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Model Independent Primordial Power Spectrum from Maxima, Boomerang, and DASI Data

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Abstract

A model-independent determination of the primordial power spectrum of matter density fluctuations could uniquely probe physics of the very early universe, and provide powerful constraints on inflationary models. We parametrize the primordial power spectrum $A_s^2(k)$ as an arbitrary function, and deduce its binned amplitude from the cosmic microwave background radiation anisotropy (CMB) measurements of Maxima, Boomerang, and DASI. We find that for a flat universe with $A_s^2(k) = 1$ (scale-invariant) for scales $k < 0.001\,\mathrm{h/Mpc}$, the primordial power spectrum is marginally consistent with a scale-invariant Harrison-Zeldovich spectrum. However, we deduce a rise in power compared to a scale-invariant power spectrum for $0.001\,h/\mathrm{Mpc} \lesssim k \lesssim 0.01\,h/\mathrm{Mpc}$. Our results are consistent with large-scale structure data, and seem to suggest that the current observational data allow for the possibility of unusual physics in the very early universe.

1. Introduction

The inflationary paradigm is presently the most plausible solution to the problems of the standard cosmology. Inflation is consistent with all current observational data. However, we are still far from establishing a definitive model of inflation. There exists a broad range of inflationary models (cf. Kolb 1997, Turner 1997), many of which appear consistent with observational data. In order to quantify what can be known about inflation one desires model-independent measurements of the primordial power spectrum of matter density fluctuations as these can provide unique and powerful constraints on inflationary models.

Although it has been conventional to take the primordial power spectrum to be a featureless power law in analyzing cosmological data, there are both theoretical and observational reasons to allow the primordial power spectrum to be a free function. That is, some inflationary models predict power spectra that are almost exactly scale-invariant (Linde 1983), or are described by a power law with spectral index less than one (Freese, Frieman, & Olinto 1990, La & Steinhardt 1991), while others predict power spectra with slowly varying spectral indices (Wang 1994), or with broken scale invariance (Holman et al. 1991ab, Randall, Soljacic, & Guth 1996, Adams, Ross, & Sarkar 1997, Lesgourgues, Polarski, & Starobinsky 1997, Lesgourgues 2000). The latter represents unusual physics in the very early universe. For example, inflation might occur in multiple stages in effective theories with two scalar fields (Holman et al. 1991ab), or in a succession of short bursts due to symmetry breaking during an era of inflation in supergravity models (Adams, Ross, & Sarkar 1997).

There is also tentative observational evidence for a peak in the power spectrum of galaxies at $k \sim 0.05\,\mathrm{Mpc}^{-1}$ (Einasto 1998, Baugh & Gaztanaga 1999, Retslaff et al. 1998, Broadhurst & Jaffe 2000, Gramann & Suhhonenko 1999, Gramann & Hütsi 2000). The simplest explanation for such a peak in the galaxy power spectrum is a new feature in the primordial power spectrum.

The cosmic microwave background radiation anisotropies (CMB) are signatures of the primordial matter density fluctuations and gravity waves imprinted at the time when photons decoupled from matter. The large-scale structure in the distribution of galaxies is a direct consequence of the power spectrum of the primordial density fluctuations. Wang, Spergel, & Strauss (1999) have explored how one can use the upcoming CMB data from the Microwave Anisotropy Probe (MAP; Bennett et al. 1997; `http://map.gsfc.nasa.gov`) and the large-scale structure data from the the Sloan Digital Sky Survey (SDSS; cf., Gunn & Weinberg 1996) to obtain a model-independent measurement of the primordial power spectrum, and to extract simultaneously the cosmological parameters.

In this paper, we implement the concept of a model-independent measurement of the primordial power spectrum from Wang et al. (1999) to extract cosmological information from the CMB data from Maxima (Hanany et al 2000), Boomerang (de Bernardis et al

2000), and DASI (Halverson et al. 2001). We parametrize the primordial power spectrum as a continuous and arbitrary function determined by its amplitude at several wavenumbers [which are equally spaced in $\log(k)$] via linear interpolation. We then measure these "binned" amplitudes from the CMB data of Maxima, Boomerang, and DASI.

2. Measurement of the Primordial Power Spectrum

We parametrize the primordial power spectrum as

$$\begin{aligned} A_s^2(k) &= \left(\frac{k_i - k}{k_i - k_{i-1}}\right) a_{i-1} + \left(\frac{k - k_{i-1}}{k_i - k_{i-1}}\right) a_i, \qquad k_{i-1} < k \leq k_i,\ i = 1, n \\ A_s^2(k) &= a_0 = 1, \qquad k \leq k_0 = k_{min}, \\ A_s^2(k) &= a_n, \qquad k \geq k_n = k_{max}, \end{aligned} \tag{1}$$

where

$$k_i = \exp\left[\frac{i}{n} \ln\left(\frac{k_n}{k_0}\right) + \ln(k_0)\right], \qquad i = 0, n$$

We impose $A_s^2(k) = 1$ for $k \leq k_0 = k_{min}$ for two reasons. First, on the largest scales, the CMB data is consistent with a scale-invariant primordial power spectrum (Smoot et al. 1992, Gorski et al. 1996), i.e., $A_s^2(k) = 1$. Second, the bin amplitude of the primordial power spectrum on the largest scales is poorly constrained by the CMB data due to the effects of cosmic variance. We also assume that $A_s^2(k) = a_n$ for $k > k_n = k_{max}$, because k_{max} is close to the scale corresponding to the angular resolution of Maxima and Boomerang. We choose $k_{min} = 0.001\,h/\text{Mpc}$ and $k_{max} = 0.1\,h/\text{Mpc}$.

We perform the parameter estimation by computing the CMB angular power spectrum $C_l(\mathbf{s})$ for a discrete set of cosmological parameters denoted as $\mathbf{s}$. To save computational time and storage space, we have assumed a flat universe and limited our parameter search to a grid in six cosmological parameters, $\{H_0, \Omega_m, \Omega_b, a_1, a_2, a_3\}$, with $\Omega_\Lambda = 1 - \Omega_m$, $\tau_{ri} = 0$. The a_i $(i = 1, 3)$ parametrize the primordial power spectrum $A_s^2(k)$ as in Eq.(1) with $n = 3$. Our assumption of a flat universe is consistent with all current observational data, and preferred by non-fine-tuned inflationary models. We use the COBE normalization for all the theoretical models.

Following the Maxima, Boomerang, and DASI teams (Jaffe et al 2001, Lange et al. 2001, Pryke et al. 2001), we use an offset lognormal likelihood function to define a χ^2 goodness of fit (Bond, Jaffe, & Knox 2000). That is, for a given theoretical model with $\mathcal{C}_l(\mathbf{s}) \equiv l(l+1)C_l(\mathbf{s})/(2\pi)$, we define

$$\chi^2(\mathbf{s}) \equiv -2\,\ln\mathcal{L} = \chi^2_d + \chi^2_{cal} + \chi^2_{beam}, \tag{2}$$

$$\chi^2_d = \sum_{i,j} (Z_i^t - Z_i^d) M_{ij}^Z (Z_j^t - Z_j^d), \tag{3}$$

$$\chi^2_{cal} = \sum_\alpha \frac{(u_\alpha - 1)^2}{\sigma_\alpha^2}, \tag{4}$$

where

$$Z_i^d \equiv \ln(D_i + x_i) \tag{5}$$

$$Z_i^t \equiv \ln\left(u_\alpha \sum_{l_{min}^i}^{l_{max}^i} f_{il}\mathcal{C}_l\, g_l + x_i\right), \tag{6}$$

