The formation of cosmic structure with modified Newtonian dynamics

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ABSTRACT

I consider the growth of inhomogeneities in a low-density baryonic, vacuum energy-dominated universe in the context of modified Newtonian dynamics (MOND). I first write down a two-field Langrangian-based theory of MOND (non-relativistic), which embodies several assumptions such as constancy of the MOND acceleration parameter, association of a MOND force with peculiar accelerations only, and the deceleration of the Hubble flow as a background field which influences the dynamics of a finite size region. In the context of this theory, the equation for the evolution of spherically symmetric over-densities is non-linear and implies very rapid growth even in a low-density background, particularly at the epoch when the putative cosmological constant begins to dominate the Hubble expansion. Small comoving scales enter the MOND regime earlier than larger scales and therefore evolve to large over-densities sooner. Taking the initial COBE-normalized power spectrum provided by CMBFAST (Seljak & Zeldarriaga 1996), I find that the final power-spectrum resembles that of the standard ΛCDM universe and thus retains the empirical successes of that model.

Subject headings: cosmology: dark matter– large scale structure– theory

1.   Introduction

A primary motivation for cosmic non-baryonic dark matter with negligible pressure is the necessity of forming the presently observed structure in the Universe without violating the constraints on temperature fluctuations in the CMB. Basically, this is because structure in the dark matter component on galaxy to super-cluster scales can begin growing via

gravitational instability considerably before hydrogen recombination (Peebles 1982, Vittorio & Silk 1984, Bond & Efstathiou 1984). This remains one of the powerful arguments against a low density baryonic universe. Any alternative cosmology not including CDM must invoke some mechanism other than conventional gravitational collapse in order to form structure. McGaugh (1999) suggests that the modified Newtonian dynamics (MOND), proposed by Milgrom (1983) as an alternative to dark matter on on galaxy and cluster scales, can provide the needed mechanism and, further, that the consistency of the observed angular structure of the temperature fluctuations in the CMB (Lange et al. 2000, Hanany et al. 2000), with a pure baryonic universe (McGaugh 2000), may be viewed as support for MOND. This speculation is based upon the general expectation that MOND, in providing stronger effective gravity in the limit of low accelerations, would assist in structure formation.

MOND is an *ad hoc* modification of Newton's law of inertia or gravity at low acceleration. The original idea is contained in the statement that when the acceleration falls below a_o, a new physical constant with units of acceleration, then the effective gravitational acceleration approaches $\sqrt{g_n a_o}$ where g_n is the usual Newtonian gravitational acceleration. Although this simple formula works remarkably well in describing galaxy rotation curves consistently with the observed distribution of detectable matter (Sanders 1996, McGaugh and de Blok 1998), it clearly lacks the generality to treat the problem of cosmological density fluctuations.

A more consistent physical description of modified dynamics is provided by the non-relativistic Langrangian-based theory of Bekenstein and Milgrom (1984, hereafter BM). An obvious procedure, when treating the growth of density fluctuations, would be to take the modified Poisson equation of BM and consider small fluctuations about a zeroth order solution as in Newtonian cosmology. The problem is that, when applied to a finite sphere as is usual in Newtonian cosmology, the zeroth order solution is not that of a linear Hubble flow– the absolute distance cannot be factored out and it is not possible to describe cosmology in terms of a universal scale factor. The cosmology is basically that described by Felten (1984) and Sanders (1998), in which MOND alters the usual Friedmann solutions; as soon as the cosmic deceleration over some physical scale falls below a_o, then that entire region begins to deviate from uniform Hubble flow. This leads to the eventual re-collapse of any finite size region regardless of its original density or expansion velocity. In this picture density fluctuations play no role. Apart from problems in principle (what determines the point or points about which MOND collapse proceeds?), this cosmology leads to clear contradictions with observations– recollapse in the present Universe occurs out to scales of 30 Mpc. One might expect that in a proper theory, the basic Hubble flow remains intact, and structure develops from the field of small density fluctuations as in standard gravitational collapse.

