## ASTRONOMY AND **ASTROPHYSICS**

# <span id="page-0-0"></span>Emission from dust in galaxies: metallicity dependence

H. Hirashita $^{1,\star},$  A. K. Inoue $^{1},$  H. Kamaya $^{1},$  and H. Shibai $^{2}$ 

<sup>1</sup> Department of Astronomy, Faculty of Science, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan (hirasita,inoue,kamaya@kusastro.kyoto-u.ac.jp)

<sup>2</sup> Division of Particle and Astrophysical Science, Graduate School of Science, Nagoya University, Chikusa-ku, Nagoya 464-8602, Japan (shibai@phys.nagoya-u.ac.jp)

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Abstract. Infrared (IR) dust emission from galaxies is frequently used as an indicator of star formation rate (SFR). However, the effect of the dust-to-gas ratio (i.e., amount of the dust) on the conversion law from IR luminosity to SFR has not so far been considered. Then, in this paper, we present a convenient analytical formula including this effect. In order to obtain the dependence on the dust-to-gas ratio, we extend the formula derived in our previous paper, in which a theoretical formula converting IR luminosity to SFR was derived. That formula was expressed as  $SFR/(M_{\odot} \text{ yr}^{-1}) = \{3.3 \times 10^{-10}(1 \eta$ )/(0.4 – 0.2f + 0.6 $\epsilon$ )}( $L_{\rm IR}/L_{\odot}$ ), where f is the fraction of ionizing photons absorbed by hydrogen,  $\epsilon$  is the efficiency of dust absorption for nonionizing photons,  $\eta$  is the cirrus fraction of observed dust luminosity, and  $L_{IR}$  is the observed luminosity of dust emission in the  $8-1000\text{-}\mu\text{m}$ range. Our formula explains the IR excess of the Galaxy and the Large Magellanic Cloud. In the current paper, especially, we present the metallicity dependence of our conversion law between SFR and  $L_{IR}$ . This is possible since both  $f$  and  $\epsilon$  can be estimated via the dust-to-gas ratio, which is related to metallicity. We have confirmed that the relation between the metallicity and the dust-to-gas ratio is applied to both giant and dwarf galaxies. Finally, we apply the result to the cosmic star formation history. We find that the comoving SFR at  $z \sim 3$  calculated from previous empirical formulae is underestimated by a factor of 4–5.

Key words: ISM: dust, extinction – galaxies: evolution – galaxies: ISM – H ii regions – Methods: analytical – Infrared: ISM: continuum

### 1. Introduction

When we want to know the entire evolutionary history of a galaxy, its color and metallicity are important diagnostic

quantities (e.g., Tinsley [1972\)](#page-7-0). The evolution of the color and the metallicity results from the superposition of successive star formation during the lifetime of the galaxy. Thus, if we wish to reveal the evolution of galaxies, we always need to estimate the star formation rate (SFR) on a galaxy-wide scale. There are many methods to estimate the SFR of galaxies from observational quantities (Kennicutt [1998a\)](#page-7-0). In this paper, we are especially interested in the conversion formula from infrared (IR) luminosity of the galaxies to their present SFR. The IR luminosity originates from dust that is heated by stellar radiation field. Kennicutt([1998b](#page-7-0)) estimated SFR from IR luminosity for starburst galaxies. Buat & Xu([1996\)](#page-7-0) adopted an empirical approach by utilizing the observed relation between the ultraviolet (UV) luminosity and the IR luminosity.

Recently, Inoue et al. [\(2000a](#page-7-0), hereafter IHK00) have derived a theoretical conversion formula from IR luminosity to the SFR by developing a standard model of H ii regions by Petrosian et al.([1972\)](#page-7-0). The present paper examines how metallicity (or dust-to-gas ratio) affects the conversion factor of IHK00. It is possible because the effect of the metallicity is well parameterized in their formula. Here, we present it as

$$
\frac{\text{SFR}}{M_{\odot} \text{ yr}^{-1}} = \frac{3.3 \times 10^{-10} (1 - \eta)}{0.4 - 0.2f + 0.6\epsilon} \frac{L_{\text{IR}}}{L_{\odot}},\tag{1}
$$

where  $f$  is the fraction of ionizing photons absorbed by hydrogen,  $\epsilon$  is the efficiency of dust absorption for nonionizing photons,  $L_{IR}$  is the observed luminosity of dust in the wavelength range of 8-1000  $\mu$ m, and  $\eta$  is the cirrus fraction of  $L_{\text{IR}}$ . In Eq. (1), f and  $\epsilon$  depend on the dust-to-gas ratio 1 . According to Hirashita (1999a, b), the dust-to-gas ratio is expressed as a function of metallicity (see also Dwek [1998;](#page-7-0) Lisenfeld & Ferrara [1998\)](#page-7-0). Therefore, we find easily that the relation between  $L_{IR}$  and SFR depends on metallicity via f and  $\epsilon$ .

Send offprint requests to: H. Hirashita

<sup>⋆</sup> Research Fellow of the Japan Society for the Promotion of Science

<sup>1</sup> In this paper, we focus our main attention on only f and  $\epsilon$ , although we should keep in mind that the  $(1 - \eta)$  may depend on the dust-to-gas ratio. The effect of  $(1 - \eta)$  on SFR is discussed in Appendix A.