$$f_{il} = \frac{W_l^i/l}{\sum_{l_{min}^i}^{l_{max}^i} W_l^i/l}, \tag{7}$$

and

$$M_{ij}^Z = M_{ij}(D_i + x_i)(D_j + x_j). \tag{8}$$

In the above equation, D_i is the measured CMB bandpower in the ith bin, x_i is an offset which depends upon experimental details, $W_l = B_l^2$ is the experimental window function (where B_l is the beam function). The weight matrix M_{ij} is given by the Fisher matrix $F_{ij} = -\frac{\partial^2 \mathcal{L}}{\partial \mathcal{C}_i \partial \mathcal{C}_j}$. The calibration uncertainty is parametrized by u_α, the factor of overall relative calibration; and which has a dispersion of $\sigma_\alpha = 0.08$ and 0.2 for Maxima-1 and Boomerang, respectively. We also consider beam uncertainties for Boomerang. Following Lange et al. (2000), we parametrize the beam uncertainty with $\exp\{-(l+0.5)^2[\Delta(\theta_s^2)]\}$, with $\langle\Delta(\theta_s^2)\rangle = 2\theta_{FWHM}\,\Delta\theta_{FWHM} = (572.0)^{-2}$, where we have used $\theta_{FWHM} = 12.9'$, and $\Delta\theta_{FWHM} = 1.4'$. Hence, we write

$$\chi^2_{beam} = \sum_i \frac{\left(\sum_{l_{min,i}}^{l_{max,i}} f_{il}\mathcal{C}_l \exp\left[-g(l)\,\delta_{beam}\right] - \sum_{l_{min,i}}^{l_{max,i}} f_{il}\mathcal{C}_l\right)^2}{\left(\sum_{l_{min,i}}^{l_{max,i}} f_{il}\mathcal{C}_l \exp\left[-g(l)\right] - \sum_{l_{min,i}}^{l_{max,i}} f_{il}\mathcal{C}_l\right)^2}, \tag{9}$$

where we have defined $\delta_{beam} \equiv 572.0^2\,\Delta(\theta_s^2)$, and $g(l) \equiv [(l+0.5)/572.0]^2$.

We use the higher precision Maxima and Boomerang data as published in Lee et al. 2001 and Netterfield et al. 2001. The relevant experimental details (beam functions, covariance matrix, and log-normal offsets x_i) have not yet been released. It has been shown that in the absence of the knowledge of x_i, Gaussian statistics describe the data better than a log-normal distribution with arbitrary x_i (Bond, Jaffe, & Knox 2000). Therefore, we replace Eq.(3) with

$$\chi^2(\mathbf{s}) = \sum_i \frac{\left[\mathcal{C}^i_{data} - \mathcal{C}^i_{BP}(\mathbf{s})\right]^2}{\sigma_i^2}, \tag{10}$$

where $\mathcal{C}^i_{data}$ are the experimental band-powers with measurement errors σ_i, and

$$\mathcal{C}^i_{BP}(\mathbf{s}) = \frac{\sum_{l_{min,i}}^{l_{max,i}} W_l \mathcal{C}_l(\mathbf{s})/l}{\sum_{l_{min,i}}^{l_{max,i}} W_l/l}, \qquad W_l = B_l^2 = e^{-(0.425\theta_{FWHM} l)^2}, \tag{11}$$

where $\theta_{FWHM} = 10'$ and $12.9'$ for Maxima and Boomerang respectively. Eq.(10) assumes symmetric error bars on $\mathcal{C}^i_{data}$. For the asymmetric errors given by Maxima, we take $\sigma_i = \sigma_{i,-}$ for $\mathcal{C}^i_{data} > \mathcal{C}^i_{BP}(\mathbf{s})$, and $\sigma_i = \sigma_{i,+}$ otherwise.

The DASI team has released the experimental details (window functions f_{il}, log-normal offsets x_i, and covariance matrix V_{ij}) together with their data (`http://astro.uchicago.edu/dasi/`). This enables us to use log-normal statistics [Eqs.(3)-(8)], with the weight matrix M_{ij} given by the inverse of the matrix $N_{ij} = V_{ij} + \sigma_\alpha D_i D_j$, where $\sigma_\alpha = 0.08$ is the calibration uncertainty of DASI.

For a given set of cosmological parameters, $\{H_0, \Omega_m, \Omega_b, a_1, a_2, a_3\}$, we marginalize the model over the calibration uncertainties of Maxima, Boomerang, and DASI, and over the beam uncertainty of Boomerang. The beam uncertainty is only marginalized for Boomerang, since only this experiment appears to have significant beam uncertainty.

3. Results

Since only five values of $H_0 = 100\,h\,\mathrm{km/s\,Mpc^{-1}}$ ($h = 0.5$, 0.6, 0.7, 0.8, 0.9) are computed for our grid, we present our results for the different values of h separately, instead of marginalizing over h.

Figs.1(a)-(c) show the data from (a) Maxima, (b) Boomerang, and (c) DASI, together with best-fit models. In each figure, the solid curve is the best-fit model to the combined Maxima, Boomerang, and DASI data, while the dotted line is the best-fit model to the (a) Maxima, (b) Boomerang, and (c) DASI data separately. The dotted error bars are the errors on each data point including calibration uncertainty. We have combined the data from Maxima, Boomerang, and DASI assuming that the three experiments are independent of each other.

Table 1 shows the best-fit models to the combined Maxima, Boomerang, and DASI data.

Table 1: Best fit models to the combined Maxima, Boomerang, and DASI data

h	Ω_m	Ω_b	a_1	a_2	a_3	u_{mx}	u_{Boom}	u_{DASI}	δ_{beam}	χ^2_{min}
0.5	0.700	0.070	2.5	1.7	1.3	0.933	1.008	0.952	0.10	28.4500
0.6	0.425	0.055	2.0	1.2	1.0	0.974	1.056	0.987	0.05	28.2064
0.7	0.250	0.040	1.5	0.9	0.9	0.978	1.056	0.994	0.05	29.3004
0.8	0.200	0.040	1.0	0.8	1.0	0.958	1.048	0.971	0.10	29.8050
0.9	0.120	0.030	0.9	0.7	1.0	0.981	1.048	0.990	0.05	29.0687

Table 2-4 show the best-fit models to the Maxima, Boomerang, and DASI data separately.

Table 2: Best fit models to the Maxima data

h	Ω_m	Ω_b	a_1	a_2	a_3	u_{mx}	χ^2_{min}
0.5	0.800	0.100	3.0	1.5	1.5	0.997	4.3313
0.6	0.600	0.100	2.5	1.0	1.5	0.997	3.2590
0.7	0.400	0.080	0.9	0.6	1.0	0 1.010	3.9188
0.8	0.300	0.070	1.0	0.6	1.2	0.994	4.5866
0.9	0.200	0.060	0.7	0.5	1.3	1.000	4.7581

Table 3: Best fit models to the Boomerang data

h	Ω_m	Ω_b	a_1	a_2	a_3	u_{B00}	δ_{beam}	χ^2_{min}
0.5	0.600	0.050	5.0	3.0	1.5	0.968	-0.05	11.4684
0.6	0.375	0.050	2.5	1.5	1.0	0.968	0.00	11.7898
0.7	0.250	0.040	2.0	1.2	0.9	0.984	0.00	12.1868
0.8	0.150	0.030	1.5	0.9	0.8	1.016	0.00	12.4757
0.9	0.120	0.030	0.9	0.8	0.9	1.008	0.05	13.5643

Table 4: Best fit models to the DASI data

h	Ω_m	Ω_b	a_1	a_2	a_3	u_{DASI}	χ^2_{min}
0.5	0.800	0.070	2.5	1.7	1.3	0.990	4.3187
0.6	0.450	0.050	4.0	1.7	1.5	1.010	3.3240
0.7	0.275	0.040	3.0	1.2	1.3	1.006	2.5608
0.8	0.200	0.040	2.0	0.9	1.3	0.994	2.6985
0.9	0.120	0.030	0.9	0.6	1.0	1.029	2.3055

We constrain parameters individually by marginalizing over all other parameters. Figs.2(a)-(e) show the likelihood functions for the combined Maxima, Boomerang, and DASI data for the set of parameters $\{\Omega_m, \Omega_b, a_1, a_2, a_3\}$, for $h = 0.5$ (dot), 0.6 (solid), 0.7 (dashed), 0.8 (long-dashed), and 0.9 (dot-dashed). The likelihood functions have been derived using χ^2 values which resulted from multi-dimensional interpolations of the χ^2 values computed for the grid of models (cf. Tegmark & Zaldarriaga 2000a).