In order to construct a reasonable MOND cosmology which has this attribute, one must supplement the BM theory with several assumptions which may reasonably follow from a more general theory. The first of these assumptions– also an aspect of the earlier MOND cosmology– is that the MOND acceleration parameter, a_o, which is comparable to the acceleration in the outer regions of galaxies ($\approx 10^{-8}$ cm/s^{-2}), does not vary with cosmic time. Numerically, $a_o \approx cH_o/6$ which suggests that modified dynamics may reflect the influence of cosmology on local particle dynamics. If a_o varies as the Hubble parameter, then the argumentation presented here would be incorrect. However, it is also possible that a_o is related to the cosmological constant (Milgrom 1999) and is independent of cosmic time. If this is true then MOND plays no role in the evolution of the early radiation-dominated Universe since cosmic deceleration greatly exceeds a_o on relevant scales (e.g. the Jeans length). In the later matter-dominated, pressureless evolution, the cosmic deceleration on co-moving scales corresponding to galaxies or clusters falls below a_o and one might expect modified dynamics to affect the formation of such structure.

The second assumption directly concerns the problem of the zeroth order Hubble flow; we wish to construct a theory in which MOND plays no role in the absence of fluctuations, and the background cosmology is essentially unaltered. In other words, MOND should apply only to peculiar accelerations– the accelerations developing from inhomogeneities– and not to the overall Hubble flow; i.e., no MOND in a homogeneous Universe. This assumption can find some justification in the context of a stratified scalar-tensor theory in which MOND phenomenology results from a scalar force that becomes dominant in the limit of low scalar field gradients (Sanders 1998).

The third assumption concerns the influence of the Hubble flow on the internal dynamics of an otherwise isolated spherical region. In modified dynamics, and any covariant extention thereof, it must be the case that the internal dynamics of a sub-system is influenced by the presence of an external field– the "external field effect" (Milgrom 1983). This is essentially an observational requirement on MOND imposed by the absence of discrepancies in Galactic star clusters. In other words, the underlying theory should not respect the equivalence principle in its strong version. With respect to cosmology, it is not clear how the external field effect would come into play, but I assume here that, for an over- or under-dense spherical region, the deceleration or acceleration of the Hubble flow is the one and only external field which influences the development of the inhomogeneity. Because the de-acceleration of the Hubble flow increases linearly with scale, fluctuations on small comoving scales are affected by MOND earlier than those on larger scale. One might expect this to lead a hierarchical scheme of structure formation, with smaller objects forming first.

2. A non-relativistic Lagrangian-based theory for MOND

It is possible to realize these three assumptions in a non-relativistic theory of modified dynamics. Following BM I write down the Lagrangian for a theory of MOND, as a modification of Newtonian gravity. Although the theory is also *ad hoc*, this approach has several advantages: As a Lagrangian-based theory it enjoys the usual properties of conservation and consistency; moreover, the assumptions described above are not arbitrarily inserted into an equation for the growth of fluctuations but are introduced at a more basic level. This means that the growth equation can be derived self-consistently from the field equations as in the Newtonian case, and the free parameters of the problem are evident. Finally, the results may actually constrain the sort of theory upon which MOND is to be ultimately based.

Because this is a MOND equivalent of Newtonian cosmology, the theory described here need not be fully covariant. The two-field scalar theory is described by the Lagrangian

$$L_f = -\frac{a_o{}^2}{\kappa}\Big[X + (\beta X^{\frac{1}{2}} + \frac{2}{3}Y^{\frac{1}{2}})Y\Big] \tag{1}$$

with

$$X = \frac{(\nabla\phi_1)^2}{a_o{}^2} \tag{2}$$

and

$$Y = \frac{(\nabla\phi_2)^2}{a_o{}^2} \tag{3}$$

where ϕ_1 is a scalar field which we wish to identify with Newtonian gravity and ϕ_2 is a second field which we identify with a MOND force. Here $\kappa = 8\pi G$ and the fundamental acceleration a_o is put in by hand but may be related to the cosmological constant in the underlying theory. This is similar to the two-field version of BM theory. The first term (X) is just the usual Lagrangian of a scalar field, but the second term is anomalous and includes the MOND Lagrangian as in BM theory $(Y^{1.5})$. However, there is an additional coupling between the two fields, $(YX^{0.5})$ where β is a parameter, of order unity, describing the strength of the coupling.