<span id="page-1-0"></span>In this paper, we quantitatively examine the importance of the metallicity to our conversion law, the conclusion being that the metallicity dependence is not always negligible when we compare SFRs of very young galaxies and those of present galaxies. First, we examine the formula by IHK00 in Sect. 2, focusing on the effect dustto-gas ratio. Then, in Sect. [3,](#page-4-0) we present the metallicity dependence of the IHK00's formula. In Sect. [4](#page-5-0), we extend our discussion to the cosmic star formation history. Finally, we summarize our conclusions in Sect. [5](#page-6-0) .

### 2. Dependence on dust-to-gas ratio

In this section, we derive the metallicity dependence of the conversion formula  $(Eq. (1))$  $(Eq. (1))$  $(Eq. (1))$ . It is convenient to define the factor  $C_{IR}$  as

$$
SFR = C_{IR}L_{IR}, \qquad (2)
$$

$$
C_{\rm IR} \equiv \frac{3.3 \times 10^{-10} (1 - \eta)}{0.4 - 0.2f + 0.6\epsilon} \left[ M_{\odot} \, \text{yr}^{-1} \, L_{\odot}^{-1} \right]. \tag{3}
$$

We focus on the dependence of  $C_{IR}$  on metallicity. The dependence of  $C_{IR}$  on dust-to-gas ratio is included through f and  $\epsilon$  as will be described in Sects. 2.1 and [2.2](#page-2-0). We will check our formula observationally in Sect. [2.6](#page-3-0). Then, using the relation between the dust-to-gas ratio and the metallicity, we will obtain the metallicity dependence of  $C_{IR}$  in the next section. We assume Case B (Baker & Menzel [1938\)](#page-7-0), where the optical depths for the Lyman series are large enough.

#### 2.1. Dependence of f on dust-to-gas ratio

The dependence of  $f$  (Eq. (3)) on dust-to-gas ratio is obtained from Spitzer [\(1978](#page-7-0), hereafter S78). We define  $\tau_{\rm S,d}$  and  $\tau_{\rm S,H}$  as the optical depths of the dust and of the neutral hydrogen atoms, respectively, for the Lymancontinuum photons over a path length equal to the Strömgren radius,  $r<sub>S</sub>$  (see S78 or Eq. (6) for the definition of  $r<sub>S</sub>$ ). Since the radius of a dust-free H II region is estimated to be  $r<sub>S</sub>$ , it is useful to normalize the length scale by  $r<sub>S</sub>$ . We assume spherical symmetry and spatial uniformity of H ii regions in this paper for the simplicity.

First, we calculate  $\tau_{S, d}/\tau_{S, H}$  for an H II region. Here, both  $\tau_{\text{S}_d}$  and  $\tau_{\text{S}_d}$  are approximated with the optical thickness at the Lyman limit  $(912 \text{ Å})$ . Then we obtain

$$
\frac{\tau_{\rm S,d}}{\tau_{\rm S,H}} = \frac{\tau_{\rm S,d}}{n_{\rm H}s r_{\rm S}} = 12 \frac{E_{B-V}}{N_{\rm H}s},\tag{4}
$$

where the dust extinction at 912 Å,  $A_{912}$ , is taken to be  $13E_{B-V}$  mag according to the Galactic extinction curve, and  $\tau_{\text{S}, d} = A_{912}/(2.5 \log_{10} e)$ . Moreover,  $n_{\text{H}}$  and  $s = 6.30 \times 10^{-18}$  cm<sup>2</sup> denote the number density of the hydrogen and the absorption cross section for a hydrogen atom in the  $n = 1$  level (Eq. (5.6) of S78), respectively, and the column density  $N_{\rm H}$  is defined as  $N_{\rm H} \equiv n_{\rm H} r_{\rm S}$ .

If the physical properties of grains such as the extinction curve are unchanged,  $E_{B-V}/N_{\rm H}$  is proportional to the dust-to-gas mass ratio  $\mathcal{D}$ . According to S78,  $\mathcal{D}$  =  $6 \times 10^{-3}$  when  $N_{\rm H}/E_{B-V} = 5.9 \times 10^{21}$  mag<sup>-1</sup> cm<sup>-2</sup> (Sects. 7.2 and 7.3 of S78). Here, we apply these values to H ii regions. That is, we assume that the dust-to-gas ratio in the H ii region is the same as that in the mean value in the interstellar space. Then, Eq. (4) reduces to

$$
\frac{\tau_{\rm S, d}}{\tau_{\rm S, H}} = \frac{1}{3100} \left( \frac{D}{6 \times 10^{-3}} \right). \tag{5}
$$

This is the same as Eq. (5.23) in S78 but an explicit expression of the dependence on D.