Fig.3 shows the primordial power spectrum $A_s^2(k)$ measured from the combined Maxima, Boomerang, and DASI data for $h = 0.6$ (solid), and 0.7 (dotted). The $\pm 1\sigma$ errors are deduced from Fig.2(c)-(e).

Note that $A_s^2(k) = 1$ corresponds to the scale-invariant Harrison-Zeldovich $n_s = 1$ spectrum, with the primordial scalar power spectrum conventionally defined as $k\, A_s^2(k) \propto k^{n_s}$. Clearly, the primordial power spectrum is marginally consistent with a scale-invariant Harrison-Zeldovich spectrum. However, there is a rise in the power at $0.001\, h\,\mathrm{Mpc}^{-1} \lesssim k \lesssim 0.01\, h\,\mathrm{Mpc}^{-1}$ compared to a scale-invariant power spectrum. Although this rise is not statistically convincing, it could be an indication of interesting physics worthy of future investigation.

4. Discussion

Due to parameter degeneracies between h and Ω_m and Ω_b, the CMB data constrain h only weakly. Fortunately, there are a number of independent methods for constraining h (Freedman et al 2001, Branch 1998, Kundic et al 1997). The current combined CMB data from Maxima, Boomerang, and DASI marginally prefer $h = 0.6$ (see Table 1).

Although the CMB data is sensitive to $\Omega_m h^2$ and $\Omega_b h^2$, the likelihood curves for different values of h do not overlap [see Figs.2(a)(b)], since these curves correspond to different values of Ω_Λ (we assume that $\Omega_\Lambda = 1 - \Omega_m$). The likelihood curves for $\Omega_b h^2$ are less sensitive to the value of h [see Fig.2(b)]. This is because varying Ω_b has a much smaller effect on Ω_Λ than varying Ω_m.

Our measurement of the primordial power spectrum [see Fig.3] is consistent with the large-scale structure data which seem to indicate a peak in the matter power spectrum at $k \sim 0.05\,\mathrm{Mpc}^{-1}$ (Einasto 1998, Baugh & Gaztanaga 1999, Retslaff et al. 1998, Broadhurst & Jaffe 2000, Gramann & Suhhonenko 1999, Gramann & Hütsi 2000). We note that the real space power spectrum of the PSCz Survey, from 0.01 to 300 h/Mpc (Hamilton & Tegmark 2000), does not show this peak. The data from 2df (Dalton et al. 2000) and SDSS (Gunn & Weinberg 1996) should help clarify any features in the large-scale structure power spectrum. However, if there is a feature at $k \lesssim 0.002\, h\,\mathrm{Mpc}^{-1}$ (corresponding to the characteristic length scale of the SDSS, see Wang, Spergel,& Strauss 1999), satelite CMB data from MAP or Planck will be required to constrain such features in the primordial power spectrum (Wang, Spergel,& Strauss 1999).

A number of authors have used Maxima and Boomerang data to derive cosmological constraints (Abazajian, Fuller, & Patel 2000, Amendola 2001, Avelino et al. 2000, Balbi et al 2000, Bento, Bertolami, & Silva 2001, Bouchet et al. 2002, Brax, Martin, & Riazuelo 2000, Bridle et al. 2001, Contaldi 2000, Durrer & Novosyadlyj 2001, Enqvist, Kurki-Suonio, & Valiviita 2000, Esposito et al. 2001, Griffiths, Silk, & Zaroubi 2001, Hannestad

2000, Hannestad & Scherrer 2001, Hu et al. 2001, Jaffe et al 2001, Kanazawa et al. 2000, Kinney, Melchiorri, Riotto 2001, Landau, Harari, & Zaldarriaga 2001, Lange et al. 2001, Lesgourgues & Peloso 2000, McGaugh 2000, Melchiorri & Griffiths 2001, Padmanabhan & Sethi 2001, Tegmark & Zaldarriaga 2000b, Tegmark, Zaldarriaga, & Hamilton 2001, White, Scott, & Pierpaoli 2001). Our work is unique in allowing the primordial power spectrum to be an arbitrary function, thus allowing the possibility for detecting new features in the primordial power spectrum.

Our results seem to indicate that the current observational data do not rule out unusual physics (such as multiple-stage inflation) in the very early universe. The upcoming data from the CMB satellite missions MAP (Bennett et al. 1997) and Planck (De Zotti et al. 1999), and the large-scale structure data from 2df (Dalton et al. 2000) and SDSS (Gunn & Weinberg 1996) should allow for a more definitive measurement of the primordial power spectrum (Wang, Spergel,& Strauss 1999). These data will more precisely constrain the possibility for such complex physics in the very early universe.

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REFERENCES

Abazajian, K., Fuller, G.M., Patel, M. 2000, hep-ph/0011048

Adams, J.A., Ross, G.G., & Sarkar, S. 1997, Nuclear Physics B, 503, 405

Amendola, L. 2001, Phys. Rev. Lett. 86, 196
in press, astro-ph/0006300

Avelino, P.P., Martins, C.J.A.P., Rocha, G., Viana, P. 2000, Phys. Rev. D62, 123508

Balbi, A. et al 2000, ApJ, 545, L1; Erratum-ibid. 558 (2001) L145

Baugh, C., & Gaztanaga, E. 1999, astro-ph/9810184 in Proceedings of the MPA/ESO Conference "Evolution of Large Scale Structure: From Recombination to Garching", ed. A. J. Banday, R. K. Sheth, & L. A. N. da Costa (Enschede: Twin), 185.

Bennett, C.L., et al., 1997, BAAS, 29, 1353

Bento, M.C., Bertolami, O., Silva, P.T. 2001, Phys. Lett. B498, 62

Bond, J.R., Efstathiou, G., & Tegmark, M. 1997, MNRAS, 291, L33

Bond, J. R.; Jaffe, A. H.; Knox, L. 2000, ApJ, 533, 19

Bouchet, F.R., Peter, P., Riazuelo, A., Sakellariadou, M. 2002, Phys.Rev. D65, 021301

Branch, D., 1998, ARA&A, 36, 17.

Brax, P., Martin, J., Riazuelo, A. 2000, Phys. Rev. D62, 103505

Bridle, S.L., et al. 2001, MNRAS, 321, 333

Broadhurst, T., & Jaffe, A. 2000, in "Clustering at High Redshift", ASP Conference Series, Vol. 200. Edited by A. Mazure, O. Le Fvre, and V. Le Brun.

Contaldi, C.R. 2000, astro-ph/0005115, submitted to PRL

Dalton, G. B.; 2dF Galaxy Redshift Survey Team 2000, AAS, 196, 5605D

de Bernardis, P. et al 2000, Nature, 404, 955

De Zotti, G., et al., 1999, in proceedings of "3K Cosmology", AIP, in press.

Durrer, R., Novosyadlyj, B. 2001, MNRAS, 324, 560

Einasto, J. 1998, astro-ph/9711321, in "Cosmological Parameters and Evolution of the Universe" (IAU Symposium 183) ed. K Sato, Kluwer Academic Publ. (1998)

Enqvist, K., Kurki-Suonio, H., & Valiviita, J. 2000, Phys.Rev. D62 (2000) 103003

Esposito, S. et al. 2001, Phys. Rev. D63, 043004

Freedman, W.L., et al. 2001, ApJ, 553, 47.

Freese, K., Frieman, J.A., & Olinto, A.V. 1990, Phys. Rev. Lett., 65, 3233

Gorski, K. M., et al. 1996, ApJ, 464L, 11

Gramann, M. & Suhhonenko, I. 1999, ApJ, 519, 433

Gramann, M. & Hütsi, G. 2000, MNRAS, 316, 631

Griffiths, L. M., Silk, J., & Zaroubi, S. 2001, MNRAS, 324, 712

Gunn, J. E., & Weinberg, D. H. 1995, in *Wide-Field Spectroscopy and the Distant Universe*, ed. S. J. Maddox and A. Aragón-Salamanca (Singapore: World Scientific), 3

Halverson, N.W. et al. 2001, astro-ph/0104489, submitted to ApJ.

