of ^8B , $^8\text{B} \rightarrow ^8\text{Be}^* + e^+ + \nu_e$, in the solar core. There is also a contribution from the *hep* reaction, $p + ^3\text{He} \rightarrow ^4\text{He} + e^+ + \nu_e$ [11].

The *third* SNP is related to the gallium experiments, GALLEX and SAGE. Formally the experimental rates measured by these groups evidence the exclusion of all contributions save the pp neutrinos produced in the reaction $p + p \rightarrow D + e^+ + \nu_e$.

As has been stated by Bahcall [11] the experimental data obtained by all five solar neutrino experimental groups, GALLEX, SAGE, HOMESTAKE, KAMIOKANDE and SUPERKAMIOKANDE, can be fitted well within the approaches involving neutrino flavour oscillations, either vacuum oscillations suggested by Gribov and Pontecorvo [17–20] or resonant matter oscillations suggested by Wolfenstein, Mikheyev and Smirnov [21] so-called MSW effect [22]. The only forthcoming of these fits is in the necessity to make

¹Everywhere in the paper we use statistical and systematic experimental errors combined in quadrature [11] $\sigma = \sqrt{(\text{stat.})^2 + (\text{syst.})^2}$

a very fine tuning of neutrino–flavour oscillation parameters: 1) the squared mass differences $\Delta m_{ij}^2 = m_i^2 - m_j^2$, the mixing angles θ_{ij} and so on where the indices run over the number of oscillating neutrino flavours $i(\text{or } j) = \nu_e, \nu_\mu, \nu_\tau, \dots$

An alternative way that does not demand a fine tuning of the neutrino–flavour oscillation parameters and leads to the solution of the SNP’s can go, for example, through the reduction of the solar neutrino fluxes in the solar core caused by the decrease of the solar core temperature. The former can be related to the enhancement of the astrophysical factor for the solar proton burning. If one would follow the SSM such an enhancement of the astrophysical factor should be restricted by the helioseismological data [10]. This constraint prohibits the possibility to reduce at once the theoretical values of the solar neutrino fluxes in agreement with experimental data. This implies that a secondary reduction is required. Such a secondary reduction of the theoretical solar neutrino fluxes but taking place already outside the solar core can be induced, for example, by neutrino–flavour oscillations. After the reduction of the solar neutrino fluxes in the solar core the final result, most likely, should not be sensitive to the mechanism of neutrino–flavour oscillations. Therefore, the secondary reduction of the theoretical solar neutrino fluxes can be carried out, for example, within a vacuum two–flavour neutrino oscillation scenario with a simplest mechanism of neutrino–flavour oscillations. Below we show that this turns out to be enough for the reduction of the theoretical values of the solar neutrino fluxes in agreement with the experimental data and does not demand a fine tuning of neutrino–flavour oscillation parameters.

The paper is organized as follows. In Section 2 we give a cursory outline of the Nambu–Jona–Lasinio model of light nuclei. In Sect. 3 we calculate the reduced value of the solar core temperature and the solar neutrino fluxes. The experimental data and theoretical results are adduced in Tables 1–5. In the Conclusion we discuss the obtained results.

2 The Nambu–Jona–Lasinio model of light nuclei

Nowadays neither the PMA nor the EFT can provide a dynamical enhancement of the astrophysical factor $S_{pp}(0)$ relative to the recommended value $S_{pp}^{\text{SSM}}(0) = 4.00 \times 10^{-25} \text{ MeVb}$ [4]. As has been pointed out in Refs. [5–9] the contributions of low–energy nuclear forces are taken into account with an accuracy better than 1%².

The required enhancement of the astrophysical factor $S_{pp}(0)$ can be obtained within the Nambu–Jona–Lasinio model of light nuclei [24–26], or differently the nuclear Nambu–Jona–Lasinio (NNJL) model, invented for the description of low–energy nuclear forces at the quantum field theoretic level. As has been shown in Ref. [24] the NNJL model is fully motivated by QCD. The deuteron appears in *the nuclear phase of QCD* as a neutron–proton collective excitation, a Cooper np–pair³, caused by a phenomenological local four–nucleon interaction. Strong low–energy interactions of the deuteron coupled to itself and other particles are described in terms of one–nucleon loop exchanges. This

²An attempt to change the value of the astrophysical factor $S_{pp}(0)$ within the EFT has been recently undertaken by Kong and Ravndal [23]. They have calculated the astrophysical factor $S_{pp}^{\text{KR}}(0) = (4.31 \pm 0.35) \times 10^{-25} \text{ MeV b}$ the meanvalue of which is increased by about 8%.

³In this connection we would like to refer to the paper by Baldo, Lombardo and Schuck [27], where there has been shown that the formation of the deuteron in heavy ion reactions at intermediate energies goes through the superfluid Cooper pair phase to a Bose deuteron gas.

allows to transfer nuclear flavours from an initial to a final nuclear state by a minimal way and to take into account contributions of nucleon–loop anomalies determined completely by one–nucleon loop diagrams. The dominance of contributions of nucleon–loop anomalies is justified in the large N_C approach to non–perturbative QCD with $SU(N_C)$ gauge group at $N_C \rightarrow \infty$, where N_C is the number of quark colours [24].

In Ref. [26] the NNJL model has been applied to the description of low–energy nuclear forces for electromagnetic and weak nuclear reactions with the deuteron. There have been calculated cross sections and astrophysical factors for the neutron–proton radiative capture (M1–capture) $n + p \rightarrow D + \gamma$, the photomagnetic disintegration of the deuteron $\gamma + D \rightarrow n + p$ and weak reactions of astrophysical interest such as: 1) the solar proton burning $p + p \rightarrow D + e^+ + \nu_e$, 2) the pep–process $p + e^- + p \rightarrow D + \nu_e$ and 3) the reactions of neutrino and anti–neutrino disintegration of the deuteron caused by charged $\nu_e + D \rightarrow e^- + p + p$, $\bar{\nu}_e + D \rightarrow e^+ + n + n$ and neutral $\nu_e(\bar{\nu}_e) + D \rightarrow \nu_e(\bar{\nu}_e) + n + p$ weak currents.

In the NNJL model the deuteron couples to itself and other particles through the nucleon axial–vector current $j^\mu(x) = -i[\bar{p}^c(x)\gamma^\mu n(x) - \bar{n}^c(x)\gamma^\mu p(x)]$ with a coupling constant g_V and the nucleon tensor current $J^{\mu\nu}(x) = \bar{p}^c(x)\sigma^{\mu\nu}n(x) - \bar{n}^c(x)\sigma^{\mu\nu}p(x)$ with a coupling constant g_T connected with g_V by the relation $g_T = \sqrt{3/8}g_V$ [24]. In turn, the coupling constant g_V is determined by the electric quadrupole moment of the deuteron Q_D : $g_V^2 = 2\pi^2 Q_D M_N^2$ [24], where $M_N = 940$ MeV is the nucleon mass.

