Steps to the Planck Function : A Centenary Reflection

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Abstract

The year 2000 marks the centenary of the Planck function for the blackbody spectral distribution. Here are traced the steps, over 100 years, along the path leading to this important formula.

1 Introduction

The Planck function was first presented at a meeting of the German Physical Society in Berlin on 19 October 1900 in a contribution entitled "On an improvement of the Wien spectral equation", and published in the proceedings of the society in a brief three-page report (Planck, 1900Va). Only the functional form was given, including two undetermined constants, derived on the basis of a tentative relation between the entropy and the energy of each of a collection of radiation resonators. The definitive formula was presented at a meeting of the society on 14 December 1900 in a paper entitled "On the theory of the law of the energy distribution in the normal spectrum" and published, in a nine-page report (Planck, 1900Vb). Herein Planck introduced his new natural constant h relating the "energy element" ϵ associated with a resonator of frequency ν by the formula $\epsilon = h\nu$, with the numerical value $h = 6.55 x 10^{-27}$ erg seconds. He only sketches the actual calculation of the entropy, now for the first time related to the probability of resonator complexions via the Boltzmann principle. Thus ended the long search for the blackbody spectral distribution function and simultaneously began the era of quantum physics. A full treatment of the derivation was later given in a paper (Planck, 1901a) entitled "On the law of the energy distribution in the normal spectrum" submitted to Annalen der Physik on 9 January 1901.

2 Thermal Radiation

That the arrival of both the visible light and the 'dark' radiant heat from the Sun are simultaneously interrupted and restored at the time of a solar eclipse, indicated that light and heat are propagated across space with the same velocity. The astronomer W. Herschel (1800), using a thermometer exposed to the prismatic spectrum of the Sun, discovered that the heating properties of light extended beyond the visible spectrum, being enhanced most especially in the infrared. He also showed that the unseen heat radiation obeyed the same laws of reflection and refraction as visible light. The similarity in other properties, including bi-refringence and polarisation, as found by others, for light and heat, illustrated a difference of degree rather than kind. Furthermore, that sources of light are also sources of heat, and *vice versa*, served to distinguish the two manifestations only by the means of detection employed. Indeed, from his measurements, Herschel presented two distributions of the (continuous) intensity of the solar rays, based respectively on their ability to illuminate and to heat. Because of the two different techniques of measurement, the relative scales are not related. If the two intensity distributions were brought to a common scale and then summed, it would represent the first wideband total intensity distribution for the solar spectrum.

Simple experiments, such as those by Leslie (1804) and others, showed that the rate of emission of thermal radiation from a heated body, expressed as the emissive power or radiant emittance E, being the power emitted per unit area, depends on the temperature and on the nature of the surface. Thus a blackened surface has a higher emittance than a polished one at the same temperature. Also when two such surfaces are subjected to incident radiant energy the fraction of energy absorbed, or absorptivity A, depends on the temperature and on the nature of the surface, a blackened surface being a more efficient absorber than a polished one.

Kirchhoff (1859), by means of a thought experiment, demonstrated that the ratio E/A is the same for all bodies at the same temperature, emitting and absorbing at the same wavelength. He considered two bodies, in the form of infinte surfaces, S_1 and S_2, backed with perfectly reflecting mirrors and arranged parallel to each other with the mirrors on the outside. S_1 is such that it can only emit and absorb radiation of wavelength Λ, while S_2 can emit and absorb all possible wavelengths λ. At wavelength Λ the emissivities are E_1 and E_2, and the absorptivities are A_1 and A_2. For all $\lambda \neq \Lambda$ all the radiation emitted by S_2 is reabsorbed by it. Therefore for temperature equilibrium S_2 must absorb as much as it emits of the radiation of wavelength $\lambda = \Lambda$, whether it comes from S_1 or S_2. Of the radiation E_1 emitted by S_1, S_2 absorbs, after repeated absorptions and emissions :

$$A_2 E_1 \sum_{n=0}^{\infty} k^n = A_2 E_1/(1-k)$$

while of the radiation E_2 emitted by S_2 itself, S_2 absorbs:

$$(1 - A_1)A_2 E_2 \sum_{n=0}^{\infty} k^n = (1 - A_1)A_2 E_2/(1 - k)$$

where $k = (1 - A_1)(1 - A_2)$. Hence :

$$E_2 = A_2[E_1 + (1 - A_1)E_2]/(1 - k)$$

giving :

$$A_1 E_2 = A_2 E_1 \text{ or } E_1/A_1 = E_2/A_2$$

The same result is obtained on applying the condition of temperature equilibrium to S_1, yielding :

$$E_1 = A_1[E_2 + (1 - A_2)E_1]/(1 - k)$$

Since the composition of the surfaces, as well as the wavelength Λ and the temperature are arbitrary, it follows that the ratio E/A, at any wavelength or temperature, is the same for all surfaces, and is a function only of wavelength and temperature, which statement is known as Kirchhoff's Law.

