Reconstructing the microwave sky using a combined maximum-entropy and mexican hat wavelet analysis

R. Belén Barreiro[1], Patricio Vielva[2,3], Michael P. Hobson[1], Enrique Martínez-González[2], Anthony N. Lasenby[1], José L. Sanz[2], and Luigi Toffolatti[4]

[1] Astrophysics Group, Cavendish Laboratory, Madingley Road,
Cambridge CB3 0HE, UK
[2] Instituto de Física de Cantabria (CSIC – UC), Fac. Ciencias,
Avda. de los Castros s/n, 39005 Santander, Spain
[3] Departamento de Física Moderna, Universidad de Cantabria,
Avda. de los Castros s/n, 39005 Santander, Spain
[4] Departamento de Física, Universidad de Oviedo, c/ Calvo Sotelo s/n,
33007 Oviedo, Spain

Abstract. We present a combined maximum-entropy method (MEM) and Mexican Hat wavelet (MHW) analysis in order to recover the different components of the microwave sky. We apply this technique to simulated observations by the ESA Planck satellite in small patches of the sky. In particular, the introduction of the MHW allows one to detect and subtract the brightest point sources present in the input data and therefore to improve the reconstructions of the CMB and foreground components achieved by MEM on its own. In addition, a point source catalogue at each Planck frequency is produced, which is more complete and accurate than those obtained by each technique independently.

1 Introduction

Cosmic Microwave Background (CMB) observations carry a wealth of information about the Universe. Indeed, an accurate knowledge of the CMB anisotropies can place tight constraints on fundamental parameters as well as to discriminate between competing theories of structure formation. Future CMB experiments such as the NASA MAP satellite and the Planck mission from ESA, will provide with multifrequency data at high resolution and sensitivity. However, these data contain not only the cosmological signal but also Galactic foregrounds, extragalactic point sources, thermal and kinetic Sunyaev-Zelodvich (SZ) emission from cluster of galaxies and instrumental noise. Therefore our capacity to recover all the valuable information encoded in the CMB will critically depend on our ability to denoise and separate the cosmological signal from the rest of components of the microwave sky.

To perform such a separation, [1] has developed a Fourier MEM algorithm. This technique is particularly successful at using multifrequency data to identify foreground emission from physical components whose spectral signatures are (reasonably) well-known. Therefore, the most problematic foreground to remove is that due to extragalactic point sources, since each source has a unique frequency spectrum and, moreover, is notoriously difficult to predict. To adress this problem, [2] extended the MEM approach to deal with point sources as an extra 'noise' contribution. A different approach was developed by [3] who showed that the Mexican Hat wavelet (MHW) is in fact the optimal pseudo-filter for detecting point sources under reasonable conditions. The application of this wavelet to realistic simulations was presented in [4] and extended in [5].

The aim of this work is to show that the MEM and MHW techniques are actually complementary and can be combined to improve the accuracy of the separation of diffuse foregrounds from the CMB and increase the number of point sources that are identified and successfully subtracted. The joint analysis has been performed on simulated observations by the Planck satellite but it could be straightforwardly applied to other multifrequency CMB experiments such as the forthcoming NASA MAP satellite or the recently performed Boomerang and MAXIMA experiments.

2 The MEM and MHW joint analysis

A detailed description of the MEM algorithm is given in [1] and [2] whereas the MHW method is explained in [4] and [5]. Therefore, we will focus on how the two approaches can be successfully combined to produce a more powerful joint analysis scheme (see also [6]).

The MEM technique presented in [2] includes point sources as part of a generalised noise vector. This has proved to be very successful at performing a full component separation with the contamination due to point sources greatly reduced in the reconstructions. Moreover, by comparing the input data maps with 'mock' data obtained from the separated components it is possible to obtain point source catalogues at each observing frequency. Since point sources are modelled as an additional noise, MEM performs well in identifying and removing a large number of point sources with low to intermediate fluxes. However, it is rather poorer at removing the contributions from the brightest point sources. These tend to remain in the reconstructed maps, although with significantly reduced amplitudes.