The reaction of the neutron–proton radiative capture for thermal neutrons $n + p \rightarrow D + \gamma$ plays an important role for nucleosynthesis in Early Universe [2]. For thermal neutrons the reaction $n + p \rightarrow D + \gamma$ is the M1–capture induced fully by the magnetic M1–transition. The M1–capture $n + p \rightarrow D + \gamma$ and the photomagnetic disintegration of the deuteron $\gamma + D \rightarrow n + p$ are related via time–reversal invariance. For the evaluation of the amplitude of the M1–capture in the NNJL model [26] we have taken into account both chiral one–meson loop corrections, obtained in Chiral perturbation theory at the quark level $(\text{CHPT})_q$ developed within the extended Nambu–Jona–Lasinio (ENJL) model with a linear realization of chiral $U(3) \times U(3)$ symmetry [28], and the $\Delta(1232)$ resonance treated as a Rarita–Schwinger field. We have shown that within the experimental uncertainties of the definition of the coupling constant of the $\pi\Delta N$ interaction off–mass shell of the $\Delta(1232)$ resonance described by the parameter Z the NNJL model fits well the experimental value of the cross section for the neutron–proton radiative capture. At $Z = 1/2$ favoured theoretically [29] we have got the result $\sigma(\text{np} \rightarrow D\gamma)(T_n) = 325.5$ mb, where $T_n = 0.0253$ eV is the kinetic energy of a thermal neutron in the laboratory frame, agreeing with the experimental value $\sigma(\text{np} \rightarrow D\gamma)(T_n) = (334.2 \pm 0.5)$ mb with an accuracy about 3% [26]. Hence, we argue that in the NNJL model as well as in the EFT, developing a quantum field theoretic approach to the description of low–energy nuclear forces but, unlike the NNJL model, in relativistically non–covariant way, all corrections and contributions to the amplitudes of low–energy nuclear reactions are under the control. We would like to note that due to the loss of relativistic covariance nucleon–loop anomalies are ill–defined in the EFT that makes impossible any application of a mechanism of nucleon–loop anomalies to the correct description of low–energy nuclear forces. In turn, in the NNJL model, the relativistically covariant quantum field theoretic model, nucleon–loop anomalies are well defined and give dominant contributions.

As has been shown in Ref.[26] the contribution of the nucleon tensor current enters to

the cross sections for weak nuclear reactions with the deuteron with an arbitrary parameter $\bar{\xi}^4$. Due to isotopical invariance of low-energy nuclear forces this parameter is the same for all low-energy weak nuclear reactions involving pp, nn or np states.

At zero contribution of the nucleon tensor current, $\bar{\xi} = 0$ ⁵, we have obtained [26]: 1) the astrophysical factor for the solar proton burning $S_{pp}(0) = 4.08 \times 10^{-25}$ MeV b agreeing well with the recommended one [4]⁶, 2) the astrophysical factor for the pep-process $S_{pep}(0)$ relative to $S_{pp}(0)$ in complete agreement with the result obtained by Bahcall and May [31]⁷, 3) the cross section for the neutrino disintegration of the deuteron $\nu_e + D \rightarrow e^- + p + p$ caused by charged weak current and 4) the cross sections for the anti-neutrino disintegration of the deuteron $\bar{\nu}_e + D \rightarrow e^+ + n + n$ and $\bar{\nu}_e + D \rightarrow \bar{\nu}_e + n + p$ caused by charged and neutral weak currents, respectively, and averaged over anti-neutrino spectrum in a reasonable agreement with recent experimental data obtained by the Reines's experimental group [32].

Hence, in the NNJL model at $\bar{\xi} = 0$ (or $\bar{\xi} = -2$ that is the same) we get a dynamics of low-energy nuclear forces for the description of weak nuclear reactions with the deuteron in agreement with the recommendations of the SSM.

Of course, one can develop a scenario of the description of low-energy nuclear forces contributing to weak nuclear reactions with the deuteron when the parameter $\bar{\xi} \neq 0$ (or that is the same $\bar{\xi} \neq -2$) and tune this parameter in such a way in order to get an enhancement of the value of the astrophysical factor $S_{pp}(0)$ relative to the recommended one $S_{pp}^{SSM}(0) = 4.00 \times 10^{-25}$ MeV b [4].

Thus, the NNJL model [24–26]⁸ gives a hint that there is a dynamical reason, a non-trivial contribution of low-energy nuclear forces induced by the nucleon tensor current coupled to the deuteron, for an enhancement of the astrophysical factor for the solar proton burning leading, in turn, to a reduction of the solar core temperature.

The main goal of this paper is to analyse the consistency of this reduction of the solar core temperature with the SSM, helioseismological data, the experimental data on the solar neutrino fluxes and experimental data on cross sections for the reactions of the anti-neutrino disintegration of the deuteron [32]. On this way in order to be close as much as it is possible to the SSM we would like to accentuate that the theoretical values of the solar neutrino fluxes, their dependence on the solar core temperature and the relationship between the changes of the astrophysical factor for the solar proton burning and the solar core temperature would be taken from the SSM [11]. Thereby, the theoretical accuracy

⁴The appearance of an arbitrary parameter in the astrophysical factor for the solar proton burning the value of which is governed by the experimental data on the cross sections for the low-energy reactions of neutrino and anti-neutrino disintegration of the deuteron has been argued recently by Kong and Ravndal [23] in the EFT approach.

⁵By checking the formulas of the cross sections for weak nuclear reactions with the deuteron calculated in Ref.[26] one can see that the value $\bar{\xi} = -2$ leads to the same result as well as the value $\bar{\xi} = 0$. Below we would call $\bar{\xi}^{PT}$ and $\bar{\xi}^{NPT}$ as a perturbative, close to $\bar{\xi} = 0$, and a non-perturbative, close to $\bar{\xi} = -2$, solution for the parameter $\bar{\xi}$, respectively.

⁶It is interesting too to compare the value of $S_{pp}(0)$ calculated in the NNJL model caused by the contribution of the nucleon axial-vector current with the old-fashioned result for $S_{pp}(0)$ obtained by Bahcall and Ulrich in 1988 [30]: $S_{pp}^{BU}(0) = 4.07 \times 10^{-25}$ MeV b.