Kirchhoff (1860) generalised the above argument, which involved only discrete wavelengths, to the continuous spectrum, by re-defining E as the emittance per unit wavelength interval. He also introduced the concept of a *perfectly black body*, being one which absorbs all incident radiation, for which $A = 1$ at all wavelengths and temperatures. By considering the temperature equilibrium of a black body enclosed in a cavity with walls made of a similarly black material, he showed that the emittance is equal to the radiant power, $F(\lambda, T)$ per unit wavelength interval, crossing unit area, within the 'black body' enclosure. Hence, by considering the temperature equilibrium of a *non-black* body within the cavity, Kirchhoff's Law takes the form $E/A = F(\lambda, T)$. Thus the absorptivity of a body is equal to its relative emittance, or the emittance of the body relative to that of a black body. Kirchhoff stressed the importance of the study of black body radiation in order to determine the function $F(\lambda, T)$. It is also worth noting here that the enunciation by Kirchhoff of the principle of detailed balance between emission and absorption precedes that of Einstein by more than half a century.

3 Integrated Properties of Black Body Radiation

Given the dependence on wavelength λ and temperature T of the black body radiation intensity $I(\lambda, T)$, its energy density u and flux F, both monochromatic and integrated, can be obtained :

$$I(T) = \int I(\lambda, T)\mathrm{d}\lambda$$

$$u(\lambda, T) = (4\pi/c)I(\lambda, T), u(T) = \int u(\lambda, T)\mathrm{d}\lambda = (4\pi/c)I(T)$$

$$F(\lambda, T) = \pi I(\lambda, T), F(T) = \int F(\lambda, T)\mathrm{d}\lambda = (c/4)u(T)$$

As indicated by Kirchhoff, the properties of black body radiation can be studied by observations on the radiation emerging from a small orifice in the wall of a black body cavity. Experiments by Stefan (1879) established that the total (integrated) emergent flux could be represented in the form :

$$F(T) = \sigma T^4$$

where $\sigma = 5.670x10^{-8}Jm^{-2}s^{-1}K^{-4}$ is the Stefan(-Boltzmann) constant, from which it follows that the energy density is given by :

$$u(T) = (4\sigma/c)T^4 = aT^4$$

with the radiation constant $a = 7.565x10^{-16}Jm^{-3}K^{-4}$. The latter form was established theoretically by Boltzmann (1884) on the basis of thermodynamics. Using for the pressure of radiation the relation :

$$P(T) = (1/3)u(T)$$

as indicated by the electromagnetic theory of Maxwell (1873), and assuming that radiation obeyed the known laws of thermodynamics, then putting $U = uV$:

$$\mathrm{d}Q = T\mathrm{d}S = \mathrm{d}(uV) + P\mathrm{d}V = (u + P)\mathrm{d}V + V\mathrm{d}u$$

from which it follows that :

$$(\partial/\partial u)_V[(u + P)/T] = (\partial/\partial V)_u[V/T]$$

which, on using $P = u/3$, gives :

$$(T/u)(\partial u/\partial T)_V = (\partial \ln u/\partial \ln T)_V = 4$$

so that :
$$u = aT^4, \quad P = (1/3)aT^4$$

in agreement with the result of Stefan. Alternatively, Stefan's result may be taken as a proof of the relation $P = u/3$. From the above expressions for u and P it follows that :
$$\mathrm{d}S = (4/3)aT^3\mathrm{d}V + 4aT^2V\mathrm{d}T$$