The MHW technique is based in the fact that the point sources are very much amplified with respect to the background in the wavelet coefficients map. Therefore, the detection of point sources is performed in wavelet space at a certain optimal scale instead of in real space. This method out-performs, in general, other tecnhiques such as SExtractor ([7]) and standard harmonic filtering ([8]). In addition, this method does not require any assumptions to be made regarding the statistical properties of the point source population or the underlying emission from the CMB (or other foreground components). The MHW is particularly efficient in detecting the brightest point sources.

Moreover, their amplitude is also accurately estimated. For weaker sources, however, the MHW performs more poorly by either inaccurately estimating the flux or failing to detect the source altogether.

The strenght and weakness of the MEM and MHW approaches clearly indicate that they are complementary and that a combined analysis might lead to improve results as compared to using each method on its own. Therefore we propose the following technique for analysing multifrequency observations of the CMB that contain point source contamination. First, the MHW is applied at each observing frequency map and the brightest point sources detected and subtracted. The processed data maps are then used as input for the MEM algorithm in order to perform a separation of the physical components as explained in [2]. This leads to more accurate reconstructed maps, mostly free from point source contamination. These reconstructions are then used to generate 'mock' data, which are subtracted from the input data to generate data residuals maps at each observing frequency. Since the diffuse components are reasonably well recovered, these residuals maps will mostly contain point sources and instrumental noise. Finally the MHW is applied on each of these maps in order to recover a more complete and accurate catalogue than those obtained by each technique independently.

3 Foreground separation

We have applied the MEM and MHW joint technique to simulated observations of the Planck satellite in small patches of the sky $(12.8^\circ \times 12.8^\circ)$. Our simulated data contain a Gaussian CDM model for the CMB with $\Omega = 0.3$ and $\Omega_\Lambda = 0.7$ for which the power spectrum was generated using CMBFAST ([9]). They also include thermal and kinetic SZ effects (following the model of [10]), Galactic foregrounds (synchrotron, dust and free-free), extragalactic point sources (simulated according to [11]) and instrumental noise at the level expected in the Planck data. A description of these simulations as well as the observational parameters used for the Planck satellite are given in [6]. Fig. 1 shows the input maps for the CMB, kinetic and thermal SZ effects and Galactic foregrounds. In order to perform the separation and reconstruction

Fig. 1. The 12.8×12.8 deg^2 realisations of the six input components used to produce the simulated Planck data. The different panels correspond to (from left to right and from top to bottom) CMB, kinetic SZ effect, thermal SZ effect, Galactic dust, Galactic free-free and Galactic synchrotron emission. Each component is plotted at 300 GHz and has been convolved with a Gaussian beam of FWHM 5 arcmin (the highest resolution expected for the Planck satellite). The map units are equivalent thermodynamic temperature in μK

of the different components we have assume knowledge of the azimuthally averaged power spectrum of these six input components (see [1] for more details). Using the model of [11] we have also introduced the power spectrum of the point sources at each frequency channel, including cross power spectra between channels. However, the recovery of the main components and point sources do not depend critically on this assumption (see [6])

The resulting reconstructions of the physical components at a reference frequency of 300 GHz are shown in Fig. 2. We see that the main input components have been faithfully recovered and no obvious visible contamination of point sources remain in the reconstructions. We give the rms reconstruction errors for each component in Table 1. For comparison, the rms error of the reconstructed maps without a previous subtraction of point sources using the MHW is also given. In particular, the reconstruction of the CMB map is very good, with a rms reconstruction error of 7.7μK which corresponds to an accuracy of ∼6.8 per cent level as compared to the rms of the input map. Even more impressive is the reconstruction of the dust map. Although the high frequency channels, where the dust is the main component, are highly contaminated by infrared sources, none of them are visible in the dust reconstructed map. The main features of the free-free emission are also recovered mostly due to its high correlation with the dust. The reconstructed synchrotron map is basicaly a lower resolution image of the input. This is expectable since the only channels that provide useful information about this component are the lowest frequency ones which also have the lowest angular resolutions. Regarding the reconstruction of the thermal SZ, most of the bright clusters have been reproduced and only a few point sources have been misidentified as clusters. At the reference frequency of reconstruction these point sources appear mostly as negative features. Finally, as expected, the

Fig. 2. As in Fig. 1 but for the reconstructed maps

Table 1. The rms in μK of the reconstruction residuals smoothed with a 5 arcmin FWHM Gaussian beam with and without the initial subtraction of bright point sources using the MHW. For comparison the rms of the input maps are also given

Component	input rms	error (with MHW)	error (without MHW)
CMB	112.3	7.68	8.62
Kinetic SZ	0.69	0.70	0.70
Thermal SZ	5.37	4.64	4.66
Dust	55.8	2.68	3.39
Free-Free	0.66	0.22	0.24
Synchrotron	0.32	0.11	0.12

reconstruction of the kinetic SZ is quite poor and only a few clusters whose corresponding thermal SZ is large have been detected.