⁷We would like to emphasize that the ratio $S_{pep}(0)/S_{pp}(0)$ does not depend on the parameter $\bar{\xi}$. Therefore, the result obtained in Ref.[26] for the ratio $S_{pep}(0)/S_{pp}(0)$ is valid for arbitrary contribution of the nucleon tensor current.

⁸As well as the result obtained by Kong and Ravndal [23] in the EFT approach.

of our results for solar neutrino fluxes should coincide with the theoretical accuracy of the SSM of the calculation of the solar neutrino fluxes. Henceforth, only in order to distinguish all theoretical quantities obtained by virtue of the reduction of the solar core temperature from those calculated in the SSM we suggest to use the label ASM that stands for the abbreviation for the Alternative Solar Model. The ASM leaves unchanged all results of the SSM save the value of the solar core temperature reduced in the ASM with respect to that recommended by the SSM [4].

3 Solar neutrino fluxes

As has been shown in Ref. [26] a non-trivial contribution of the nucleon tensor current [24] caused by the interaction

$$\delta\mathcal{L}_{\text{npD}}(x) = \sqrt{\frac{3}{8}} \frac{g_V}{2M_N} [\bar{p}^c(x)\sigma^{\mu\nu}n(x) - \bar{n}^c(x)\sigma^{\mu\nu}p(x)] D_{\mu\nu}^\dagger(x) + \text{h.c.} \quad (3.1)$$

changes⁹ the astrophysical factor $S_{\text{pp}}(0)$ for the solar proton burning $p + p \rightarrow D + e^+ + \nu_e$ as follows

$$S_{\text{pp}}^{\text{ASM}}(0) = (1 + \bar{\xi})^2 \times 4.08 \times 10^{-25} \text{ MeV b}, \quad (3.2)$$

where $\bar{\xi}$ is an arbitrary parameter (see Appendix of Ref. [26])¹⁰.

Below we develop a scenario of a dynamics of low-energy nuclear forces providing $(1 + \bar{\xi})^2 \geq 1$ and analyse a consistency of this dynamics with the helioseismological data [10], the experimental data on the solar neutrino fluxes [12–16] and the cross sections for the reactor anti-neutrino disintegration of the deuteron [32]

The changes ΔT_c of the solar core temperature T_c and $\Delta S_{\text{pp}}(0)$ of the astrophysical factor $S_{\text{pp}}(0)$ for the solar proton burning with respect to the values $T_c^{\text{SSM}} = 1.574 \times 10^7 \text{ K}$ [9] and $S_{\text{pp}}^{\text{SSM}}(0) = 4.00 \times 10^{-25} \text{ MeV b}$ recommended by the SSM are related by [9]

$$\frac{\Delta T_c}{T_c^{\text{SSM}}} = -0.15 \frac{\Delta S_{\text{pp}}(0)}{S_{\text{pp}}^{\text{SSM}}(0)}. \quad (3.3)$$

According to the helioseismological data [10] the maximum value of the astrophysical factor can be equal to $S_{\text{pp}}^{\text{max}}(0) = 1.18 \times S_{\text{pp}}^{\text{SSM}}(0) = 1.18 \times 4.00 \times 10^{-25} \text{ MeV b} = 4.72 \times 10^{-25} \text{ MeV b}$. From Eq.(3.3) it is seen that the maximum value of the astrophysical factor defines the minimum value of the solar core temperature. The helioseismological data give also the lower bound on the astrophysical factor [10]: $S_{\text{pp}}^{\text{min}}(0) = 0.94 \times S_{\text{pp}}^{\text{SSM}}(0) = 0.94 \times 4.00 \times 10^{-25} \text{ MeV b} = 3.76 \times 10^{-25} \text{ MeV b}$. The minimum value of the astrophysical factor corresponds to the maximal solar core temperature which is greater than that predicted by the SSM, $T_c^{\text{SSM}} = 1.574 \times 10^7 \text{ K}$. Since for the temperatures higher than

⁹Here $D_{\mu\nu}^\dagger(x) = \partial_\mu D_\nu^\dagger(x) - \partial_\nu D_\mu^\dagger(x)$ and $D_\nu^\dagger(x)$ is the operator of the interpolating field of the deuteron, $\bar{p}^c(x) = p^T(x)C$ and $\bar{n}^c(x) = n^T(x)C$ with C is a charge conjugation matrix and T is a transposition.

¹⁰The same factor $(1 + \bar{\xi})^2$ appears in the astrophysical factor for the pep-process $p + e^- + p \rightarrow D + \nu_e$ and, due to the isotopical invariance of low-energy nuclear forces, in the cross sections for neutrino and anti-neutrino disintegration of the deuteron $\nu_e + D \rightarrow e^- + p + p$, $\bar{\nu}_e + D \rightarrow e^+ + n + n$ and $\bar{\nu}_e(\nu_e) + D \rightarrow \bar{\nu}_e(\nu_e) + n + p$ [26].

T_c^{SSM} solar neutrino fluxes become increased with respect to those predicted by the SSM, so that we would consider only temperatures lower than T_c^{SSM} and astrophysical factors greater than the recommended one $S^{\text{SSM}} = 4.00 \times 10^{-25}$ MeV b.

In the NNJL model due to the isotopical invariance of low-energy nuclear forces the contribution of the nucleon tensor current enters into the cross sections for the neutrino and anti-neutrino disintegration of the deuteron with the same parameter $\bar{\xi}$ [26]. This means that the enhancement of the astrophysical factor $S_{\text{pp}}(0)$ in the NNJL model should be restricted not only by the helioseismological data but also the experimental data on the cross sections for the neutrino and anti-neutrino disintegration of the deuteron at low energies [26].

First, let us find out the maximal reduction of the solar neutrino fluxes caused by the cooling of the solar core up to the minimal temperature. By equating the astrophysical factor calculated in the NNJL model to the maximum value $S_{\text{pp}}^{\text{ASM}}(0) = S_{\text{pp}}^{\text{max}}(0) = 4.72 \times 10^{-25}$ MeV b allowed by the helioseismic data [10], we obtain the minimal value of the solar core temperature

$$T_c^{\text{ASM}} = 1.533 \times 10^7 \text{ K.} \quad (3.4)$$

Thus, due to the helioseismological constraint the minimal value of the solar core temperature cannot be less than the recommended one by about 2.7%¹¹.