Next, we calculate  $\tau_{\rm S,H} = n_{\rm H} s r_{\rm S}$ . The Strömgren radius is determined as

$$
r_{\rm S} = 1.4 \left(\frac{N_{\rm u}}{10^{48} \text{ s}^{-1}}\right)^{1/3} \left(\frac{n_{\rm H}}{10^2 \text{ cm}^{-3}}\right)^{-2/3} \text{ pc},\tag{6}
$$

where  $N_u$  represents the number of ionizing photons emitted from central stars per second. In this equation, we have assumed that the temperature of the H ii region is 8000 K (i.e., the recombination coefficient to the  $n = 2$  level is  $\alpha^{(2)} = 3.09 \times 10^{-13}$  cm<sup>3</sup> s<sup>-1</sup>; see Eq. (5.14) and Table 5.2 of S78) and that  $n_e = n_H$ , where  $n_e$  is the number density of electrons, is satisfied within the Strömgren radius. Using the estimation of the Strömgren radius above, we obtain

$$
\tau_{\rm S,H} = 2.7 \times 10^3 \left( \frac{n_{\rm H}}{10^2 \text{ cm}^{-3}} \right)^{1/3} \left( \frac{N_{\rm u}}{10^{48} \text{ s}^{-1}} \right)^{1/3},\tag{7}
$$

Combining this with Eq.  $(5)$ ,  $\tau_{S,d}$  is estimated as

$$
\tau_{\rm S, d} = 0.87 \left( \frac{\mathcal{D}}{6 \times 10^{-3}} \right) \left( \frac{n_{\rm H}}{10^2 \, \rm cm^{-3}} \right)^{1/3} \times \left( \frac{N_{\rm u}}{10^{48} \, \rm s^{-1}} \right)^{1/3} . \tag{8}
$$

We note that  $\tau_{\rm S, d}$  becomes large as  $N_{\rm u}$  increases. This is because the probability that the dust grains absorb the photons inside an H ii region increases as the size of the region becomes larger.

Next, we estimate the fraction of the ionizing photons absorbed by dust grains. Due to the grain absorption, the size of an H  $\scriptstyle\rm II$  region is smaller than  $r_{\rm S}$ . In other words, if we define  $y_i$  as the ionization radius normalized by  $r<sub>S</sub>$ ,  $y_i < 1$ . (Without dust grains,  $y_i = 1$ .) A useful relation between  $\tau_{\rm S, d}$  and  $y_i$  is given in Table 5.4 of S78, where  $y_i$ is determined from the following expression (Eq. (5.29) of S78):

$$
3\int_0^{y_i} y^2 e^{y\tau_{\rm S,d}} \, \mathrm{d}y = 1. \tag{9}
$$

Using  $y_i$ , the fraction of the ionizing photons absorbed by hydrogen,  $f$ , is expressed as

$$
f = y_i^3. \tag{10}
$$

<span id="page-2-0"></span>This is the same as  $f$  in Eq. [\(1](#page-0-0)). In Fig. 1a, we show  $f$  as a function of  $\tau_{\rm S, d}$ .

When we consider the dependence of f on the dust-togas ratio, a large ambiguity exists: The number of ionizing photons per H ii region is unknown, since the typical number and mass function of OB stars in an H ii region is difficult to determine exactly. Fortunately, for the purpose of finding the dependence on  $\mathcal{D}$ , this is resolved by calibrating the "Galactic" f with the value of Orion Nebula (Petrosian et al. 1972). According to them,  $f = 0.26$  in the Nebula. They also commented that the value explains the IR emission from H II regions. Adopting  $f = 0.26$  as the typical value of the Galaxy (see also Inoue et al. [2000b\)](#page-7-0), we obtain  $\tau_{S, d} = 2.7$  for the typical Galactic H II regions (we consider  $\mathcal{D} \simeq 6 \times 10^{-3}$  for such regions) from Fig. 1a. Hence we write

$$
\tau_{\rm S, d} = 2.7 \left( \frac{\mathcal{D}}{6 \times 10^{-3}} \right). \tag{11}
$$

This is consistent with Eq. [\(8](#page-1-0)) if we assume  $n_{\rm H}$  =  $10^2 \text{ cm}^{-3}$  and  $N_u \simeq 3.0 \times 10^{49} \text{ s}^{-1}$ . Thus, the net effects of mass function and number of OB stars are included in the value of  $N_u$  by adopting Eq. (11). This simplicity is meaningful for our motivation to find the dependence of  $C_{IR}$ on D. Combining Eq. (11) with Eq. ([9\)](#page-1-0), we obtain f as a function of  $D$  as shown in Fig. 1b.

#### 2.2. Dependence of  $\epsilon$  on dust-to-gas ratio

In IHK00,  $\epsilon$  in Eq. [\(3](#page-1-0)) is defined by

$$
\epsilon \equiv 1 - e^{-\tau_{\text{nonion}}},\tag{12}
$$

where  $\tau_{\text{nonion}}$  is the mean optical depth of dust for nonionizing photons. That is,  $\epsilon$  represents the efficiency of the dust absorption of nonionizing photons. Inoue et al. ([2000c\)](#page-7-0) estimated  $\epsilon = 0.9$  ( $\tau_{\text{nonion}} = 2.3$ ) from the averaged visual extinction of Usui et al.([1998\)](#page-7-0)'s sample  $(A_V \simeq 1$  mag) and the Galactic extinction curve be-tween1000 Å and 4000 Å by Savage & Mathis ([1979\)](#page-7-0). This wavelength range is fit for our purpose, since most of the nonionizing photons from OB stars are emitted in the wavelength range much shorter than 4000 Å. The efficient extinction in this short wavelength makes  $\epsilon$  large (i.e., nearly unity).