Hamilton, A. J. S., & Tegmark, M. 2000, MNRAS, 312, 285

Hanany, S. et al 2000, ApJ, 545, L5

Hannestad, S. 2000, Phys. Rev. Lett. 85, 4203

Hannestad, S., Scherrer, R.J. 2001, Phys. Rev. D63, 083001

Holman, R., Kolb, E.W., Vadas, S.L., & Wang, Y. 1991a, Phys. Rev., D43, 3833

Holman, R., Kolb, E.W., Vadas, S.L., & Wang, Y. 1991b, Phys. Lett., B269, 252

Hu, W., Fukugita, M., Zaldarriaga, M., & Tegmark, M. 2001, ApJ, 549, 669

Jaffe, A.H., et al 2001, Phys. Rev. Lett. 86, 3475

Jungman, G., Kamionkowski, M., Kosowsky, A., & Spergel, D.N. 1996, Phys. Rev., D54, 1332

Kanazawa, T., Kawasaki, M., Sugiyama, N., Yanagida, T. 2000, astro-ph/0006445

Kinney, W.H., Melchiorri, A., Riotto, A. 2001, Phys. Rev. D63, 023505

Kolb, E.W. 1997, in *Current Topics in Astrofundamental Physics*, ed. N. Sanchez & Z. Zichichi (Singapore: World Scientific), 162

Kolb, E.W., & Turner, M.S. 1990, *The Early Universe* (Addison-Wesley Publishing Company)

Kundic, T., et al. 1997, ApJ, 482, 75.

La, D., & Steinhardt, P.J. 1991, Phys. Rev. Lett., 62, 376

Landau, S., Harari, D., Zaldarriaga, M. 2001, Phys. Rev. D 63, 083505

Lange, A. E., et al. 2001, Phys.Rev. D63, 042001

Lee, A.T. et al. 2001, ApJ, 561, L1

Lesgourgues, J., Polarski, D., & Starobinsky, A.A. 1997, Nuclear Physics B, 497, 479

Lesgourgues, J. 2000, Nucl. Phys. B582, 593

Lesgourgues, J., & Peloso, M. 2000, Phys. Rev. D 62, 81301

Lidsey, J.E., Liddle, A.R., Kolb, E.W., Copeland, E.J., Barriero, T. & Abney, M. 1997, Rev. Mod. Physics, 69, 373

Linde, A.D. 1983, Phys. Lett., 129B, 177

McGaugh, S. 2000, ApJ, 541, L33

Melchiorri, A., Griffiths, L.M. 2001, New Astronomy Reviews, 45, Issue 4-5, 2001

Netterfield, C.B. et al. 2001, astro-ph/0104460, submitted to ApJ.

Orito, M., Kajino, T., Mathews, G.J. , Boyd, R.N., 2001, Nucl. Phys., A688, 17.

Padmanabhan, T., Sethi, S.K. 2001, ApJ, ApJ, 555, 125

Pryke, C., Halverson, N.W., Leitch, E.M., Kovac, J., Carlstrom, J. E., Holzapfel, W.L., & Dragovan, M. 2001, astro-ph/0104490, submitted to ApJ.

Randall, L., Soljacic, M., & Guth, A. 1996, Nucl. Phys. B472, 377

Retslaff, J., Borgani, S., Gottlöber, S., Klypin, A., & Müller, V. 1998, New A., 3, 631

Seljak, U., & Zaldarriaga, M. 1996, ApJ, 469, 437

Smoot, G. F., et al. 1992, ApJ, 396L, 1

Tegmark, M., Zaldarriaga, M. 2000, ApJ, 544, 30

Tegmark, M., Zaldarriaga, M. 2000, Phys. Rev. Lett., 85, 2240

Tegmark, M., Zaldarriaga, M., Hamilton, A.J.S. 2001, Phys. Rev. D63, 043007

Turner, M.S. 1997, in *Generation of Cosmological Large-Scale Structure*, ed. D.N. Schramm & P. Galeotti (Dordrecht: Kluwer), 153

Wang, Y. 1994, Phys. Rev., D50, 6135

Wang, Y., Spergel, D.N., and Strauss, M.A. 1999, ApJ, 510, 20

White, M., Scott, D., & Pierpaoli, E. 2001, ApJ, 545, 1

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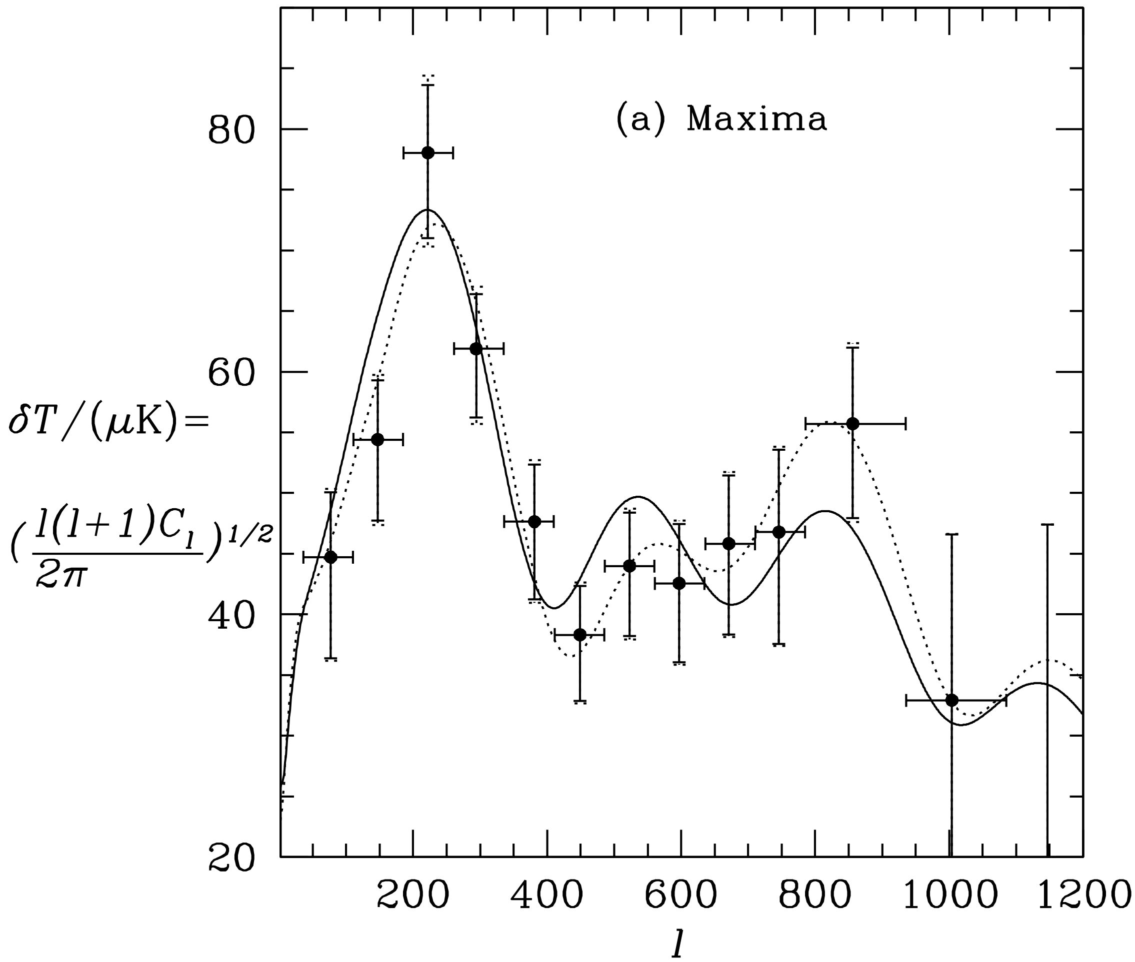
Fig. 1.— The data from (a) Maxima, (b) Boomerang, and (c) DASI, together with best-fit models. In each figure, the solid curve is the best-fit model to the combined Maxima, Boomerang, and DASI data, while the dotted line is the best-fit model to the (a) Maxima, (b) Boomerang, and (c) DASI data separately. The dotted error bars are the errors on each data point including calibration uncertainty.