In Tables 1 and 2 we adduce the experimental data on the solar neutrino fluxes and the theoretical values of the solar neutrino fluxes predicted by the SSM [3,11] and normalized to the recommended value of the astrophysical factor [4]. The theoretical accuracies of the solar neutrino fluxes predicted by the SSM (see Table 2) are equal to (+15.6%, -13.0%), (+6.2%, -4.7%) and (+19.4%, -13.6%) for the HOMESTAKE, GALLEX, SAGE and SUPERKAMIOKANDE experiments, respectively. This implies that the theoretical solar neutrino fluxes reduced by virtue of the reduction of the solar core temperature should be defined with the same accuracy. Thereby, the description of the experimental data within the ASM inheriting the accuracy of the SSM cannot be carried out with an accuracy better than (+15.6%, -13.0%), (+6.2%, -4.7%) and (+19.4%, -13.6%) for the data by HOMESTAKE, GALLEX, SAGE and SUPERKAMIOKANDE Collaborations, respectively.

By using a temperature dependence of the solar neutrino fluxes obtained by Bahcall and Ulmer [33]: $\Phi(\text{pp}) \propto T_c^{-1.1}$, $\Phi(\text{pep}) \propto T_c^{-2.4}$, $\Phi(^7\text{Be}) \propto T_c^{10}$, $\Phi(^8\text{B}) \propto T_c^{24}$, $\Phi(^{13}\text{N}) \propto T_c^{24.4}$ and $\Phi(^{15}\text{O}) \propto T_c^{27.1}$ we can calculate the solar neutrino fluxes for the reduced solar core temperature Eq.(3.4). The new values of the solar neutrino fluxes we have adduced in Table 3.

It is seen that the solar neutrino fluxes calculated for the solar core temperature $T_c^{\text{ASM}} = 1.533 \times 10^7$ K are still not enough decreased in order to satisfy the experimental data. The next step for the reduction of the solar neutrino fluxes taking place outside the solar core is in the attraction of neutrino-flavour oscillations [17–20]. We would follow a simplest scenario of vacuum two-flavour neutrino oscillations [17,18]. By virtue of the vacuum two-flavour neutrino oscillations $\nu_e \leftrightarrow \nu_\mu$ the theoretical solar neutrino fluxes

¹¹For the parameter $\bar{\xi}$ we get perturbative $\bar{\xi}^{\text{PT}} = 0.077$ and non-perturbative $\bar{\xi}^{\text{NPT}} = -2.077$ solutions. The results of the SSM can be restored at $\bar{\xi}^{\text{PT}} = 0$ and $\bar{\xi}^{\text{NPT}} = -2$, respectively.

should be multiplied by a factor [17–20]

$$P_{\nu_e \rightarrow \nu_e}(E_{\nu_e}) = 1 - \frac{1}{2} \sin^2 2\theta \left(1 - \cos \frac{\Delta m^2 L}{2E_{\nu_e}} \right), \quad (3.5)$$

where $\Delta m^2 = m_{\nu_\mu}^2 - m_{\nu_e}^2$, L is the distance of the neutrino's travel, E_{ν_e} is a neutrino energy and θ is a neutrino–flavour mixing angle [17]. After the averaging over energies and keeping L of order of the Sun–Earth distance the theoretical solar neutrino fluxes calculated for the reduced solar core temperature Eq.(3.4) become multiplied by a factor [18]

$$\overline{P_{\nu_e \rightarrow \nu_e}(E_{\nu_e})} = 1 - \frac{1}{2} \sin^2 2\theta. \quad (3.6)$$

The result of the integration over energies Eq.(3.6) can be valid only if the quantity $\Delta m^2 L / 2E_{\nu_e}$ obeys the constraint

$$\frac{\Delta m^2 L}{2E_{\nu_e}} \gg 1. \quad (3.7)$$

If we would like to get the factor Eq.(3.6) for all solar neutrino fluxes including the ^8B neutrinos, the upper bound on the neutrino energies should coincide with the upper bound on the ^8B neutrino energy spectrum equal to $E_{\nu_e} = 15 \text{ MeV}$ [1]. As the Sun–Earth distance L amounts to $L = 1.496 \times 10^{13} \text{ cm} = 7.581 \times 10^{23} \text{ MeV}^{-1}$, the inequality Eq.(3.7) gives the lower bound on Δm^2 :

$$\Delta m^2 \gg 4 \times 10^{-11} \text{ eV}^2. \quad (3.8)$$

Hence, in order to get a correct agreement with the experimental data on the solar neutrino fluxes measured by HOMESTAKE, GALLEX and SAGE we do not need to make a *fine tuning of the neutrino–flavour oscillation parameter* Δm^2 [34] and to have much more information about Δm^2 save that given by Eq.(3.8).

The value of the mixing angle $\sin^2 2\theta$ we can get fitting, for example, the meanvalue of the neutrino flux measured by HOMESTAKE Collaboration. This gives $\sin^2 2\theta = 0.838$. The solar neutrino fluxes reduced by virtue of vacuum two–flavour neutrino oscillations are adduced in Table 4. One can see that the theoretical solar neutrino fluxes fit well the experimental data by GALLEX and SAGE Collaborations.

The theoretical value of the solar neutrino flux $\Phi_{\text{SK}}^{\text{ASM}}(^8\text{B})$ fitting the experimental data of SUPERKAMIOKANDE Collaboration is related to the solar ^8B neutrino flux $\Phi^{\text{ASM}}(^8\text{B})$. This relation can be derived by following Bahcall *et al.* [35]. With an accuracy better than 2% the theoretical expression for the solar neutrino flux $\Phi_{\text{SK}}^{\text{ASM}}(^8\text{B})$ is given by

$$\Phi_{\text{SK}}^{\text{ASM}}(^8\text{B}) = \left(1 - \frac{4 \sin^2 2\theta \sin^2 \theta_{\text{W}}}{(1 + 2 \sin^2 \theta_{\text{W}})^2} \right) \Phi^{\text{ASM}}(^8\text{B}) = (1.73_{-0.24}^{+0.34}) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}, \quad (3.9)$$

where θ_{W} is the Weinberg's mixing angle of the Standard electroweak model equal to $\sin^2 \theta_{\text{W}} = 0.225$ [16]. The theoretical value $\Phi_{\text{SK}}^{\text{ASM}}(^8\text{B}) = (1.73_{-0.24}^{+0.34}) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$ is comparable with the experimental one $\Phi_{\text{SK}}^{\text{exp}}(^8\text{B}) = (2.40 \pm 0.09) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$ [16] with an accuracy about 30%. In order to get the theoretical solar neutrino fluxes fitting the experimental data with accuracies (+15.6%, –13.0%), (+6.2%, –4.7%) and (+19.4%, –13.6%) for

HOMESTAKE, GALLEX (SAGE) and SUPERKAMIOKANDE Collaborations, respectively, it is sufficient to diminish the value of the mixing angle up to $\sin^2 2\theta = 0.607$. This yields: $S_{\text{Cl}}^{\text{ASM}} = (3.07_{-0.40}^{+0.49})$ SNU, $S_{\text{Ga}}^{\text{ASM}} = (77.98_{-3.67}^{+4.83})$ SNU and $\Phi_{\text{SK}}^{\text{ASM}}(^8\text{B}) = (2.00_{-0.27}^{+0.39}) \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$. The errors of the theoretical values of the solar neutrino fluxes coincide with the theoretical errors of the SSM inherited by the ASM.