It is obvious that  $\tau_{\text{nonion}}$  depends on the dust-to-gas ratio. If the column density of gas contributing to the absorption of nonionizing photons is fixed,  $\tau_{\text{nonion}}$  is proportional to the dust-to-gas ratio. Since we are interested in the dependence of  $C_{IR}$  on  $\mathcal{D}$ , we simply adopt  $\tau_{\text{nonion}} \propto \mathcal{D}$ . Here, we determine the numerical value of  $\tau_{\text{nonion}}$  as

$$
\tau_{\text{nonion}} = 2.3 \left( \frac{\mathcal{D}}{6 \times 10^{-3}} \right),\tag{13}
$$

so that  $\tau_{\text{nonion}}$  becomes 2.3 for the Galactic dust-to-gas ratio. By combining Eqs. (12) and (13), we obtain  $\epsilon$  as a function of  $D$  as shown in Fig. [2](#page-3-0).

Fig. 1. a Fraction of the ionizing photons absorbed by hydrogen, f, as a function of  $\tau_{S,d}$  (optical depth of dust over a distance equal to the Strömgren radius). **b** Fraction of the ionizing photons absorbed by hydrogen, f, as a function of the dust-to-gas ratio, D.

#### 2.3. Cirrus Fraction

The fraction of the cirrus component,  $\eta$ , remains to be determined. In this paper, an empirical value of  $\eta$  is simply adopted. According to Lonsdale Persson & Helou([1987\)](#page-7-0),  $\eta \sim 0.5$ –0.7 for their sample spiral galaxies. We use  $\eta = 0.5$ as adopted in IHK00 (the value averaged for Usui et al. 1998's sample) for the current estimate for our SFR or  $C_{IR}$ . As a first step, we adopt a constant  $\eta$ , since a theoretically reasonable determination of it is very difficult. The variation of the cirrus fraction by the change of  $D$ will be briefly considered in Appendix A. We note that the range of  $C_{IR}$  becomes larger if we consider the metallicity dependence of the cirrus fraction (Appendix A).

#### 2.4. Absorption of Lyman- $\alpha$  photons by dust

Since IHK00's derivation of Eq. [\(1](#page-0-0)) is based on Case B,  $Ly\alpha$  photons are assumed to be easily trapped in an H  $\scriptstyle\rm II$ region (Osterbrock [1989](#page-7-0)). Thus, during the resonant scatterings in an H  $\scriptstyle\rm II$  region, all the Ly $\alpha$  photons are assumed



<span id="page-3-0"></span>

Fig. 2. Fraction of the nonionizing photons absorbed by dust,  $\epsilon$ , as a function of the dust-to-gas ratio,  $\mathcal{D}$ .

to be absorbed by grains in IHK00, which this paper is based on.

If the dust-to-gas ratio is significantly smaller than the Galactic value, the dust grains might not efficiently absorb the  $Ly\alpha$  photons in H II regions. However, H I envelopes on a galactic scale generally exists around H ii regions. A  $Ly\alpha$  photon is absorbed in the H I envelopes soon after it escapes from an H ii region (Osterbrock [1961](#page-7-0)). Hence, even if there is only a small amount of dust, we expect a sufficient chance for dust to absorb the  $Ly\alpha$  photons in the H i envelope. We check this point below.

First of all, we define a path length of the  $Ly\alpha$  photons as  $l_{\text{path}}$ . It is estimated as being  $l_{\text{path}} \sim \tau_{\text{Ly}\alpha}^2 l_{\text{Ly}\alpha}$ , where  $\tau_{\text{Ly}\alpha}$  means the optical depth for Ly $\alpha$  photons and  $l_{\text{Ly}\alpha}$ is the mean free path of the Ly $\alpha$  photons. The square to  $\tau_{\text{Lvo}}$  means that the resonant scattering is assumed to be a random-walk process of photons. Next, we define an optical depth of dust grains for  $Ly\alpha$  photons over the length of  $l_{\text{path}}$  as  $\tau_{\text{dust}}$ . It is estimated as  $\tau_{\text{dust}} \sim \pi a^2 l_{\text{path}} n_{\text{dust}}$ , where  $a$  is the size of grains and  $n_{\text{dust}}$  is their number density. Here, we will discuss whether the  $Ly\alpha$  photons have a chance to escape from galaxies. We are interested in a star-forming region surrounded by H i gas envelopes. The scale length of the H  $\scriptstyle\rm I$  envelope,  $\scriptstyle\rm L$ , may be estimated to be about 100 pc, which corresponds to the thickness of the disk of spiral galaxies. In such a case,  $\tau_{\text{Ly}\alpha}$  is estimated to be about  $L/l_{\text{Ly}\alpha}$ . Then, we find  $\tau_{\text{dust}} \sim \pi a^2 n_{\text{dust}} L^2/l_{\text{Ly}\alpha}$ . Using  $\mathcal{D}$ , it is expressed as

$$
\tau_{\rm dust} \sim \frac{\pi a^2 n_{\rm H} \mathcal{D} L^2 m_{\rm p}}{l_{\rm Ly\alpha} m_{\rm dust}}\,,\tag{14}
$$

where  $m_{\rm p}$  is the proton mass and  $m_{\rm dust}$  is the dust mass for an assumed spherical dust with mass density of 3 g cm<sup>-3</sup>. Adopting  $n_{\rm H} = 0.1$  cm<sup>-3</sup> for a diffuse H I medium,  $a = 0.1 \mu \text{m}, \mathcal{D} = 6 \times 10^{-5} \text{ (0.01 times smaller than the)}$ Galactic value),  $L = 100$  pc, and the resonant-scattering cross section of Ly $\alpha$  photons  $2 \times 10^{-13}$  cm<sup>2</sup> (i.e.,  $l_{Ly\alpha} = 5 \times$  $10^{13}$  cm), we find  $\tau_{\text{dust}} \sim 5 \times 10^{2}$ . Thus,  $\tau_{\text{dust}}$  is much larger

than unity. This means that most of the  $Ly\alpha$  photons are absorbed even if the dust-to-gas ratio is as small as 0.01 times the Galactic value.