The coupling of these two fields to matter is described by the interaction Lagrangian,

$$L_i = -\Big[(\rho - 2\rho_\Lambda)\phi_1 + \delta\rho\phi_2\Big] \tag{4}$$

where ρ is the actual density and $\delta\rho$ is the deviation of the density from its mean value (i.e., $\rho = \sum m_i \delta^3(\mathbf{r} - \mathbf{r_i}(t))$; and $\delta\rho = \rho - \Pi(\tau)$ where Π is the cosmological density which is only a function of cosmic time τ). The first term describes the coupling of Newtonian gravity to ordinary matter and the vacuum energy density. The second term describes

coupling of the anomalous field to density fluctuations in the ordinary matter and is also a modification of BM theory to suit the requirements of cosmology. The non-standard coupling to fluctuations embodies the second assumption described in the Introduction: there is no anomalous MOND force in the absence of fluctuations. This *ansatz*– the most questionable step in the present procedure– probably would not be necessary in a fully covariant theory where spatial gradients in a scalar field develop only in the presence of density gradients.

The theory is complete when we include the usual matter Lagrangian

$$L_m = \sum_i m_i \dot{r_i}^2 \delta^3(\mathbf{r} - \mathbf{r_i}(t)) \tag{5}$$

where r_i is the position of a given particle. Then, the dynamics of the theory come from the action

$$S = \int (L_f + L_i + L_m) d^3x. \tag{6}$$

Taking the extremum of the action with respect to variations in the fields, as usual, one finds the field equations

$$\nabla \cdot \left[\frac{1}{2} \beta a_o \hat{u}_1 \left(\frac{\nabla \phi_2}{a_o} \right)^2 + \nabla \phi_1 \right] = 4\pi G(\rho - 2\rho_\Lambda) \tag{7}$$

$$\nabla \cdot \left\{ \left[\frac{|\nabla \phi_2|}{a_o} + \beta \frac{|\nabla \phi_1|}{a_o} \right] \nabla \phi_2 \right\} = 4\pi G \delta\rho. \tag{8}$$

Here $\hat{u}_1$ is a unit vector in the direction of $\nabla \phi_1$. Similarly, a stationary action with respect to variations in particle position yields the usual particle equation of motion:

$$\frac{d^2 \mathbf{r_i}}{dt^2} = -\nabla \phi_1(\mathbf{r}_i) - \nabla \phi_2(\mathbf{r}_i); \tag{9}$$

Note that this is MOND as a modification of gravity and not of inertia.

This is a true one-parameter theory of MOND. One might think that the strength of the coupling of ϕ_2 to matter might also be adjustable through the introduction of some additional unitless parameter, α (i.e., the interaction Lagrangian would then contain $\alpha \delta\rho\phi_2$). But then α can always be absorbed in a rescaling of the acceleration parameter a_o; in fact, this must happen because of the requirement of MOND that the force about a point mass asymptotically approach $\sqrt{GMa_o/r^2}$. Therefore β is the only possible free parameter of the theory; this is a "minimalist" MOND theory in the sense that there are no arbitrary functions and only one adjustable parameter.