giving for the entropy :
$$S = (4/3)aT^3V$$

4 Spectrum of Black Body Radiation

The challenge, first presented by Kirchhoff (1859), to determine the spectral distribution of black body radiation, was not to be completely answered until the turn of the century. Experiments by Draper (1847) and by Weber (1887) showed that, with increasing temperature, the maximum intensity in the light emitted by incandescent solid bodies, progressed across the spectrum, in the direction from the red to the violet. Bartoli (1876) had pointed out that, in order not to contradict the second law of thermodynamics, radiation must exert a pressure, and Boltzmann (1884) showed that work must be done against this pressure in compressing a volume occupied by radiation alone. Wien (1893), developing this argument, made the first important steps in the determination of the black body radiation spectrum. On the basis of the Doppler principle, applied to radiation within a rectangular parallelepiped and reflected (normally) from a wall moving with velocity v, where the relative wavelength change is given by $\delta\lambda/\lambda = 2v/c$ (twice the ordinary Doppler shift), then radiation of wavelength λ_0 and frequency ν_0 in a volume V_0, subjected to an adiabatic compression to volume V, undergoes a change to wavelength λ and frequency ν where :

$$\lambda^3 V^{-1} = \lambda_0^3 V_0^{-1} \quad \text{and} \quad \nu^3 V = \nu_0^3 V_0$$

The change in energy of the radiation, supplied by the work done in the compression against the radiation pressure, results in a change in temperature from T_0 to T. Since for an adiabatic change $VT^3 = V_0 T_0^3$ it follows that :

$$\lambda T = \lambda_0 T_0 \quad \text{and} \quad \nu/T = \nu_0/T_0$$

giving the earliest expression of the Wien Displacement Law, which states that with changing temperature the black body spectrum is shifted so that for each wavelength the product of the temperature and the wavelength remains constant. Similarly each interval of wavelength and frequency is changed according to :

$$T\mathrm{d}\lambda = T_0 \mathrm{d}\lambda_0 \quad \text{and} \quad \mathrm{d}\nu/T = \mathrm{d}\nu_0/T_0$$

Since the (spatial) energy density obeys the relation :

$$u(\lambda, T)\mathrm{d}\lambda/u(\lambda_0, T_0)\mathrm{d}\lambda_0 = u(\nu, T)\mathrm{d}\nu/u(\nu_0, T_0)\mathrm{d}\nu_0 = T^4/T_0^4$$

then :

$$u(\lambda, T)/u(\lambda_0, T_0) = (T/T_0)^5 = (\lambda_0/\lambda)^5$$
$$u(\nu, T)/u(\nu_0, T_0) = (T/T_0)^3 = (\nu/\nu_0)^3$$

in agreement with the observations of Weber. It also follows that the wavelength $\lambda_m(T)$ and frequency $\nu_m(T)$ at which the wavelength and frequency spectral distributions reach their maxima satisfy the relations :

$$T\lambda_m(T) = T_0\lambda_m(T_0) \quad \text{and} \quad \nu_m(T)/T = \nu_m(T_0)/T_0$$

representing another expression of the Wien Displacement Law. Therefore given the spectral distribution for one temperature T_0, that for any other temperature T can be determined.

Wien (1894) re-derived all the above relations when the radiation is enclosed in a spherical volume of variable radius. He also showed that the spectral distributions for different temperatures cannot cross. The condition for this to be true is that :

$$[\partial \ln u(\lambda, T)/\partial \ln \lambda]_T > -5$$

which is trivially true for the rising branch of the curve, and also obviously true for the falling branch. In terms of the frequency distribution the condition becomes :

$$[\partial \ln u(\nu, T)/\partial \ln \nu]_T < +3$$

4.1   The Wien Formula

Wien (1896) pointed out that, since the black body radiation spectrum corresponded to thermal equilibrium, it should be completely determined from the principle of maximum entropy, although he did not do this himself. Instead, adopting a radiation model based on analogy with the Maxwell velocity distribution, he obtained a spectrum of the form :

$$u(\lambda, T) = f_1(\lambda) \exp[-f_2(\lambda)/T]$$

where $f_1(\lambda) = c_1/\lambda^\alpha$, ($c_1$ and α being constants), and $f_2(\lambda)$ are functions to be determined. In order to satisfy the Stefan-Boltzmann T^4 dependence of the energy density, it is necessary for $\alpha = 5$ and for $f_2(\lambda)/T$ to be a function of the product λT, thus giving a more general mathematical expression to the Wien Displacement Law. Putting, specifically, $f_2(\lambda)/T = c_2/(\lambda T)$, where c_2 is a constant, gives the form :

$$u(\lambda, T) = c_1 \lambda^{-5} \exp[-c_2/(\lambda T)]$$

known as Wien's formula, which attains its maximum value at a wavelength λ_m with $\lambda_m T = c_2/5$. The formula was in good accord with the observations of Paschen (1896), with $c_2 = 5\lambda_m T = 14455\mu$ K, or $\lambda_m T = 2.891x10^{-3}$ m K.