## 2.5. Dependence of  $C_{IR}$  on dust-to-gas ratio

In the above subsections, we have expressed f and  $\epsilon$  as a function of the dust-to-gas ratio  $(D)$ , while  $\eta$  is treated as a constant ( $\eta = 0.5$ ). We also assume a typical starforming region whose mean density of gas and production rate of ionizing photons are about  $10^2 \text{ cm}^{-3}$  and  $3.0 \times 10^{49}$  $s^{-1}$ , respectively. Then, we can express  $C_{IR}$  defined in Eq. ([3\)](#page-1-0) as a function of the dust-to-gas ratio. In Fig. [3,](#page-4-0) we present  $C_{IR}$  as a function of  $D$ . From this figure, we find that the coefficient of the conversion from IR light to SFR becomes 4–5 times smaller for  $\mathcal{D} \sim 6 \times 10^{-3}$  (the Galactic value) than that for  $\mathcal{D} \sim 6 \times 10^{-5}$  (0.01 times the Galactic value).

 $C_{IR}$  is also changed if we adopt a different extinction curve. In the previous discussions, we have adopted the extinction curve of the Galaxy. The current paragraph examines how the relation between  $C_{IR}$  and  $D$  changes when we adopt the extinction curve of the Small Magellanic Cloud (SMC) with the other quantities adopted in this paper unchanged. In the SMC-type extinction law, the ratio of the optical depth in Eq.([4\)](#page-1-0) becomes two times larger, because the UV extinction is enhanced in the SMC extinction. This indicates that the same estimation performed in Sect. [2](#page-1-0) is possible if we make the normalization of  $D$  half. Thus, we can find a rough dependence of  $f$  on  $\mathcal D$  from Fig. [1b](#page-2-0) by doubling  $\mathcal D$  (i.e., the line in Fig. 1b is moved to the left by  $0.3 \text{ dex}$ . Since  $\mathcal D$  of SMC is about  $5 \times 10^{-4}$  (Issa et al. [1990](#page-7-0)), we find  $f \simeq 0.75$  for the SMC extinction case, while  $f \approx 0.85$  if we adopt the extinction law of the Galaxy. We can apply the same scaling to  $\epsilon$ . For  $\mathcal{D} \simeq 5 \times 10^{-4}$ ,  $\epsilon \simeq 0.4$  for the SMC extinction case, while  $\epsilon \simeq 0.2$  for the Galactic extinction case. As a result, at  $\mathcal{D} \simeq 5 \times 10^{-4}$ ,  $C_{\text{IR}} \simeq 3.4 \times 10^{-10} \ M_{\odot} \text{ yr}^{-1} \ L_{\odot}^{-1}$  for the SMC extinction, while  $C_{IR} \simeq 4.7 \times 10^{-10} M_{\odot}$  yr<sup>-1</sup>  $L_{\odot}^{-1}$ for the Galactic extinction. Thus, we should be aware of the effect of extinction law on our conversion formula, but the effect is small (a factor of 1.4). If we remember that the uncertainty owing to the IMF is a factor of 2 (e.g., IHK00), we find that the dependence of  $C_{IR}$  on the the extinction curve is not very important. We note that we can estimate  $C_{IR}$  for the SMC-type extinction by moving the line in Fig. [3](#page-4-0) to the left by 0.3 dex (a factor of 2).

#### 2.6. Observational check

Here, we examine whether the values of f and  $\epsilon$  that we adopted for the Galactic value (i.e.,  $f = 0.26$  and  $\epsilon = 0.9$ ) are consistent with the properties of actual H ii regions, by using the ratio between the ionizing-photon luminosity and the IR luminosity. This ratio was extensively studied in 1970s (e.g., Harper & Low [1971\)](#page-7-0).

<span id="page-4-0"></span>

Fig. 3.  $C_{IR}$  (conversion coefficient from the IR luminosity of dust to the star formation rate) as a function of the dust-to-gas ratio, D.