In the limit where $\beta \to 0$, eq. 7 reduces to the Poisson equation, and ϕ_1 can be identified with the Newtonian potential. In the absence of fluctuations ($\delta\rho = 0$), $\nabla \phi_2 = 0$

and the theory becomes entirely Newtonian. Thus combined with the equation of motion for a finite uniform sphere, eq. 7 yields the usual Friedmann equation for the time evolution of the dimensionless scale factor of the sphere. Assuming, as in Newtonian cosmology, that the scale factor of the finite sphere is identical to that of the cosmology, the usual linear Hubble flow is recovered. However, in the presence of fluctuations, the second field, the MOND force, contributes to peculiar accelerations. Because of the coupling of the two fields (eq. 8), the zeroth order Hubble de-acceleration over a finite size region appears as a background field in the determination of the MOND peculiar accelerations (the external field effect).

Eqs. 7 and 8 can be readily solved for $\nabla\phi_1$ and $\nabla\phi_2$ in the case of a spherically symmetric mass distribution representing a bound object such as galaxy (with $\delta\rho = \rho$). Reasonable rotation curves result when $2 < \beta < 4$. For smaller values of β, rotation curves are not flat but gradually decline to the asymptotic MOND value ($(GMa_o)^{0.25}$) and for larger values, rotation curves rapidly decline and then rise to this asymptotic value.

3. The growth of fluctuations

Here I use the notation and units from Sanders (1998): x is the dimensionless scale factor in terms of the present scale factor; $\dot{x}/x$ is the Hubble parameter in terms of the present Hubble parameter (H_o); and the cosmic time is in units of the Hubble time, $1/H_o$. The Friedmann equation for the evolution of the scale factor (derivable from the above theory in the absence of fluctuations) may then be written

$$\left(\frac{\dot{x}}{x}\right)^2 = \frac{\Omega_r}{x^4} + \frac{\Omega_m}{x^3} - \frac{k}{x^2} + \Omega_\Lambda, \tag{10}$$

where Ω_r is the present density parameter in radiation (and other relativistic particles), Ω_m is that of ordinary matter (baryonic and CDM), Ω_Λ in vacuum energy density, and $k = \Omega_r + \Omega_m + \Omega_\Lambda - 1$ is the curvature constant.

In the Newtonian treatment of the development of density fluctuations, one may consider the time-evolution of a single Fourier component (δ_k) of the fluctuation field in isolation. This works, as long as the fluctuations are small, because Newtonian theory is linear; with modified dynamics, which is fundamentally non-linear, this is not obviously the case. Therefore, I consider the evolution of a sub-horizon spherical region in the Universe having mass M and an average over (under)-density $\Delta = <\delta\rho/\rho>_r$. This mean over-density may be also identified with the mass variance over some comoving scale– a quantity which is calculable from the power spectrum of Gaussian fluctuations (Padmanabhan 1993). I

take the total radius of the region to be $r_o + r_1$ where r_o is the radius in absence of the over-density. To first order

$$\Delta = -\frac{3r_1}{r_o}. \tag{11}$$

Differentiating this expression twice with respect to time we find

$$\ddot{\Delta} + 2\frac{\dot{x}}{x}\dot{\Delta} + \frac{\ddot{x}}{x}\Delta = -3\frac{\ddot{r}_1}{r_o}. \tag{12}$$

With Newtonian dynamics the peculiar acceleration would be given by

$$\ddot{r}_1 = -\frac{2GM}{3r_o{}^2}\Delta + \Omega_\Lambda r_1 \tag{13}$$

while, in the late matter and vacuum energy-dominated regime, the Hubble flow deceleration is

$$\frac{\ddot{x}}{x} = -\frac{GM}{r_o{}^3} + \Omega_\Lambda. \tag{14}$$

Combining eqs. 13 and 14 in eq. 12 we find the usual Newtonian expression for the linear evolution of fluctuations:

$$\ddot{\Delta} + 2\frac{\dot{x}}{x}\dot{\Delta} = \frac{3\Omega_m}{2x^3}\Delta. \tag{15}$$