In a series of studies, from 1897 onwards, on the entropy of radiation, treating the radiation field as an ensemble of monochromatic resonators, Planck sought to realize the hope of Wien that the blackbody radiation spectrum could be derived on the basis of the maximization of the entropy. Indeed Planck (1899) believed (erroneously) that he had shown that the Wien spectral distribution was a necessary consequence of the principle of entropy maximization, and that its range of validity coincided with that of the (two) laws of thermodynamics. Such support for the Wien formula led to it being referred to as the Wien-Planck distribution. However, experimental measurements by Lummer and Pringsheim

(1899), Rubens (1899) and others pointed to systematic departures of the Wien-Planck formula from the experimental results particularly at long wavelengths. Thiesen (1900), following the lead of Wien, expressed the spectral distribution in the, only slightly different, general form :

$$u(\lambda, T) = T^5 \psi(\lambda T)$$

where $\psi(\lambda T)$ is a still unknown function. Thiesen also suggested that the function ψ, when determined, would accommodate the deviations between theory and experiment by differing from the Wien-Planck form, leading to a modification of the Wien displacement law.

Planck advanced his support of the Wien formula in two further papers (Planck, 1900Aa and 1900Ab). In the first paper he adopted, by way of definition, $s = -(w/a\nu)\ln(w/eb\nu)$, where s and w are the entropy and (average) energy per radiation resonator of frequency ν, and a and b are universal positive constants, while e is the base of natural (Napierian) logarithms. Using the thermodynamic relation $ds/dw = 1/T$ where T is the temperature, it follows that $w = b\nu \exp(-a\nu/T)$, from which, on multiplying by the density $8\pi\nu^2/c^3$ of resonators in ν-space, the Wien formula follows. From the measurements of Paschen (1899) the constants a and b were determined to be $a = 4.818x10^{-11}$ sec K and $b = 6.885x10{-}27$ erg sec. The value of b here therefore represents the first determination of what later became the Planck constant h, while the value of a is that of h/k. In the second paper Planck sought to justify his earlier assumed definition of the entropy s, by arguing that maximization required $d^2s/dw^2 = -F(w)$, where $F(w)$ is a positive function of w. Using a further relation (of limited validity) $F(nw) = F(w)/n$, where n is the number of resonators, Planck adopted the solution $F(w) \propto 1/w$. Thence putting $d^2s/dw^2 = -\alpha/w$, it follows that $s = -\alpha w \ln(\beta w/e)$, where now α and β are positive constants which depend only on the frequency ν. Then $w = [1/e\beta(\nu)]\exp[-1/T\alpha(\nu)]$, leading again to a spectral distribution of the Wien form. Furthermore the Wien displacement law required both α and β to be proportional to ν^{-1}, thus permitting the identifications $\alpha = 1/a\nu$ and $\beta = 1/eb\nu$, where a and b are the quantities introduced earlier. However further work by Lummer and Pringsheim (1900) showed that both a and b were not constant but increased with wavelength, leading them to conclude the invalidity of the Wien-Planck spectral distribution.

4.2 The Rayleigh Formula

To Rayleigh (1900) the Wien formula was difficult to accept on the grounds it implied that, with increasing temperature, the radiation density at a given wavelength approached a limit. Using an argument based on the still disputed Maxwell-Boltzmann principle of the equipartition of energy, in which each degree of freedom has an energy proportional to the temperature T, and in analogy

with the theory of sound, taking for the density of vibrational modes $n(\lambda) \propto \lambda^{-4}$, he obtained :

$$u(\lambda, T) = KT\lambda^{-4}$$

where K is a constant, which has the form prescribed by Thiesen, suggesting that it may be correct for large values of λT. However the formula tends to infinity at short wavelengths, leading Rayleigh to attach the Wien exponential factor $\exp[-c_2/(\lambda T)]$.