Fig. 2.— The likelihood functions for the combined Maxima, Boomerang, and DASI data for the set of parameters $\{\Omega_m, \Omega_b, a_1, a_2, a_3\}$, for $h = 0.5$ (dot), 0.6 (solid), 0.7 (dashed), and 0.8 (long-dashed).

Fig. 3.— The primordial power spectrum $A_s^2(k)$ measured from the combined Maxima, Boomerang, and DASI data for $h = 0.6$ (solid), and 0.7 (dotted). The $\pm 1\sigma$ errors are estimated from Fig.2(c)-(e).



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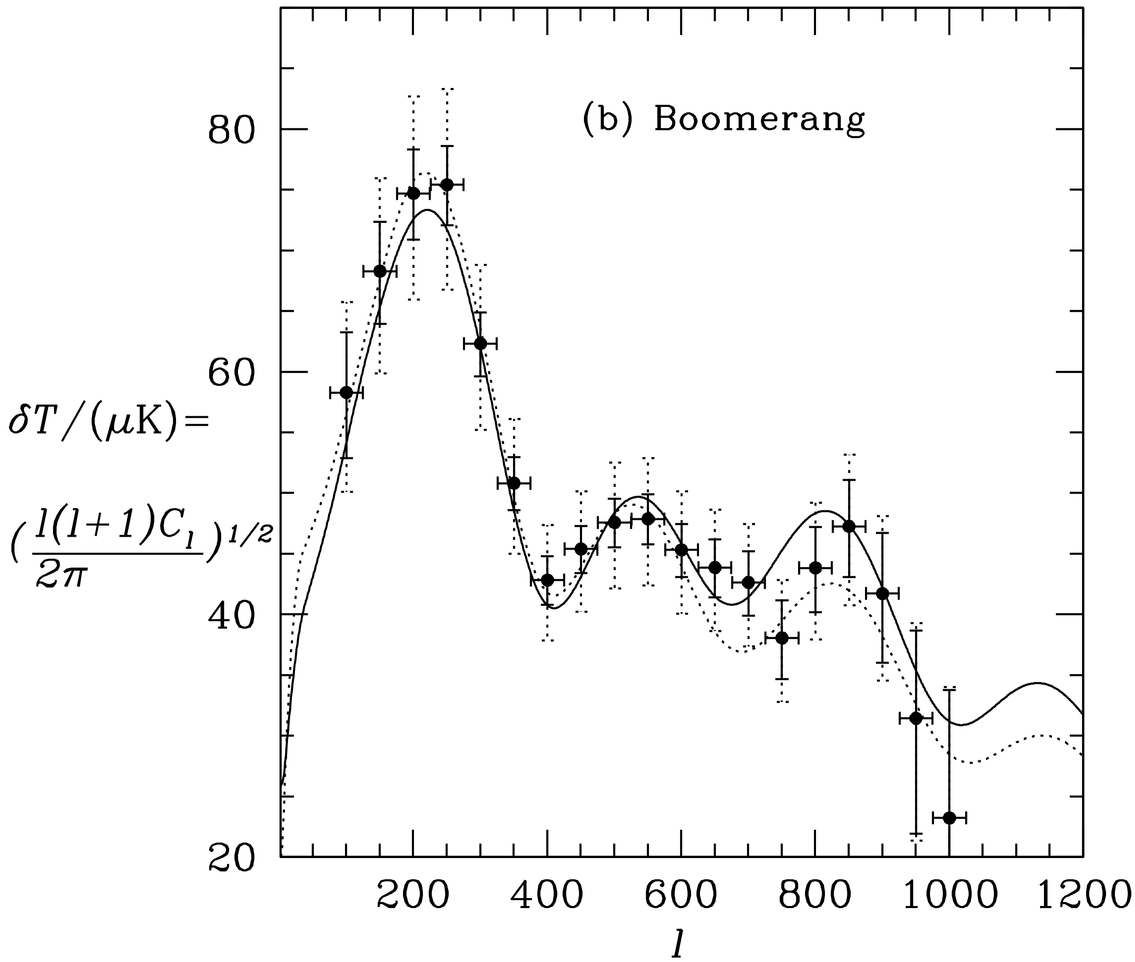


Fig. 1.— (b) Boomerang data.

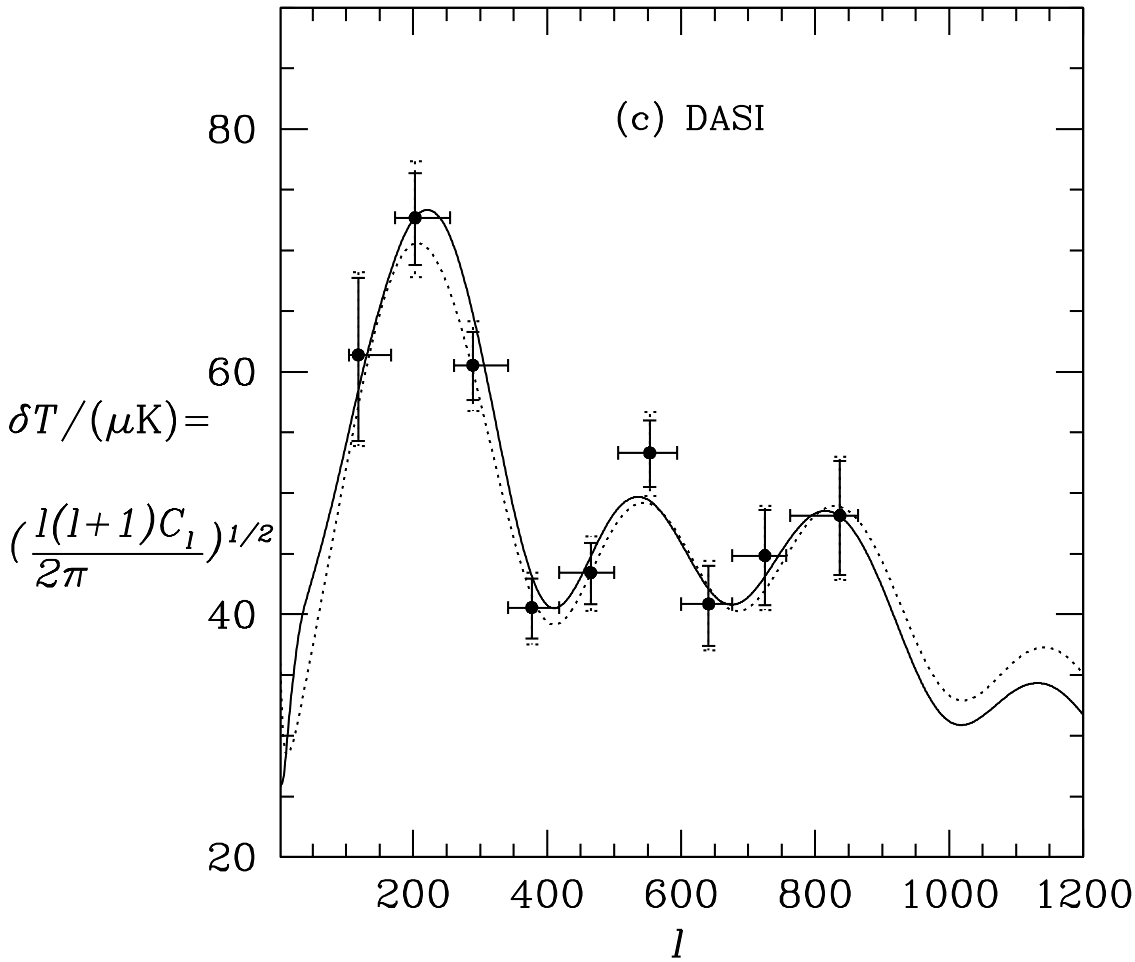


Fig. 1.— (c) DASI data.