We start with the relation by Petrosian et al.([1972\)](#page-7-0) as

$$
L_{\rm IR} = L(\text{Ly}\alpha) + (1 - f)\langle h\nu \rangle_{\rm ion} N_{\rm u} + \epsilon L_{\rm nonion} \,, \tag{15}
$$

where  $\langle h\nu \rangle_{\text{ion}}$  is the mean energy of an ionizing photon, and  $L_{IR}$ ,  $L(Ly\alpha)$ , and  $L_{nonion}$  are luminosities of IR dust emission,  $Ly\alpha$ , and nonionizing photons, respectively. We note that the meanings of f and  $\epsilon$  in Eq. (15) are the same asthose in Eq. ([3\)](#page-1-0). IHK00 derived  $L_{\text{nonion}} = 1.5 \langle h\nu \rangle_{\text{ion}}N_{\text{u}}$ by adopting the Salpeter's initial mass function (Salpeter [1955](#page-7-0)) with the stellar mass range of 0.1–100  $M_{\odot}$  and the mass–luminosity relation of stars (Table 3.13 of Binney & Merrifield [1998\)](#page-7-0). Then, according to Mezger [\(1978](#page-7-0)), we define the IR excess (IRE) as

$$
IRE \equiv \frac{L_{\rm IR}}{h\nu_{\rm Ly\alpha}fN_{\rm u}} = a + (1 - f + 1.5\epsilon) \frac{\langle h\nu\rangle_{\rm ion}}{h\nu_{\rm Ly\alpha}f},\qquad(16)
$$

where  $\nu_{Ly\alpha}$  is the frequency of the Ly $\alpha$  radiation, and a is the fraction of  $Ly\alpha$  photons that reach the 2p state and go down to 1s (i.e.,  $L(Ly\alpha) = ah\nu_{Ly\alpha}fN_u$ )<sup>2</sup>. According to S78,  $a \approx 0.67$ . Adopting  $f = 0.26$  and  $\epsilon = 0.9$  for the Galactic values (Sects. [2.1](#page-1-0) and [2.2\)](#page-2-0), we obtain  $IRE = 11$ , where we have assumed that  $\langle h\nu \rangle$ <sub>ion</sub> is equal to  $h\nu$  at the Lymanlimit (912 Å). According to Harper  $\&$  Low ([1971\)](#page-7-0),  $5 \leq \text{IRE} \leq 10$  (see also Aannestad [1978;](#page-7-0) Mezger 1978; Maihara et al. [1981\)](#page-7-0) for Galactic H ii regions. Fig. 7.3 in S78 showed that the IR luminosity of H ii regions is larger than predicted from  $L_{\text{V}\alpha}$  luminosity by an order of magnitude (i.e., IRE  $\sim$  10). Thus, f and  $\epsilon$  adopted in this paper are almost consistent with the properties of the Galactic H ii regions within the scatter of observed IRE.

Another test of our model is possible for the case of H ii regions in the Large Magellanic Cloud (LMC) by using the observational results of DeGioia-Eastwood([1992\)](#page-7-0). From Table 3 of DeGioia-Eastwood (1992), we obtain systematically lower IRE for the LMC H ii regions; IRE ranges from 1 to 4, and the median is 1.6. This can be naturally explained by our model with a lower value of the dust-togas ratio in the LMC. If we adopt  $\mathcal{D} = 1.2 \times 10^{-3}$  for the LMC (one-fifth of that of the galaxy) according to Issa et al. (1990), we obtain  $f \approx 0.7$  and  $\epsilon \approx 0.4$ . Then we obtain  $\mathrm{IRE} \simeq 2.4$  for the LMC. This value of IRE for the LMC lies in the range of the observed IRE. Thus, we consider Eqs.([11\)](#page-2-0) and([13\)](#page-2-0) to be applicable and useful in spite of simplification in their derivation. Therefore, we discuss the metallicity dependence of  $C_{IR}$  and its effect on the cosmic star formation history by using the relation between  $C_{IR}$ and  $D$  as shown in Fig. 3 in the following sections.

### 3. Metallicity dependence

In order to obtain the metallicity dependence of the conversion formula (Eq.([1\)](#page-0-0)), which depends on the dust-togas ratio,  $D$ , as shown in the previous section, we must relate  $D$  and the metallicity. Indeed,  $D$  and metallicity are positively correlated (Issa et al. [1990;](#page-7-0) Schmidt & Boller [1993;](#page-7-0) Lisenfeld & Ferrara [1998\)](#page-7-0). Here, we adopt the latest relation proposed by Hirashita (1999a, b). He has constructed a new evolution model of dust in the galactic environment, emphasizing the importance that the grains can grow via the accretion of the metal elements in the cool and neutral components of interstellar medium (ISM; see also Dwek [1998](#page-7-0)). Then, his relation between  $\mathcal D$  and metallicity has explained the observational relation for both the giant and dwarf galaxies. Here, we adopt the relation as shown in Fig. [4](#page-5-0) (solid line), since this seems to reproduce the relation for dwarf galaxies and spiral galaxies<sup>3</sup>. The metallicity is represented by  $[O/H]$ . We note that  $[O/H] = x$  means that the abundance of oxygen is  $10^x$ times the solar value (the solar oxygen abundance is assumed to be 0.013 in mass; e.g., Whittet [1992\)](#page-7-0). In Fig. [4,](#page-5-0) observational data are also shown: The observed relations for nearby spiral galaxies are presented with the square (Issa et al. [1990](#page-7-0)), and the area marked with dwarfs represents a typical locus of dwarf irregular galaxies and blue compact dwarf galaxies (Lisenfeld & Ferrara [1998\)](#page-7-0). We hereafter adopt the solid line in Fig. [4](#page-5-0) as the relation between the dust-to-gas ratio and the metallicity.

Once we accept the relation in Fig. [4](#page-5-0), we relate  $C_{IR}$  and [O/H] by using the relation between the dust-to-gas ratio and  $C_{IR}$  (Fig. 3). The relation between  $C_{IR}$  and [O/H] is presented in Fig. [5](#page-5-0). We see that if [O/H] in the ISM evolves from −2 to 0 via chemical evolution, the coefficient of the

<sup>2</sup> The other photons decay to the ground state with a twophoton process.