Since the theoretical solar neutrino fluxes fit reasonably well the experimental data, so that this evidences the solution of the SNP's in the form formulated by Bahcall [11].

We would like to emphasize that the agreement between the theoretical solar neutrino fluxes and the experimental ones has been reached without a fine tuning of the neutrino-flavour oscillation parameters Δm^2 and $\sin^2 2\theta$.

Now we have to analyse how the maximal enhancement of the astrophysical factor allowed by the helioseismological data and providing the solution of the SNP's affects the theoretical values of the cross sections for the reactor anti-neutrino disintegration of the deuteron [26,32]. On this way one can find that the enhancement of the astrophysical factor up to the value $S_{\text{pp}}^{\text{ASM}}(0) = 4.72 \times 10^{-25} \text{ MeV b}$ leads to the theoretical values of the cross sections for the reactions $\bar{\nu}_e + \text{D} \rightarrow \text{e}^+ + \text{n} + \text{n}$ and $\bar{\nu}_e + \text{D} \rightarrow \bar{\nu}_e + \text{n} + \text{p}$ fitting the meanvalues of the experimentally measured cross sections with an accuracy 1.75σ and 1.55σ , respectively [26,32]. Such an agreement being reasonable in principle can be improved by diminishing the value of the astrophysical factor $S_{\text{pp}}^{\text{ASM}}(0) = 4.72 \times 10^{-25} \text{ MeV b}$.

One of the ways for the optimization of the enhancement of the astrophysical factor can be, for example, the following. Let the theoretical solar neutrino fluxes be functions of two variables $(\sin^2 2\theta, \lambda)$, where λ is related to the theoretical solar neutrino flux $\Phi_{\text{SK}}^{\text{ASM}}(^8\text{B}) = \lambda \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$ fitting the experimental data by SUPERKAMIOKANDE Collaboration. From Eq.(3.9) the theoretical solar ^8B neutrino flux $\Phi^{\text{ASM}}(^8\text{B})$ can be expressed in terms of $\Phi_{\text{SK}}^{\text{ASM}}(^8\text{B}) = \lambda \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$ and $\sin^2 2\theta$. At $\sin^2 \theta_{\text{W}} = 0.225$ we obtain

$$\Phi^{\text{ASM}}(^8\text{B}) = \lambda \times 10^6 (1 - 0.428 \sin^2 2\theta)^{-1} \text{ cm}^{-2}\text{s}^{-1}. \quad (3.10)$$

The ratio of the solar core temperatures $T_c^{\text{ASM}}/T_c^{\text{SSM}}$ is then defined by

$$\frac{T_c^{\text{ASM}}}{T_c^{\text{SSM}}} = 0.934 \lambda^{1/24} (1 - 0.428 \sin^2 2\theta)^{-1/24}. \quad (3.11)$$

The solar neutrino fluxes as functions of $\sin^2 2\theta$ and λ measured in $10^{10} \text{ cm}^{-2}\text{s}^{-1}$ read [11]:

$$\begin{aligned} \Phi^{\text{ASM}}(\text{pp}) &= 6.403 \lambda^{-1.1/24} (1 - 0.428 \sin^2 2\theta)^{1.1/24}, \\ \Phi^{\text{ASM}}(\text{pep}) &= 1.638 \times 10^{-2} \lambda^{-1/10} (1 - 0.428 \sin^2 2\theta)^{1/10}, \\ \Phi^{\text{ASM}}(^7\text{Be}) &= 2.425 \times 10^{-1} \lambda^{10/24} (1 - 0.428 \sin^2 2\theta)^{-10/24}, \\ \Phi^{\text{ASM}}(^8\text{B}) &= 1.000 \times 10^{-4} \lambda (1 - 0.428 \sin^2 2\theta)^{-1}, \\ \Phi^{\text{ASM}}(^{13}\text{N}) &= 1.144 \times 10^{-2} \lambda^{24.4/24} (1 - 0.428 \sin^2 2\theta)^{-24.4/24}, \\ \Phi^{\text{ASM}}(^{15}\text{O}) &= 0.836 \times 10^{-2} \lambda^{27.1/24} (1 - 0.428 \sin^2 2\theta)^{-27.1/24}. \end{aligned} \quad (3.12)$$

The theoretical expressions for the solar neutrino fluxes measured by HOMESTAKE, GALLEX and SAGE experiments are given by [11]

$$S_{\text{Cl}}^{\text{ASM}} = (1 - 0.5 \sin^2 2\theta)$$

$$\begin{aligned}
& \times [0.236 \lambda^{-1/10} (1 - 0.428 \sin^2 2\theta)^{1/10} + 0.582 \lambda^{10/24} (1 - 0.428 \sin^2 2\theta)^{-10/24} \\
& + 0.019 \lambda^{24.4/24} (1 - 0.428 \sin^2 2\theta)^{-24.4/24} + 0.063 \lambda^{27.1/24} (1 - 0.428 \sin^2 2\theta)^{-27.1/24} \\
& + 1.146 \lambda (1 - 0.428 \sin^2 2\theta)^{-1}], \\
S_{\text{Ga}}^{\text{ASM}} &= (1 - 0.5 \sin^2 2\theta) \\
& \times [75.043 \lambda^{-1.1/24} (1 - 0.428 \sin^2 2\theta)^{1.1/24} + 3.300 \lambda^{-1/10} (1 - 0.428 \sin^2 2\theta)^{1/10} \\
& + 17.380 \lambda^{10/24} (1 - 0.428 \sin^2 2\theta)^{-10/24} + 0.700 \lambda^{24.4/24} (1 - 0.428 \sin^2 2\theta)^{-24.4/24} \\
& + 0.943 \lambda^{27.1/24} (1 - 0.428 \sin^2 2\theta)^{-27.1/24} + 2.408 \lambda (1 - 0.428 \sin^2 2\theta)^{-1}], \quad (3.13)
\end{aligned}$$

where the factor $(1 - 0.5 \sin^2 2\theta)$ takes into account the contribution of vacuum two-flavour neutrino oscillations.