Table 2. The point source catalogues obtained using the MHW alone (MHWc), MEM alone (MEMc) and the joint analysis method (M&Mc). For each Planck observing frequency, we list the number of detected sources, the flux limit of the catalogue and the mean percentage error for the amplitude estimation

	MHWc			MEMc			M&Mc		
Freq. (GHz)	No. detect.	Min Flux (Jy)	E_{abs} (%)	No. detect.	Min Flux (Jy)	E_{abs} (%)	No. detect.	Min Flux (Jy)	E_{abs} (%)
30	4	0.46	12.1	21	0.10	12.1	19	0.10	12.3
44	3	0.58	6.7	11	0.24	8.6	11	0.24	8.4
70	5	0.28	21.0	19	0.12	10.5	18	0.15	8.1
100 (L)	3	0.59	6.8	16	0.13	15.6	14	0.13	10.0
100 (H)	7	0.27	7.7	33	0.08	13.4	33	0.07	12.9
143	4	0.40	13.6	1	0.10	14.2	8	0.06	24.2
217	5	0.25	9.9	1	0.10	23.4	8	0.06	19.0
353	10	0.07	34.6	6	0.24	41.4	9	0.24	8.2
545	29	0.26	20.4	13	0.23	39.4	41	0.24	14.4
857	86	0.58	10.4	107	0.41	17.6	150	0.31	13.8

4 Point source catalogues

A main aim of the Planck mission is also to produce accurate point source catalogues at each of the observing frequencies. In this section we will focus on how the combination of the MEM and MHW techniques can improve the catalogues obtained by each of them independently.

The MHW catalogue (MHWc) is produced in the way explained in [5]. The MEM and joint analysis catalogues (MEMc and M&Mc) are constructed applying the MHW to the data residuals maps obtained as explained in §2. Table 2 gives the number of point sources, minimum flux reached and average error in the estimation of the amplitude for the three catalogues.

In the low frequency channels, the MEM and M&M catalogues contain a similar number of point sources, being much more complete than the MHWc. In this case, the contribution of the MHW is only to improve the amplitude estimation of a few bright point sources. The MHW can only detect a few point sources in these channels due to the large beam size, what means that CMB and point sources have a similar characteristic scale.

The improvement is far more noticeable in the intermediate and high frequency channels. On the one hand, a larger number of point sources are detected with the combined technique than with each of the methods independently. This is due to the complementary nature of the two approaches, so that bright sources are detected by the MHW whereas fainter sources are identified by MEM. On the other hand, the amplitude of the point sources is more accurately estimated in the M&Mc. When MEM is used without previously applied the MHW, the bright point sources, which are not well characterised by a generalised noise, tend to remain in the reconstructions. This produces a bias in the estimation of the amplitude of the bright point sources which are underestimated. This problem is solved with the combined technique. We also point out that, although the average error in the amplitude estimation can be higher in the M&Mc due to the detection of a larger number of faint point sources, those point sources present in all three catalogues are, in average, better estimated with the joint analysis.

5 Conclusions

We have presented a combined analysis of the maximum-entropy method and the Mexican Hat wavelet to separate and reconstruct the physical components of the microwave sky from multifrequency observations of the CMB that contain point sources. We have applied this technique to simulated data of the Planck satellite pointing out the improvements achieved due to the complementary nature of both approaches. Bright point sources are identified and subtracted by the MHW whereas MEM is able to deal with fainter point sources as a generalised noise. As a result, the reconstructions of the CMB, SZ effects and Galactic foregrounds are improved and mostly free of contaminating point sources. Moreover, using the joint analysis more complete and accurate point source catalogues are produced at each observing frequency as compared to those obtained by each of the techniques independently.

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