As for the parameters in Hirashita (1999a, b), we choose the values as  $f_{\text{in, O}} = 0.1$  and  $\beta_{\text{acc}} = 2\beta_{\text{SN}} = 10$ , where  $f_{\text{in, O}}$ is the dust mass fraction in the material injected from stars, and  $\beta_{\text{acc}}$  and  $\beta_{\text{SN}}$  are defined as gas consumption timescale (gas mass divided by star formation rate) normalized by dust growth timescale in clouds and by dust destruction timescale in supernova shocks, respectively (see Hirashita 1999a for details).

<span id="page-5-0"></span>

**Fig. 4.** Relation between the dust-to-gas ratio  $\mathcal{D}$  and oxygen abundance [O/H]. The solid line represents the wellfitting model by Hirashita (1999a, b). The observed relations for nearby spiral galaxies are presented with the square (Issa et al. 1990). The area marked with dwarfs represents a typical locus of dwarf irregular galaxies and blue compact dwarf galaxies (Lisenfeld & Ferrara [1998\)](#page-7-0).



**Fig. 5.**  $C_{IR}$  (conversion coefficient from the IR luminosity of dust to the star formation rate) as a function of the oxygen abundance, [O/H].

conversion from IR light to SFR becomes about 4–5 times smaller, again. Thus, when we would like to determine the SFR precisely within a factor of 4–5, we should not neglect the effect of metallicity.

## 4. Comments on the cosmic star formation history

The cosmological evolution of SFR is derived from the comoving density of galactic light. For example, Madau et al.([1996\)](#page-7-0) applied the conversion formula from UV light to the SFR and showed that the SFR as a function of the redshift z seems to have a peak at  $z \sim 1-2$ . The cosmic

**Table 1.** Metallicity and  $C_{IR}$  as functions of z.

$\tilde{z}$	$\log(Z/Z_{\odot})$	$C_{\rm IB}$
		$[M_{\odot} \text{ yr}^{-1} L_{\odot}^{-1}]$
0.0	0.00	$1.7 \times 10^{-10}$
0.5	$-0.22$	$2.0 \times 10^{-10}$
1.0	$-0.48$	$2.9 \times 10^{-10}$
2.0	$-1.02$	$5.6 \times 10^{-10}$
3.0	$-1.50$	$7.1 \times 10^{-10}$
4.0	$-1.79$	$7.6 \times 10^{-10}$
5.0	$-1.88$	$7.8 \times 10^{-10}$

SFR as a function of z has been revised and discussed by many authors (e.g., Steidel et al. [1999\)](#page-7-0), commonly referred to as the Madau plot. Takeuchi et al. [\(2000\)](#page-7-0) applied the formula by IHK00 to the determination of the cosmic star formation history from the number-count data of IR galaxies.

The cosmic SFR is also determined from the dust emission at the sub-millimeter (sub-mm) observational wavelength (Hughes et al. [1998](#page-7-0); Barger et al. [1998\)](#page-7-0). We can convert the sub-mm luminosity to the SFR by applying the formula proposed in IHK00. If we take into account the chemical evolution of galaxies on a cosmological timescale  $(e.g., Pei \& Fall 1995), however, we must examine the de (e.g., Pei \& Fall 1995), however, we must examine the de (e.g., Pei \& Fall 1995), however, we must examine the de$ pendence of the conversion formula on metallicity as seen in the previous sections.

According to such a motivation, we apply the results obtained in the previous sections, especially Fig. 5 and its related discussions, to the cosmic star formation history. We consider the evolution of our conversion law along the metal enrichment with the aid of previous research that determined the metallicity as a function of z. We adopt Pei et al.([1999\)](#page-7-0) as a recent modeling of the cosmic chemical evolution.

Pei et al.([1999\)](#page-7-0) modeled the cosmic star formation history and chemical evolution and calculated the evolution of dust amount. Considering the absorption and reprocessing of light by dust, they determined the cosmic chemical evolution after the calibration with the observed comoving density of light. The resultant metallicity evolution as a function of z is shown in the second column of Table 1. Here we adopt the solid line of Fig. 8 in Pei et al. [\(1999](#page-7-0)). Though their treatment of the evolution of the dust-to-gas ratio is not just the same as our treatment, for readers' qualitative understanding, we present a relation between the metallicity and the redshift z from their Fig. 8.

We also present  $C_{IR}$  in the third column of Table 1 calculated from the relation between metallicity  $(Z)$  and  $C_{\text{IR}}$  as shown in Fig. 5. Here we assume that  $\log(Z/Z_{\odot}) =$  $[O/H]$ , where  $Z_{\odot}$  is the solar metallicity and Z is measured in the galactic scale. From Table 1, we see that  $C_{IR}$  is 4–5 times larger at  $z \sim 3$  than that at  $z \sim 0$ . Thus, we should carefully consider the chemical evolution of the galaxies if we determine the cosmic star formation history from

<span id="page-6-0"></span>the dust emission within a factor of 4–5. Deriving the cosmic star formation history from IR–sub-mm observation will be made possible by the future observational projects (e.g., Takeuchi et al. [1999](#page-7-0)). Our formulation, which takes into account the metallicity dependence, will be useful in determining the cosmic star formation history.