For the fit of experimental data on the solar neutrino fluxes by the theoretical expressions Eq.(3.13) we can feel ourselves to be constrained only by the requirement of the description of the experimental data by GALLEX and SAGE Collaborations with an accuracy not worse than (+6.2%, -4.7%) inherited from the SSM, as the solar neutrino fluxes for HOMESTAKE and SUPERKAMIOKANDE Collaborations are determined in the SSM with a much worse accuracy.

The theoretical solar neutrino fluxes fitting the experimental data by GALLEX and SAGE Collaborations with an accuracy (+6.2%, -4.7%) can be obtained at $\lambda = 2.12$ and $\sin^2 2\theta = 0.780$: $\Phi_{\text{SK}}^{\text{ASM}}(^8\text{B}) = (2.12_{-0.29}^{+0.41}) \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$, $S_{\text{Cl}}^{\text{ASM}} = (3.11_{-0.40}^{+0.49}) \text{ SNU}$ and $S_{\text{Ga}}^{\text{ASM}} = (70.56_{-3.32}^{+4.38}) \text{ SNU}$. The errors are the theoretical uncertainties of the calculation of the solar neutrino fluxes in the SSM [11]. The solar neutrino fluxes given by Eqs.(3.12) and (3.13) and calculated at $\lambda = 2.12$ and $\sin^2 2\theta = 0.780$ are adduced in Table 5.

If there would be allowed to fit the experimental data by GALLEX and SAGE Collaborations with an accuracy worse than (+6.2%, -4.7%), the region of variables $(\sin^2 2\theta, \lambda)$ would become much broader. For example, at the maximum mixing angle $\sin^2 2\theta = 1$ and $\lambda = 2.12$ we obtain: $\Phi_{\text{SK}}^{\text{ASM}}(^8\text{B}) = (2.12_{-0.29}^{+0.41}) \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$, $S_{\text{Cl}}^{\text{ASM}} = (2.90_{-0.38}^{+0.45}) \text{ SNU}$ and $S_{\text{Ga}}^{\text{ASM}} = (59.64_{-2.80}^{+3.70}) \text{ SNU}$. Our prediction for the low-energy solar neutrino flux agrees with the experimental data by GALLEX and SAGE Collaborations with an accuracy about 20% (see Table 1) and fits well the experimental data by GNO Collaboration. The experimental value of the high-energy solar neutrino flux measured by HOMESTAKE Collaboration is fitted with an accuracy about 12%.

If there would be set $\sin^2 2\theta = 1$ and $\lambda = 2.40$ that corresponds to the meanvalue of the experimental flux measured by SUPERKAMIOKANDE Collaboration, there would have been obtained the following theoretical predictions: $\Phi_{\text{SK}}^{\text{ASM}}(^8\text{B}) = (2.40_{-0.33}^{+0.47}) \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$, $S_{\text{Cl}}^{\text{ASM}} = (3.24_{-0.42}^{+0.51}) \text{ SNU}$ and $S_{\text{Ga}}^{\text{ASM}} = (61.30_{-2.90}^{+3.80}) \text{ SNU}$ fitting the experimental data on the high-energy (HOMESTAKE) and low-energy (GALLEX and SAGE) solar neutrino fluxes with an accuracy about 25%. In turn, at $\sin^2 2\theta = 0.780$ and $\lambda = 2.40$ one obtains: $\Phi_{\text{SK}}^{\text{ASM}}(^8\text{B}) = (2.40_{-0.33}^{+0.47}) \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$, $S_{\text{Cl}}^{\text{ASM}} = (3.46_{-0.45}^{+0.54}) \text{ SNU}$ and $S_{\text{Ga}}^{\text{ASM}} = (72.33_{-3.04}^{+4.49}) \text{ SNU}$.

This testifies a consistency of the experimental solar neutrino fluxes with theoretical fluxes given by Eqs.(3.12) and (3.13) calculated in the SSM with a reduced solar core temperature and supplemented by the scenario of vacuum two-flavour solar neutrino oscillations.

The reduced solar core temperature T_c^{ASM} , the astrophysical factor for the solar proton burning $S_{\text{pp}}^{\text{ASM}}(0)$ and the enhancement factor $S_{\text{pp}}^{\text{ASM}}(0)/S_{\text{pp}}^{\text{SSM}}(0)$ calculated at $\lambda = 2.12$

and $\sin^2 2\theta = 0.780$ are equal to

$$\begin{aligned} T_c^{\text{ASM}} &= (1.549_{-0.016}^{+0.005}) \times 10^7 \text{ K}, \\ S_{\text{pp}}^{\text{ASM}}(0) &= (4.42_{-0.08}^{+0.30}) \times 10^{-25} \text{ MeV b}, \\ \frac{S_{\text{pp}}^{\text{ASM}}(0)}{S_{\text{pp}}^{\text{SSM}}(0)} &= 1.11_{-0.02}^{+0.07}, \end{aligned} \quad (3.14)$$

where the errors are defined by the theoretical uncertainties of the solar ^8B neutrino flux calculated in the SSM ¹².

The theoretical values of the cross sections for the anti-neutrino disintegration of the deuteron related to the astrophysical factors corresponding to the theoretical solar neutrino fluxes calculated at $(\sin^2 2\theta = 0.780, \lambda = 2.12)$, $(\sin^2 2\theta = 1, \lambda = 2.12)$, $(\sin^2 2\theta = 1, \lambda = 2.40)$ and $(\sin^2 2\theta = 0.780, \lambda = 2.40)$ fit the meanvalues of the experimentally measured cross sections for the reaction $\bar{\nu}_e + \text{D} \rightarrow \text{e}^+ + \text{n} + \text{n}$ and $\bar{\nu}_e + \text{D} \rightarrow \bar{\nu}_e + \text{n} + \text{p}$ with an accuracy better than 1.32σ and 1σ , respectively [26,32].

4 Conclusion

We have shown that the scenario of a dynamics of low-energy nuclear forces leading to the reduction of the solar core temperature provides a reasonable theoretical foundation for the solution of the SNP's in the form formulated by Bahcall [11]. Really, the SSM with the reduced solar core temperature and supplemented by the scenario of vacuum two-flavour neutrino oscillations $\nu_e \leftrightarrow \nu_\mu$ during the travel of solar neutrinos to the Earth proposed by Gribov and Pontecorvo [17] admits a possibility to calculate the theoretical solar neutrino fluxes fitting experimental data without a fine tuning of neutrino-flavour oscillation parameters Δm^2 and the mixing angle $\sin^2 2\theta$. As has been shown due to the reduction of the solar neutrino fluxes in the solar core for the simultaneous description of the experimental data on the solar neutrino fluxes with an accuracy not worse than the theoretical accuracy of the SSM it is sufficient to know only that $\Delta m^2 \gg 4 \times 10^{-11} \text{ eV}^2$ and $\sin^2 2\theta \geq 0.780$. The former makes the application of neutrino-flavour oscillations to be much more flexible with respect to different experimental constraints on the parameters of the neutrino-flavour oscillations [36].