To consider how MOND may alter the growth of fluctuations, I make use of the two-field theory described in the previous section to determine the peculiar acceleration in eq. 12 ($\ddot{r}_1$). For the spherical mass distribution, application of the Gauss theorem to eq. 7 yields

$$\frac{\beta}{2}\frac{g_2{}^2}{a_o}\hat{u}_1 + g_1 = -\frac{4\pi G\rho}{3}(r_o + r_1)(1 + \Delta) + \frac{8\pi G\rho_\Lambda}{3}(r_o + r_1). \tag{16}$$

where $g_1 = -d\phi_1/dr$ and $g_2 = -d\phi_2/dr$. I set $g_1 = g_b + \delta g_1$ where the g_b is the zeroth-order background field given by

$$g_b = \frac{4}{3}\pi G r_o(2\rho_\Lambda - \rho) \tag{17}$$

and δg_1 is the small fluctuation about this background. Given the zeroth order Hubble expansion and the usual definitions of density parameters, eq. 17 may also be written

$$g_b = -\lambda_c H_o{}^2\left[\frac{\Omega_m}{2x^2} - \Omega_\Lambda x\right] \tag{18}$$

where λ_c is the comoving scale corresponding to r_o (i.e., $r_o = x\lambda_c$).

Using eq. 11 to eliminate r_1, eq. 16, to first order in Δ becomes

$$\delta g_1 + \frac{\beta}{2}\frac{g_2{}^2}{a_o}\hat{u}_1 = -\frac{8\pi G r_o\Delta}{9}(\rho + \rho_\Lambda) \tag{19}$$

Because of the coupling of ϕ_2 to inhomogeneities, the fluctuations in the MOND field g_2 are always small. Therefore, to lowest order, eq. 19 reduces to

$$\delta g_1 = -A\Delta \tag{20}$$

where

$$A = \frac{\lambda_c H_o{}^2}{3}\left[\frac{\Omega_m}{x^2} + \Omega_\Lambda x\right] \tag{21}$$

This is just the usual expression for the perturbed Newtonian force.

Application of the Gauss theorem to eq. 8 yields

$$|g_2|g_2 + \beta|g_1|g_2 = -\frac{4\pi a_o G}{3}(r_o + r_1)\Delta = -B\Delta \tag{22}$$

where

$$B = \frac{a_o \Omega_m \lambda_c H_o^2}{2x^2}\Delta \tag{23}$$

to first order in Δ. With eq. 20, this reduces to

$$|g_2|g_2 + \beta g_2\left|g_b - A\Delta\right| = -B\Delta \tag{24}$$

Thus we have two algebraic expressions for the perturbed force fields δg_1 and g_2 in terms of the over- or under-density Δ: eq. 20 for the Newtonian field and eq. 24 for the MOND field. We cannot immediately dismiss the second order term in eq. 24. Due to the presence of the cosmological constant, the background field, g_b, will become vanishingly small over some range of scale factor, and then the left-hand-side of eq. 24 only contains second order terms. However, this quadratic equation for g_2 is readily solved:

$$g_2 = \pm 0.5\beta\left|g_b - A\Delta\right| \mp 0.5\sqrt{\beta^2(g_b - A\Delta)^2 \pm 4B\Delta} \tag{25}$$

where the upper sign applies to over-densities ($\Delta > 0$) and the lower sign to under-densities ($\Delta < 0$).

From eq. 9, the total acceleration of a spherical shell is

$$\ddot{r}_o + \ddot{r}_1 = g_b + \delta g_1 + g_2. \tag{26}$$

Given that $\ddot{r}_o = g_b$ (the Friedmann equation), then

$$\ddot{r}_1 = \delta g_1 + g_2. \tag{27}$$

Substituting eqs. 27, 21 and 20 into eq. 12, the growth equation becomes

$$\ddot{\Delta} + 2\frac{\dot{x}}{x}\dot{\Delta} = \frac{3\Omega_m}{2x^3}\Delta - \frac{3g_2}{x\lambda_c} \tag{28}$$

with g_2 given by eq. 25 supplemented by eqs. 23, 21 and 18. This is the basic equation for the evolution of spherically symmetric over- or under-dense regions of the Universe in the context of the assumed two-field theory of modified dynamics. If $a_o \to 0$, then the final term (the MOND field g_2) vanishes and the equation is identical to eq. 15 for the Newtonian growth of perturbations. The second term dominates only when $|g_b| < a_o$.