4.3   The Planck Formula

In the light of the increasing evidence for the inadequacy of the Wien blackbody spectral distribution, Planck (1900Va) now addressed the problem of improving upon it. He recalled his earlier conclusion that the maximization of entropy was not of itself sufficient to derive the correct distribution, and recognised that his support of the Wien function had depended upon his adoption of the further special condition $d^2s/dw^2 = -\alpha/w$, applicable only in the case of indefinitely small values of w and s. Accordingly he considered instead the condition $d^2s/dw^2 = -\alpha/w(\beta + w)$, which he described as by far the next simplest relation giving s as a logarithmic function of w, while at the same time reducing to the former relation for vanishingly small values of w. It is readily shown that, ignoring constants of integration, $ds/dw = -(\alpha/\beta) \ln\{w/(\beta + w)\}$, and $s = -(\alpha/\beta)\{w \ln w - (\beta + w) \ln(\beta + w)\}$. Then, using $ds/dw = 1/T$, it follows that $w = \beta/\{\exp(\beta/\alpha T) - 1\}$. Noting that the Wien Displacement Law requires (see later) that s be a function of w/ν, where ν is the frequency, (demanding that β be proportional to ν), then enabled Planck to express the spectral distribution in the form $u(\lambda, T) = C_1\lambda^{-5}/\{\exp(C_2/\lambda T) - 1\}$, where C_1 and C_2 are constants, thus introducing what is now known as the Planck Function. Rubens and Kurlbaum (1900) pointed out that the new Planck formula included those of Wien and Rayleigh in the limiting cases of short and long wavelengths respectively, while their experimental measurements favoured it over all others.

However, the derivation was largely phenomenological since the entropy had not been calculated from first principles. Therefore Planck (1900Vb) next sought to follow the "Boltzmann prescription" whereby entropy involves disorder through the relation $S = k \ln W$, where W is the number of ways or states in which the physical system may be realized. This use of this famous relation, never given by Boltzmann himself yet inscribed on his tombstone, is the first occurrence known to the writer. Since the total radiation energy is distributed among all the radiation resonators involving a number of frequencies, it is first necessary to consider the distribution of an energy E among the N resonators with the same frequency ν. Now if E were infinitely divisible the distribution would be possible in infinitely many ways. Hence Planck considered - and this is the essential point of the whole calculation - E to be made up of a fixed number P of finite equal parts, or energy elements, each of magnitude $\epsilon = h\nu$, where h

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is a new constant of nature, so that $E = P\epsilon = Nw$, where w is the average energy per resonator. This therefore represents the first appearance of the Planck constant, both as to its symbol and definition. Planck also proceeded to use combination theory to calculate the total number of possible complexions :

$$W = (N + P - 1)!/(N - 1)!/P! \approx (N + P)^{N+P}/N^N/P^P$$

on making use of the Stirling formula whereby for large n, $n! \approx n^n$. He did not however give an expression for the entropy S, but gave his final result for the spectral distribution in its definitive form :

$$u(\nu, T) = (8\pi h\nu^3/c^3)/[\exp(h\nu/kT) - 1]$$

where now the Boltzmann constant k appears explicitly as well as the new (Planck) constant h. From the formula, using the measurements of Kurlbaum (1898) and of Lummer and Pringsheim (1900), the values of h and k were determined to be $h = 6.55x10^{-27}$ erg sec and $k = 1.346x10^{-16}$ erg grad.

Very shortly afterwords, Planck (1901a) completed his derivation with details of the calculation of the entropy giving :

$$\ln W = N[(1 + P/N)\ln(1 + P/N) - (P/N)\ln(P/N)]$$

so that, since $P/N = w/\epsilon$, the entropy per resonator becomes :

$$s = S/N = k[(1 + w/\epsilon)\ln(1 + w/\epsilon) - (w/\epsilon)\ln(w/\epsilon)] = f(w/\epsilon)$$

which for small values of w/ϵ (small w and large ϵ or ν) leads to the earlier (Wien) result. Now it follows that :

$$\mathrm{d}s/\mathrm{d}w = 1/T = (k/\epsilon)\ln(1 + \epsilon/w)$$

giving :
$$w = \epsilon/\{\exp(\epsilon/kT) - 1\}$$

Then since :
$$u(\nu, T) = 8\pi(\nu^2/c^3)w$$

and, according to the Wien Displacement Law :

$$u(\nu, T) = \nu^3 f_\nu(\nu/T)$$

where f_ν is a still undetermined function :