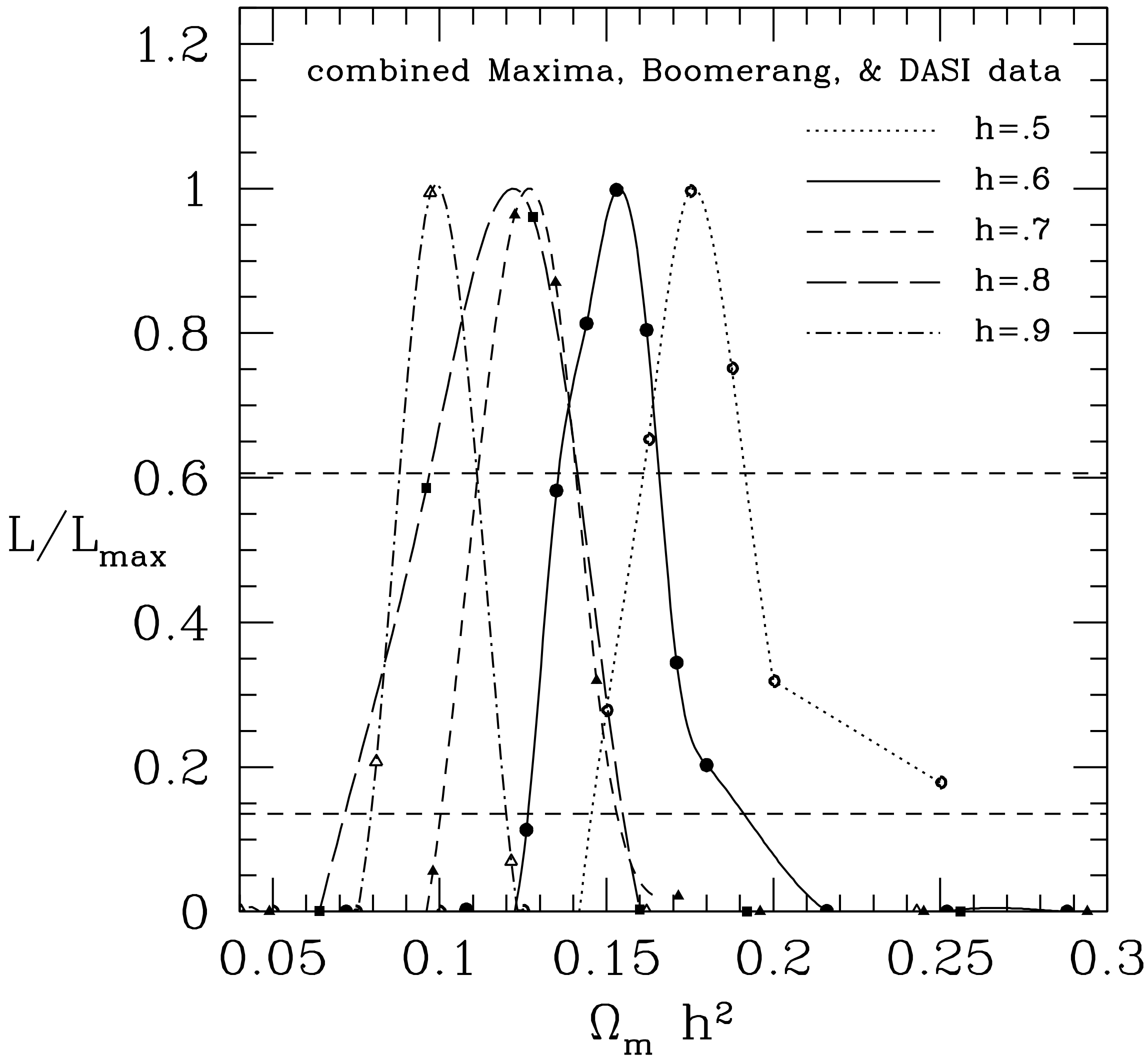


Fig. 2.— The likelihood functions for the combined Maxima, Boomerang, and DASI data for the set of parameters $\{\Omega_m, \Omega_b, a_1, a_2, a_3\}$, for $h = 0.5$ (dot), 0.6 (solid), 0.7 (dashed), and 0.8 (long-dashed). (a) $\Omega_m h^2$.

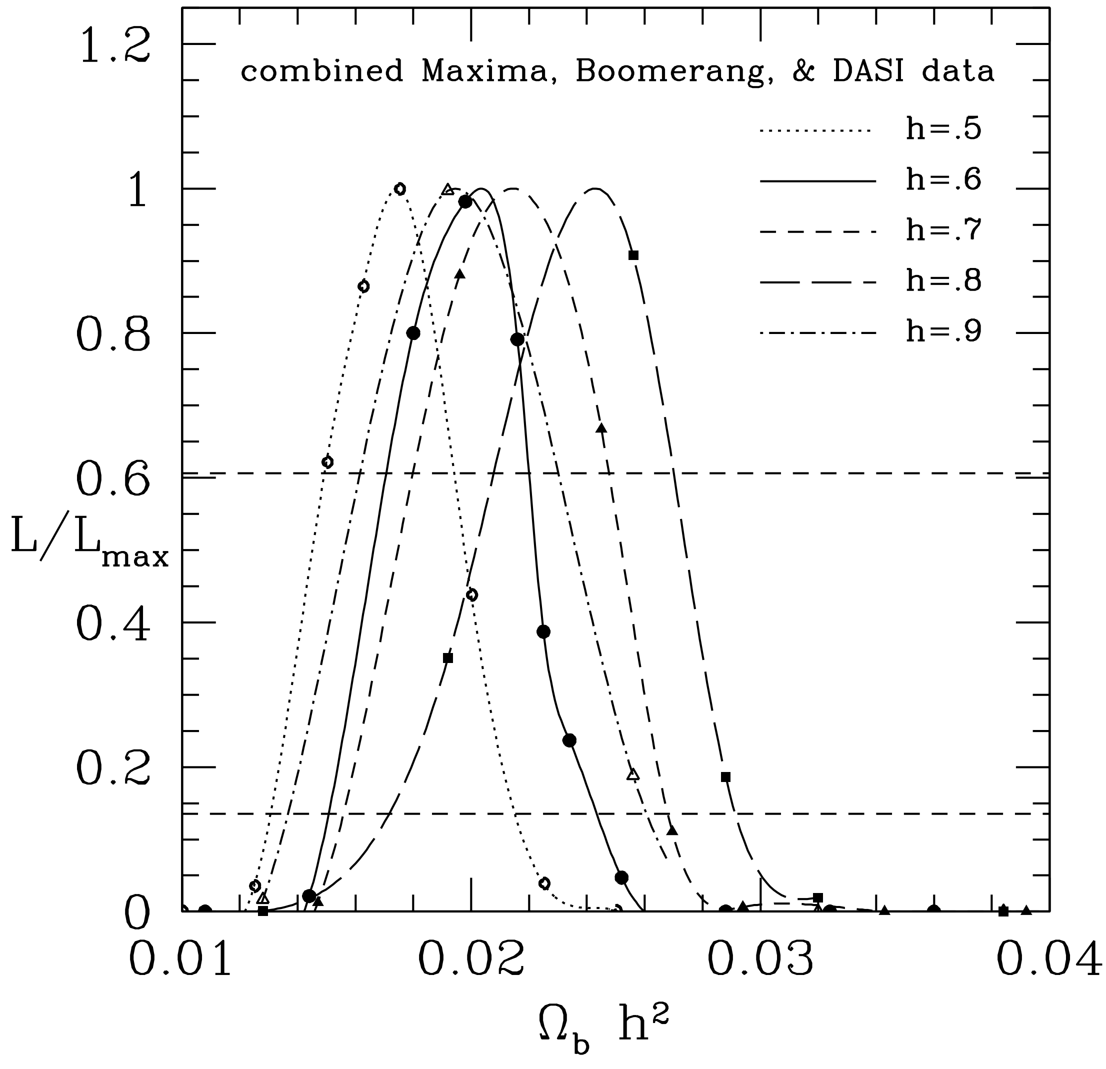


Fig. 2.— (b) $\Omega_b h^2$.