The theoretical solar neutrino fluxes given by Eqs.(3.12) and (3.13) and calculated at $\Delta m^2 \gg 4 \times 10^{-11} \text{ eV}^2$ and $\sin^2 2\theta = 0.780$ read: $S_{\text{Cl}}^{\text{ASM}} = (3.11_{-0.40}^{+0.49}) \text{ SNU}$, $S_{\text{Ga}}^{\text{ASM}} = (70.56_{-3.32}^{+4.38}) \text{ SNU}$ and $\Phi_{\text{SK}}^{\text{ASM}}(^8\text{B}) = (2.12_{-0.29}^{+0.41}) \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$. For the ratios of Experiment : Theory we obtain the numbers

$$\begin{aligned} \frac{S_{\text{Cl}}^{\text{exp}}}{S_{\text{Cl}}^{\text{ASM}}} &= 0.82_{-0.13}^{+0.15}, \\ \frac{S_{\text{Ga}}^{\text{exp}}}{S_{\text{Ga}}^{\text{ASM}}} &= 1.05_{-0.11}^{+0.12}, \\ \frac{\Phi_{\text{SK}}^{\text{exp}}(^8\text{B})}{\Phi_{\text{SK}}^{\text{ASM}}(^8\text{B})} &= 1.13_{-0.14}^{+0.22} \end{aligned} \quad (4.1)$$

¹²For the parameter $\bar{\xi}$ we obtain $\bar{\xi}^{\text{PT}} = 0.041_{-0.009}^{+0.031}$ and $\bar{\xi}^{\text{NPT}} = -2.041_{-0.009}^{+0.031}$.

that are comparable with unity.

This reconciles the experimental data of the solar neutrino fluxes by HOMESTAKE, GALLEX, SAGE and SUPERKAMIOKANDE Collaborations with the theoretical predictions and solves the SNP's in the form formulated by Bahcall [11].

The agreement with the experimental data by KAMIOKANDE Collaboration $\Phi_{\text{exp}}^{\text{K}} = (2.80 \pm 0.36) \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$ can be obtained within an accuracy about 15%, where the error is defined by $\sigma = \sqrt{(\text{stat.})^2 + (\text{syst.})^2}$.

The theoretical solar ^8B and ^7Be neutrino fluxes defined by Eq.(3.12) and calculated at $\sin^2 2\theta = 0.780$ for the HOMESTAKE experiments are equal to $\Phi_{\text{Cl}}^{\text{ASM}}(^8\text{B}) = (2.22_{-0.30}^{+0.43}) \text{ SNU}$ and $\Phi_{\text{Cl}}^{\text{ASM}}(^7\text{Be}) = (0.57_{-0.08}^{+0.11}) \text{ SNU}$ and agree reasonably well with recent data by HOMESTAKE [14] (Lande, Neutrino 2000): $\Phi_{\text{Cl}}^{\text{exp}}(^8\text{B}) = 2.16 \text{ SNU}$ and $\Phi_{\text{Cl}}^{\text{exp}}(^7\text{Be}) = 0.4 \text{ SNU}$.

Thus, we have shown that the experimental data on the solar neutrino fluxes measured by HOMESTAKE, GALLEX, SAGE, KAMIOKANDE and SUPERKAMIOKANDE Collaborations are consistent with both each other and the theoretical solar neutrino fluxes calculated in the SSM for the reduced solar core temperature and supplemented by a scenario of vacuum two-flavour neutrino oscillations.

The enhancement of the astrophysical factor being necessary for the reduction of the solar neutrino fluxes in the solar core is caused by the contribution of the nucleon tensor current coupled to the deuteron and depends on the parameter $\bar{\xi}$ [26]. As has been shown in Ref.[26] this parameter enters to the cross sections for the neutrino and anti-neutrino disintegration of the deuteron at low energies [26]. Thus, the NNJL model provides a theoretical foundation for a new check of a value of the astrophysical factor for the solar proton burning in terrestrial laboratories. At present the enhancement of the astrophysical factor given by Eq.(3.14) does not contradict to the available experimental data on the cross sections for the disintegration of the deuteron by reactor anti-neutrinos [32]. The theoretical cross sections fit the meanvalues of the experimentally measured cross sections for the reactions $\bar{\nu}_e + \text{D} \rightarrow \bar{\nu}_e + \text{n} + \text{p}$ and $\bar{\nu}_e + \text{D} \rightarrow \text{e}^+ + \text{n} + \text{n}$ [32] with an accuracy 1σ and 1.32σ , respectively [26].

The value of the astrophysical factor $S_{\text{pp}}^{\text{KR}}(0) = (4.31 \pm 0.35) \times 10^{-25} \text{ MeV b}$ having been recently calculated by Kong and Ravndal [23] in the EFT agrees with the value $S_{\text{pp}}^{\text{ASM}}(0) = (4.42_{-0.08}^{+0.30}) \times 10^{-25} \text{ MeV b}$ given by Eq.(3.14). The astrophysical factor in the Kong–Ravndal approach depends on the unknown counter-term that seems to be very similar to our parameter $\bar{\xi}$. *Assuming the counter-term to have a natural magnitude* [23], Kong and Ravndal have obtained the value $S_{\text{pp}}^{\text{KR}}(0) = (4.31 \pm 0.35) \times 10^{-25} \text{ MeV b}$. As has been stated by Kong and Ravndal [23] the true value of the counter-term can be determined from precise measurements of the cross sections for neutrino disintegration of the deuteron at low energies. This is just that we have pointed out above and earlier in Ref.[26] concerning the parameter $\bar{\xi}$.

For the discussion of the solar *hep* neutrino problem [11,37] appeared due to recent experiments by SUPERKAMIOKANDE Collaboration [16] we need to calculate in the NNJL model the astrophysical factor for the *hep* reaction $\text{p} + ^3\text{He} \rightarrow ^4\text{He} + \text{e}^+ + \nu_e$. The former demands, in turn, the extension of the NNJL model by the inclusion of the light nuclei ^3He , ^3H and ^4He . This work is in progress [24].

Acknowledgement

One of the authors (A.N. Ivanov) thanks Acad. G.T.Zatsepin and the participants of the seminar of the Lebedev Physical Institute of Academy of Sciences, where this paper has been reported, for fruitful discussions.