$$w = \nu[(c^3/8\pi)f_\nu(\nu/T)]$$

Thus :
$$1/T = \mathrm{d}s/\mathrm{d}w = \nu^{-1}(8\pi/c^3)f_\nu^{-1}(w/\nu)$$

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so that, on integrating :
$$s = F(w/\nu)$$

and therefore, in order to agree with the result that $s = f(w/\epsilon)$, it follows that $\epsilon = h\nu$ giving :

$$w = h\nu/[\exp(h\nu/kT) - 1] = (hc/\lambda)/[\exp(hc/kT\lambda) - 1]$$

where h is again the Planck constant. Thus one finally obtains the Planck formula :

$$u(\nu, T) = (8\pi h\nu^3/c^3)/[\exp(h\nu/kT) - 1]$$

$$u(\lambda, T) = (8\pi hc/\lambda^5)/[\exp(hc/kT\lambda) - 1]$$

The integrated radiation energy density is now $u(T) = aT^4$ with (Planck, 1901a) :

$$a = (8\pi k^4/h^3 c^3) \int_0^\infty z^3/(\exp z - 1)\mathrm{d}z = 8\pi^5 k^4/15c^3 h^3$$

in which the integral evaluates to $3!\zeta(4) = \pi^4/15$ giving $a = 7.565 x 10^{-16} m^{-3} K^{-4}$. From the expression for w, giving $s = k[z/(\exp z - 1) + \ln\{\exp z/(\exp z - 1)\}]$, where $z = h\nu/kT$, the integrated entropy density is $S(T) = (4/3)aT^3 V$, as has already been derived from thermodynamics.

The maximum of the wavelength distribution occurs for $x_m = hc/kT\lambda_m$ where :

$$x_m = 5[1 - \exp(-x_m)] = 4.965114...$$

while the maximum of the frequency distribution occurs for $y_m = h\nu_m/kT$ where :

$$y_m = 3[1 - \exp(-y_m)] = 2.821439...$$

giving $\nu_m\lambda_m/c = y_m/x_m = 0.568252...$, indicating that $\lambda_m < c/\nu_m$ and $\nu_m < c/\lambda_m$. Hence also :

$$\nu_m/T = (k/h)x_m = 5.8785 x 10^{10} Hz K^{-1}$$

$$\lambda_m T = (hc/k)/y_m = 2.89779 x 10^{-3} mK$$

In the low frequency/long wavelength limit $h\nu/kT = hc/kT\lambda \ll 1$ the Rayleigh formula is obtained, on expanding the exponential in the denominator :

$$u(\nu, T) = (8\pi h\nu^2/c^3)kT$$

$$u(\lambda, T) = 8\pi kT/\lambda^4$$

while in the high frequency/short wavelength limit $h\nu/kT = hc/kT\lambda \gg 1$ the Wien formula is obtained, on ignoring the 1 in the denominator :

$$u(\nu, T) = (8\pi h\nu^3/c^3)\exp(-h\nu/kT)$$

$$u(\lambda, T) = (8\pi hc/\lambda^5) \exp(-hc/kT\lambda)$$

Further experimental measurements by Lummer and Pringsheim (1901) showed that the values of $\lambda_m T$ were in excellent agreement with the predictions of Planck. Finally, Planck (1901b) returned to the question of the entropy of radiation, to demonstrate that the expression obtained from his theory satisfied the maximization principle. Thus in his stumbling search for the blackbody radiation spectral distribution, Planck, in forging the hitherto missing link between radiation and thermodynamics, not only closed the era of classical physics but also opened up the era of quantum physics.

5   Biographical Note

Max (Karl Ernst Ludwig) Planck was born on 23 April 1858 in Kiel, and died on 4 October 1947 in Göttingen. He attended the universities of Munich and Berlin (in the latter he studied under Kirchhoff and Helmholtz), and received his doctorate from the University of Munich in 1879. Subsequently he taught in the universities of Munich (1880-1885), Kiel (1885-1889) and Berlin (1889-1928). In 1894 he became a member of the Prussian Academy of Science, and the permanent secretary of its Section of Mathematics and Natural Science in 1912. He was awarded the Nobel Prize for Physics in 1918. In 1930 he became president of the Kaiser Wilhelm Society in Berlin, later to be known as the Max Planck Society.

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