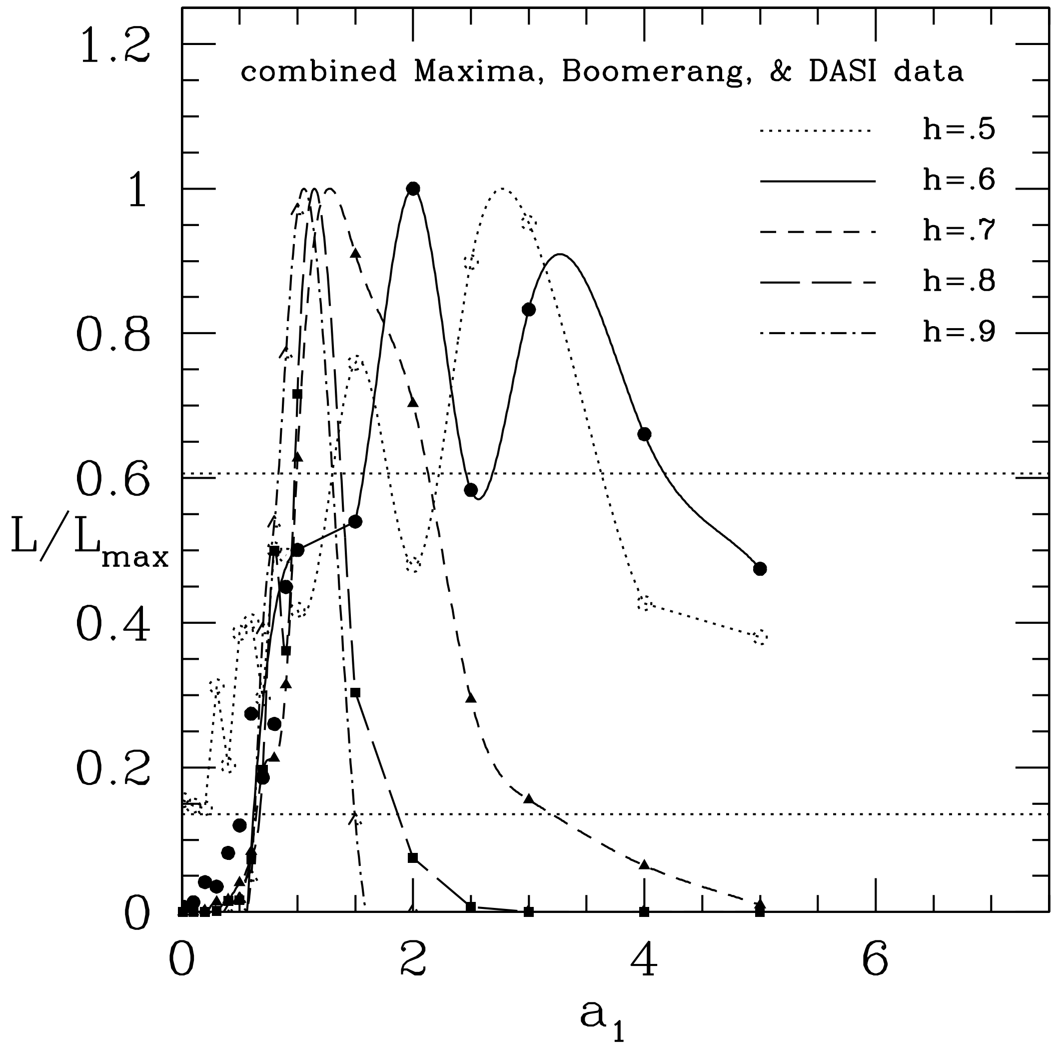

combined Maxima, Boomerang, & DASI data
h=.5
h=.6
h=.7
h=.8
h=.9
1.2
1
0.8
0.6
0.4
0.2
0
L/L$_{max}$
0
2
4
6
a$_1$

Fig. 2.— (c) a_1.

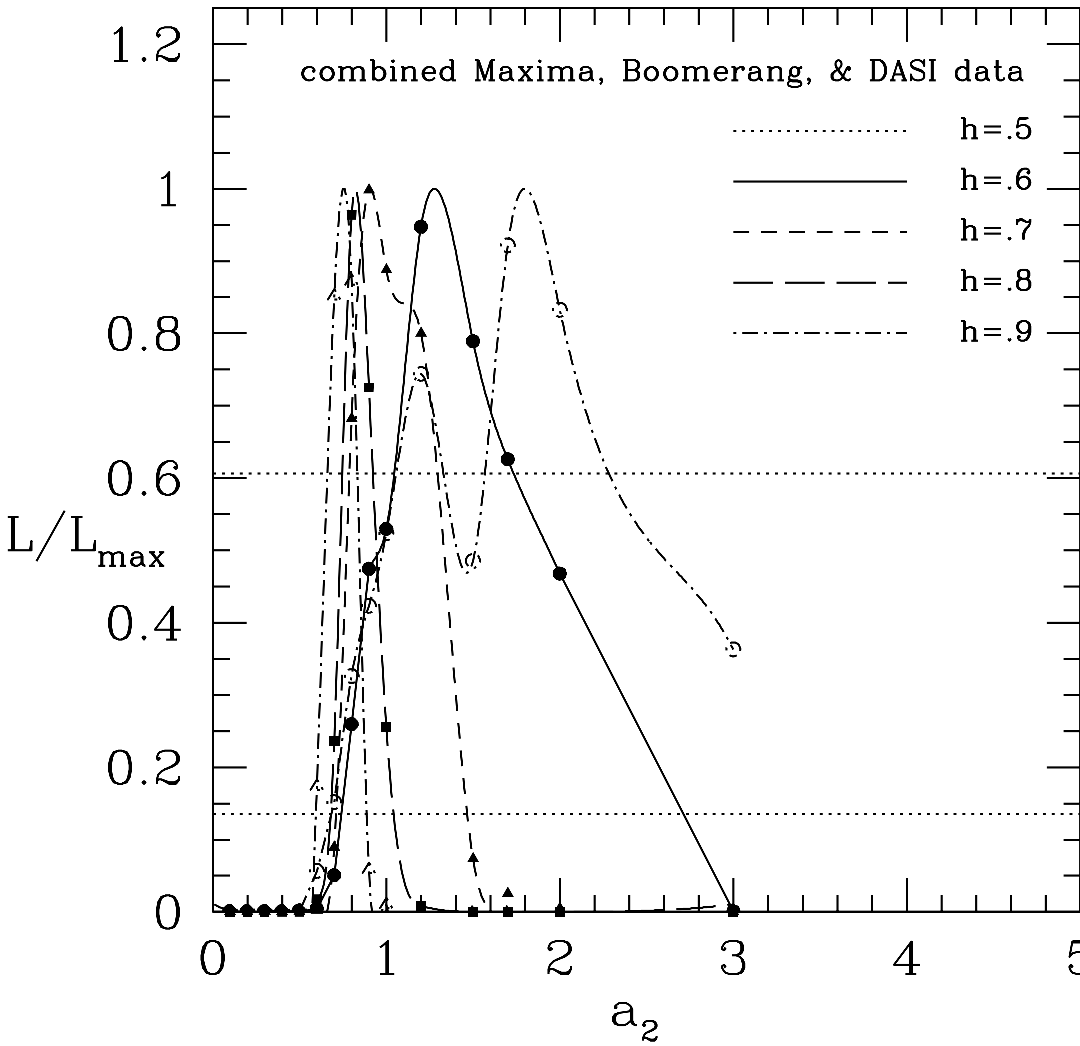

combined Maxima, Boomerang, & DASI data
h=.5
h=.6
h=.7
h=.8
h=.9
L/L$_{max}$
a_2
0
0.2
0.4
0.6
0.8
1
1.2
0
1
2
3
4
5

Fig. 2.— (d) a_2.