We should mention that Pei et al.([1999\)](#page-7-0) treated averaged quantities for each redshift and focused only on the redshift dependence. Thus, our Table [1](#page-5-0) must be applied to the data averaged for each z. In other words, we should not apply them to each individual galaxy at a certain z. For each galaxy, Fig. [5](#page-5-0) should be used instead after their metallicity is known. Finally, we should keep in mind the possibility that far-IR (FIR) or sub-mm sample of galaxies is biased to metal-enriched system because metal-poor (i.e., dust-poor) galaxies are not easily observed in these wavelengths.

## 5. Conclusion

Based on IHK00's formulation, we consider the factor  $C_{IR}$  in the conversion formula between the IR luminosity andthe SFR (Eqs.  $(2)$  $(2)$  and  $(3)$ ). The factor  $C_{IR}$  becomes  $1.7 \times 10^{-10}$ ,  $5.6 \times 10^{-10}$ , and  $7.9 \times 10^{-10}$   $M_{\odot}$  yr<sup>-1</sup>  $L_{\odot}^{-1}$ for the metallicity of 1, 0.1, 0.01 times the solar value, respectively. Thus,  $C_{IR}$  differs by a factor of 4–5 in the range. Importantly, applicability of our formula is observationally confirmed by IREs of the Galaxy and the LMC. Applying our result to the cosmic star formation history, we have found that  $C_{IR}$  may be about 4–5 times larger at  $z \sim 3$  than that at  $z \sim 0$ . This factor of 4–5 is larger than the uncertainty caused by the initial mass function of stars (a factor of 2). Thus, we should carefully consider the chemical evolution of galaxies if we determine the cosmic star formation history from the dust emission within a factor of 4–5.

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## Appendix A: dependence of cirrus fraction on metallicity

In the main body of this paper, we have assumed that the cirrus fraction of the IR luminosity,  $\eta$ , is 0.5. Here, we examine how  $C_{IR}$  is affected owing to the change of  $\eta$ . There is a large uncertainty about observational estimate of  $\eta$ . But, theoretically, we consider some simple cases where the change in  $\eta$  cannot be neglected in our conversion formula.

Fig. A.1.  $C_{IR}$  (conversion coefficient from the IR dust luminosity to the star formation rate) as a function of the oxygen abundance, [O/H], for starburst galaxies. The cirrus fraction  $\eta$  is assumed to be 0.

The assumption of  $\eta = 0.5$  breaks if we are interested in starburst galaxies. When we examine a sample of starburst galaxies, indeed, it is reasonable to assume  $\eta \simeq 0$ . This is because young stars dominate the radiation field that heats the dust, and the optical depth of dust is so large that almost all of the bolometric luminosity is emitted in the IR (Soifer et al. [1987;](#page-7-0) Kennicutt [1998b\)](#page-7-0). For the starburst galaxies, thus, the dependence of  $C_{IR}$  on the metallicity is obtained by putting  $\eta = 0$  into Eq. [\(3](#page-1-0)), while the dependence of f and  $\epsilon$  on the metallicity is the same as described in Sect. [3.](#page-4-0) The result is shown in Fig. [A.1.](#page-2-0) We see that  $C_{IR}$  in this figure is larger by a factor of 2 than that in Fig. [5.](#page-5-0)

Next, we consider a possible change in  $\eta$  of spiral galaxies as a function of metallicity. The cirrus fraction of spiral galaxies is not negligible at the present epoch ( $\eta \simeq 0.5$  for nearby spiral galaxies; Lonsdale Persson & Helou [1987\)](#page-7-0). In the beginning of a spiral galaxy's evolution, when only a little metal is produced,  $\eta = 0.0$  may be reasonable. Thus, the first approximation for the dependence of  $\eta$  on the dust-to-gas ratio is

$$
\eta = 0.5 \left( \frac{\mathcal{D}}{6 \times 10^{-3}} \right). \tag{A.1}
$$

If we adopt this relation, we obtain Fig. [A.2](#page-3-0) as the relation between  $C_{IR}$  and metallicity. We note that our simple treatment as Eq. (A.1) is only applicable for [O/H]  $\leq 0.1$ , because  $\eta > 1$  does not make sense. This means that we need a nonlinear modeling of the relation between  $\eta$  and  $\mathcal D$ at the present epoch. We may need to consider a complex mode of evolution of ISM, whose nonlinearity causes intermittent star formation history (Kamaya & Takeuchi [1997;](#page-7-0) Hirashita & Kamaya [2000;](#page-7-0) Takeuchi & Hirashita [2000\)](#page-7-0). Comparing  $C_{IR}$  at [O/H] = -2 with that at [O/H] = 0, we see that  $C_{IR}$  changes by an order of magnitude as the chemical enrichment proceeds. Thus, if we want to know



<span id="page-7-0"></span>

Fig. A.2.  $C_{IR}$  (conversion coefficient from the IR dust luminosity to the star formation rate) as a function of the oxygen abundance, [O/H], for spiral galaxies. The cirrus fraction  $\eta$  is assumed to be proportional to the dust-to-gas ratio.

the realistic cosmic evolution of galaxies from  $L_{IR}$ , the metallicity evolution of cirrus must also be examined. We will try this very difficult theme in the near future.

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