This second order differential equation is non-linear, even in the regime where Δ is small. The non-linear term in eq. 28 (or in g_2 given by eq. 25) only becomes important when $g_b \to 0$, but this does happen within a Hubble time in a Universe having a cosmological constant comparable to $H_o{}^2$. It is also noteworthy that, unlike the Newtonian case, the growth of an over-density depends upon the comoving scale. This is primarily because smaller comoving scales enter the MOND regime earlier; specifically, the scale factor at which a fluctuation enters the MOND regime, given by the condition that $|g_b| = a_o$, is

$$x_c = \sqrt{\frac{\Omega_m \lambda_c}{2a_o}} H_o \qquad (29)$$

in the matter-dominated regime.

4. Numerical results

4.1. The growth of inhomogeneities

Using a fourth order Runge-Kutta technique, I have numerically integrated eq. 28 for the growth of over-densities in McGaugh's (2000) vacuum energy dominated baryonic Universe: $\Omega_m = \Omega_b = 0.034$, $\Omega_\Lambda = 1.01$, h = 0.75. The calculations are initiated at $x = 1.37 \times 10^{-3}$ or at a redshift of 730 which is roughly the epoch of matter-radiation equality (in this low density Universe matter-radiation equality occurs after hydrogen recombination). In all calculations I take $a_o = 1.2 \times 10^{-8}$ cm s^{-2} (scaled to $H_o = 75$ km s^{-1} Mpc^{-1}) as determined by fitting to the observed rotation curves of nearby galaxies (Begeman et al. 1991). Smaller values of the coupling between the Newtonian and MOND fields, β, yield more rapid growth. As $\beta \to 0$ the background field effect vanishes and only the peculiar accelerations enter the MOND equation. In that case the non-linear term in the growth equation always dominates and the growth of fluctuations is much too rapid. With respect to matching the form of galaxy rotation curves β should lie between 2 and 4. In all calculations described below I take $\beta = 3.5$.

Fig. 1 illustrates the growth of spherically symmetric over-densities having comoving radii of 20, 40 and 80 Mpc. The initial over-density is assumed to be $\Delta_i = 2 \times 10^{-5}$ which

is comparable to the COBE-normalized fluctuation amplitude on these scales in standard cosmology (Bunn & White 1997). The solid lines follow the MOND evolution of Δ as a function of scale factor (eq. 28). The dotted line is the standard Newtonian evolution (eq. 15).

It is evident that, in the standard model of structure formation by gravitational instability, a region with this initial over-density could not grow to the non-linear regime by the present epoch. The COBE-normalized fluctuations in a baryonic universe, with Ω_b of a few 10^{-2}, could not possibly develop into the observed large scale structure. On the other hand, with modified dynamics, the growth becomes nearly exponential from the time a region enters the low acceleration regime up to the point where the cosmological term dominates in the Friedmann equation. The most rapid growth occurs at the point where $g_b \to 0$ and the non-linear term dominates; thus the cosmological constant, by permitting the background cosmological acceleration to vanish, actually promotes the rapid growth of fluctuations.

The development of non-linear structure on the scale of tens of Mpc would certainly seem possible. The fact that the smaller scale fluctuations enter the non-linear regime earlier is consistent with a hierarchical scheme (with galaxies forming before clusters), as suggested in Sanders (1998). However, unlike the scenario sketched there, the underlying Hubble flow remains intact.