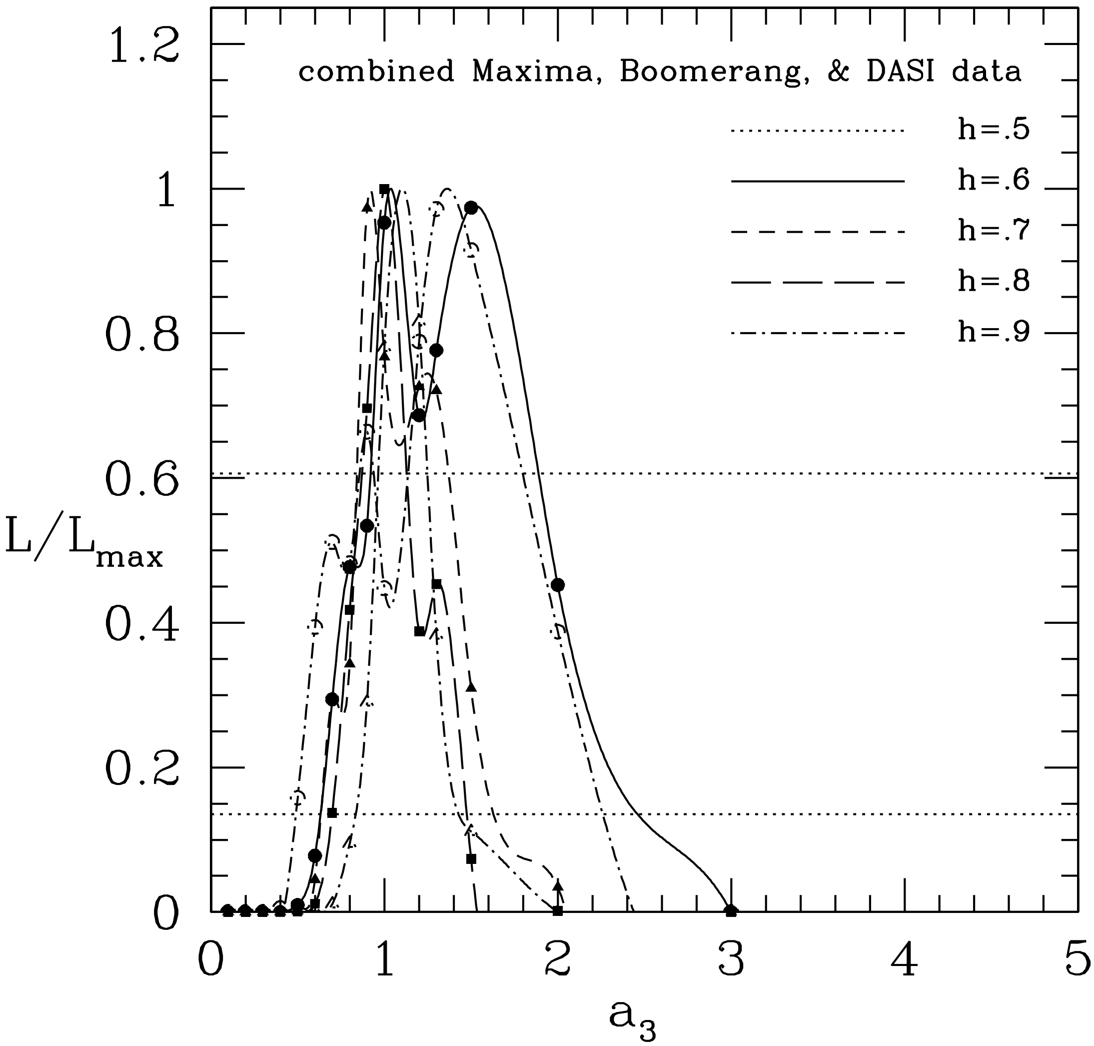
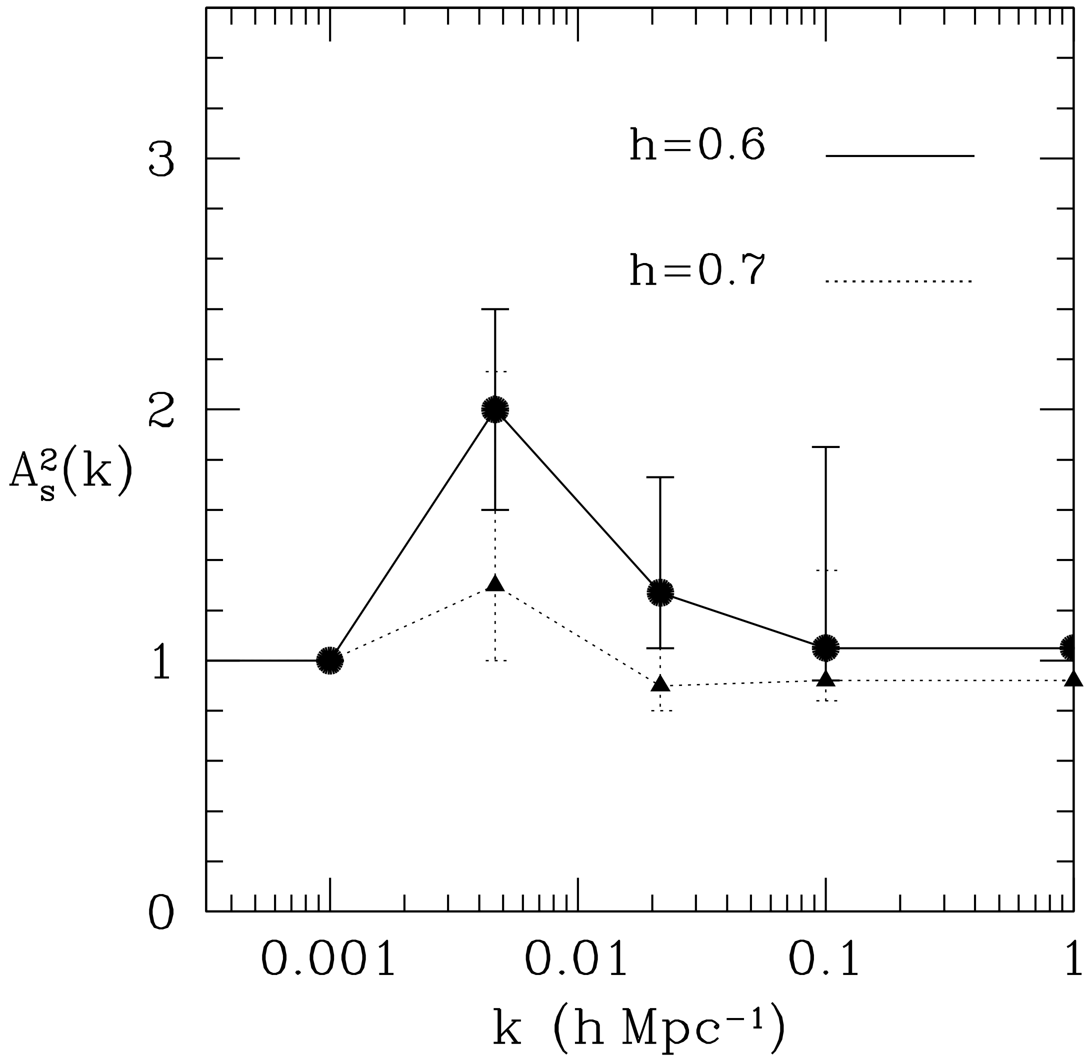

combined Maxima, Boomerang, & DASI data
h=.5
h=.6
h=.7
h=.8
h=.9
L/L$_{max}$
a$_3$
0
0.2
0.4
0.6
0.8
1
1.2
0
1
2
3
4
5

Fig. 2.— (e) a_3.



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