Table 1. Solar neutrino data, $1 \text{ SNU} = 10^{-36} \text{ events}/(\text{atoms} \cdot \text{s})$. The error is defined as $\sigma = \sqrt{(\text{stat.})^2 + (\text{syst.})^2}$

Experiment	Data $\pm \sigma$	Units
HOMESTAKE $\nu_e + {}^{37}\text{Cl} \rightarrow e^- + {}^{37}\text{Ar}$ $E_{\text{th}} = 0.81 \text{ MeV}$	2.56 ± 0.23	SNU
SAGE $\nu_e + {}^{71}\text{Ga} \rightarrow e^- + {}^{71}\text{Ge}$ $E_{\text{th}} = 0.23 \text{ MeV}$	75.4 ± 7.6	SNU
GALLEX $\nu_e + {}^{71}\text{Ga} \rightarrow e^- + {}^{71}\text{Ge}$ $E_{\text{th}} = 0.23 \text{ MeV}$	77.5 ± 7.7	SNU
GNO $\nu_e + {}^{71}\text{Ga} \rightarrow e^- + {}^{71}\text{Ge}$ $E_{\text{th}} = 0.23 \text{ MeV}$	65.8 ± 10.5	SNU
GALLEX + GNO $\nu_e + {}^{71}\text{Ga} \rightarrow e^- + {}^{71}\text{Ge}$ $E_{\text{th}} = 0.23 \text{ MeV}$	74.1 ± 6.8	SNU
KAMIOKANDE $\nu + e^- \rightarrow \nu + e^-$ $E_{\text{th}} = 7.0 \text{ MeV}$	2.80 ± 0.38	$10^6 \text{ cm}^{-2} \text{ s}^{-1}$
SUPERKAMIOKANDE $\nu + e^- \rightarrow \nu + e^-$ $E_{\text{th}} = 5.5 \text{ MeV}$	2.40 ± 0.09	$10^6 \text{ cm}^{-2} \text{ s}^{-1}$

Table 2. Standard Solar Model predictions for the solar neutrino fluxes normalized to the recommended value of the astrophysical factor $S_{\text{pp}}(0) = 4.00 \times 10^{-25} \text{ MeV b}$ [11].

Source	Flux ($10^{10} \text{ cm}^{-2} \text{ s}^{-1}$)	Cl (SNU)	Ga (SNU)	SK ($10^6 \text{ cm}^{-2} \text{ s}^{-1}$)
pp	$5.94(1.00^{+0.01}_{-0.01})$	0.0	69.6	$5.15^{+1.0}_{-0.7}$
pep	$1.39 \times 10^{-2}(1.00^{+0.01}_{-0.01})$	0.2	2.8	
${}^7\text{Be}$	$4.80 \times 10^{-1}(1.00^{+0.09}_{-0.09})$	1.15	34.4	
${}^8\text{B}$	$5.15 \times 10^{-4}(1.00^{+0.19}_{-0.14})$	5.9	12.4	
${}^{13}\text{N}$	$6.05 \times 10^{-2}(1.00^{+0.19}_{-0.13})$	0.1	3.7	
${}^{15}\text{O}$	$5.32 \times 10^{-2}(1.00^{+0.22}_{-0.15})$	0.4	6.0	
		$7.7^{+1.2}_{-1.0}$	129^{+8}_{-6}	$5.15^{+1.0}_{-0.7}$

Table 3. The NNJL model predictions for the solar neutrino fluxes normalized to astrophysical factor $S_{pp}(0) = 4.72 \times 10^{-25}$ MeV b caused by the non-trivial contribution of the nucleon tensor current.

Source	Flux ($10^{10} \text{ cm}^{-2}\text{s}^{-1}$)	Cl (SNU)	Ga (SNU)	SK ($10^6 \text{ cm}^{-2}\text{s}^{-1}$)
pp	6.10	0.0	71.49	2.69
pep	1.48×10^{-2}	0.21	2.98	
^7Be	3.66×10^{-1}	0.88	26.23	
^8B	2.69×10^{-4}	3.08	6.46	
^{13}N	3.13×10^{-2}	0.05	1.91	
^{15}O	2.56×10^{-2}	0.19	2.89	
		4.41	111.96	

Table 4. The solar neutrino fluxes normalized to $S_{pp}^{\text{ASM}}(0) = 1.18 S_{pp}(0) = 4.72 \times 10^{-25}$ MeV b. The theoretical values of experimentally measured neutrino fluxes are calculated within a scenario of vacuum two-flavour neutrino oscillations at $\sin^2 2\theta = 0.838$. The error is defined as $\sqrt{(\text{stat.})^2 + (\text{syst.})^2}$

Source	Flux ($10^{10} \text{ cm}^{-2}\text{s}^{-1}$)	Cl (SNU)	Ga (SNU)	SK ($10^6 \text{ cm}^{-2}\text{s}^{-1}$)
pp	6.10	0.0	41.54	1.73
pep	1.48×10^{-2}	0.13	1.73	
^7Be	3.66×10^{-1}	0.50	15.25	
^8B	2.69×10^{-4}	1.79	3.74	
^{13}N	3.13×10^{-2}	0.03	1.11	
^{15}O	2.56×10^{-2}	0.11	1.68	
		$2.56^{+0.40}_{-0.33}$	$65.05^{+4.03}_{-3.06}$	$1.73^{+0.34}_{-0.24}$
		2.56 ± 0.23	74.1 ± 6.8	2.40 ± 0.09

Table 5. The solar neutrino fluxes normalized to $S_{pp}^{\text{ASM}}(0) = 4.42 \times 10^{-25}$ MeV b. The theoretical values of experimentally measured neutrino fluxes are calculated within a scenario of vacuum two-flavour neutrino oscillations at $\sin^2 2\theta = 0.780$. The error is defined as $\sqrt{(\text{stat.})^2 + (\text{syst.})^2}$

Source	Flux ($10^{10} \text{ cm}^{-2}\text{s}^{-1}$)	Cl (SNU)	Ga (SNU)	SK ($10^6 \text{ cm}^{-2}\text{s}^{-1}$)
pp	6.07	0.0	43.40	2.12
pep	1.46×10^{-2}	0.13	1.79	
^7Be	3.93×10^{-1}	0.57	17.18	
^8B	3.18×10^{-4}	2.22	4.67	
^{13}N	3.71×10^{-2}	0.05	1.39	
^{15}O	3.09×10^{-2}	0.14	2.13	
		$3.11^{+0.49}_{-0.40}$	$70.56^{+4.38}_{-3.32}$	$2.12^{+0.41}_{-0.29}$
		2.56 ± 0.23	74.1 ± 6.8	2.40 ± 0.09

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