4.2. The power spectrum of fluctuations

Through use of eq. 28 it is straightforward to determine the present power spectrum of fluctuations generated by MOND growth. The initial COBE-normalized power spectra is provided by CMBFAST (Seljak & Zeldarriaga 1996) at the epoch of matter-radiation equality (z = 730) in the McGaugh universe (assuming the primordial fluctuation spectrum to be standard Harrison-Zeldovitch, n=1). A measure of over- (under-) density on various scales is the mass-variance, $\Delta_k{}^2$, per unit interval in log wavenumber; this is related to the power spectrum as

$$\Delta_k{}^2 = 2\pi^2 k^3 P(k) \tag{30}$$

(Padmanabhan 1993). I follow the evolution of these over-densities with amplitude Δ_k on a sub-horizon scale again by numerical integration of eq. 28. Super-horizon growth of large regions is followed crudely by considering the density evolution of sub-universes with the curvature constant appropriate to a given initial Δ_k. When a spherical region enters the horizon, eq. 28 is immediately applied for the subsequent growth. This method is not precise for those regions which enter the horizon in the matter dominated era ($z < 700$)

but should yield an accurate representation of the MOND power spectrum for smaller scale fluctuations ($k > 0.01$).

I convert the final over-densities back to the power spectrum by again applying eq. 30. The resulting power spectrum at the present epoch is shown by the solid curve in Fig. 2. The oscillations with wave number are the relic of the acoustic oscillations frozen into the plasma at the epoch of recombination (in dark matter-dominated models these are suppressed). Also shown in the same figure is the power spectrum that would result from the identical initial spectrum but with only Newtonian growth (dotted curve). The abrupt decrease in power where $k > 0.05$ is due to Silk damping (photon diffusion) of the fluctuations on scales smaller than about 20 Mpc by the epoch of recombination (Silk 1968).

Even with Silk damping there is large amplification of the power spectrum at these small scales due to effect of modified dynamics. On scales smaller than 10 Mpc the structure has already become highly non-linear ($\Delta_k > 1$), and the calculated power spectrum is not to be trusted.

Also shown on the same figure is the zero redshift power spectrum of fluctuations in the context of the standard ΛCDM model. This, less the oscillations, is very similar to the MOND power spectrum on currently measurable scales ($k > 0.01$). In other words, MOND in a pure baryonic universe mimics quite closely the power spectrum in this favored dark matter model on scales of 10 to 100 Mpc; i.e., the phenomenological successes of the standard model are retained. In particular, the usual measure of the amplitude of inhomogeneities in the present Universe, σ_8 is found to be 1.5 from the MOND power spectrum, which is about 50% larger than that of the ΛCDM model.

4.3. Peculiar velocities

One may also consider the predicted peculiar velocities as a test of this scenario (bearing in mind the obvious dangers of assumed spherical symmetry and non-linearity on the scale of 10 to 20 Mpc).

The estimation of peculiar velocities follows from the continuity equation as in the usual treatment (Peebles 1999). Taking the first time derivative of eq. 11, we find

$$\dot{r}_1 = -\frac{r_o \dot{\Delta}}{3} - \frac{\Delta r_o H_o}{3}. \tag{31}$$

The peculiar velocity at $r = r_o + r_1$ is then just

$$V_p = (\dot{r}_o + \dot{r}_1) - H_o(r_o + r_1), \tag{32}$$

which, again after use of eq. 11, becomes

$$V_p = -r_o \dot{\Delta}/3. \tag{33}$$

On the comoving scale of the Virgo cluster ($Hr_o \approx 1300$ km/s), the over-density of galaxies is roughly $\Delta \approx 2$ (Strauss et al. 1992). Choosing an initial Δ on this comoving scale to match the present observed over-density of galaxies, I find from numerical integration of eq. 28 that $v_p = 450$ km/s. This is larger than present estimates, although widely disparate values have been reported (Tonry et al. 2000). Although somewhat larger values of the coupling parameter β can result in less vigorous stirring of the local Universe, it is probably premature to tinker with the crude MOND theory described here.

5. Conclusions

Here I show that MOND provides the possibility of overcoming the problem of structure growth in a low-density baryonic Universe. In the context of the simple two-field non-relativistic theory of modified dynamics presented here, we see that when the background deceleration of the Hubble flow over a given scale falls below the critical MOND acceleration, a_o, then the growth of structure on that scale is greatly enhanced relative to the Newtonian expectation. The growth of over-densities on smaller scale is even more enhanced due to the fact that smaller regions enter the MOND regime earlier; the early growth of small-scale fluctuations can compensate for the effect of Silk damping on these scales. Thus the resulting power spectrum, apart from the oscillations, closely resembles that of the favored ΛCDM cosmology. On the scale of galaxies (1.5 Mpc), even though the typical initial over-density is on the order of 2×10^{-10}, the fluctuation grows to the non-linear regime by a redshift of 2.5. Thus MOND would appear not only to explain the observed large scale structure, but also provide a mechanism for early galaxy formation. This is all achieved with the value of a_o determined from galaxy rotation curves. The minimalist MOND theory has not been fine-tuned in any sense to match the observed power spectrum; the single adjustable parameter β lies within the range which is consistent with the observed form of galaxy rotation curves.

Of course, these conclusions depend upon the approximate validity of the assumed theory described in section 2. This theory, and assumptions embodied therein, guarantee that the early, radiation-dominated evolution of the universe is identical to that of the standard model, that the basic Hubble flow is unaffected by MOND, and that fluctuations on the scale of galaxies to super-clusters enter the MOND regime, determined by the background Hubble flow, sufficiently early (but not too early) to assure growth to the present amplitude.

Because of the necessity of such an *ad hoc* theory in the absence of a more fundamental covariant theory, it is perhaps premature to compare in detail the predicted power spectrum or peculiar velocities with observations. In particular, the oscillations (Fig. 2) may not actually be evident in the evolved power spectrum due to non-linear aspects of the theory which are ignored here– specifically, not only individual Fourier components but also over-dense spherical regions cannot be considered in isolation (larger scale peculiar accelerations contribute to the background field). The detailed results shown in Figs. 1 and 2 should be taken as a demonstration that MOND, in a low-density baryonic universe, can provide a vigorous growth of fluctuations– growth which is sufficiently rapid to lead to the large scale structure observed at the current epoch.

Finally, I re-emphasize that the presence of a dynamically significant cosmological constant plays a necessary role in the rapid growth of structure with this version of modified dynamics. The MOND growth of inhomogeneities accelerates at the epoch when $g_b \approx 0$ due to the dominance of the non-linear term in eq. 28 (only possible with a cosmological constant comparable to $H_o{}^2$). This adds a new aspect to an anthropic argument originally given by Milgrom (1989): we are observing the Universe at an epoch when Ω_Λ has only recently emerged as the dominant term in the Friedmann equation because it is only then that structure formation proceeds rapidly.

If the evidence in support of a baryonic-Λ universe continues with further observations of the CMB angular power spectrum, then some unconventional mechanism for the formation of structure must be invoked. Here it is evident that modified dynamics, with a well-documented success in explaining the kinematic observations of galaxies and clusters without dark matter, may also successfully address the problem of structure formation in a low-density baryonic universe.

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Fig. 1.— The growth of spherically symmetric over-densities, Δ, in a low-density baryonic universe as a function of scale factor in the context of the two-field Langrangian theory of MOND. The solid curves correspond to regions with comoving radii of 20, 40 and 80 Mpc. The dotted line is the corresponding Newtonian growth. With MOND smaller regions enter the low-acceleration regime sooner and grow to larger final amplitude. The vertical dashed line indicates the epoch at which the cosmological constant begins to dominate the Hubble expansion in the assumed model cosmology

Fig. 2.— The present power spectrum of fluctuations in the low density baryonic universe where the growth is determined by the MOND modified gravity theory (solid curve). The initial amplitudes are taken from the COBE-normalized provided by CMBFAST at the epoch of matter-radiation equality. Also shown (dotted curve) is the power spectrum which would result from Newtonian growth of the same initial fluctuation spectrum. The dashed curve is the power spectrum in the standard ΛCDM cosmology. In all cases h=0.75.