Weakly nonlinear theory of the Jeans instability in disk galaxies of stars

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Abstract

The reaction of collective oscillations excited in the interaction between aperiodically growing Jeans-type gravity perturbations and stars of a rapidly rotating disk of flat galaxies is considered. An equation is derived which describes the change in the main body of equilibrium distribution of stars in the framework of the nonresonant weakly nonlinear theory. Certain applications of the theory to the problem of relaxation of the Milky Way at radii where two-body relaxation is not effective are explored. The theory, as applied to the Solar neighborhood, accounts for observed features, such as the shape for the velocity ellipsoid of stars and the increase in star random velocities with age.

Subject headings: galaxies: kinematics and dynamics — galaxies: structure — instabilities

1. Introduction

Relaxation of stellar distribution in galaxies is not completely understood yet. Lynden-Bell (1967) and later Shu (1978) proposed violent relaxation in spherical-like (that is, nonrotating) protogalaxies in not virial equilibrium. The associated relaxation for an individual to gain or lose energy occurs on the exceedingly short time much smaller than one typical radial period and well before the rotating galaxy disk is formed. The relaxation, however, does not stop at this stage. There are numerous observations clearly showing that there exists ongoing slow relaxation (on the time scale of $10-20$ rotation periods or even larger) in the rapidly rotating disk of Milky Way's Galaxy (Wielen 1977; Binney & Tremaine 1987, p. 470; Gilmore, King, & van der Kruit 1990). This slow relaxation of the distribution of young stars which were born in the equilibrium disk of the Galaxy results in a randomization of the velocity distribution ("Maxwellianization") and a monotonic increase of the stellar random velocity dispersion ("heating") from about 15 $\mathrm{km\,s^{-1}}$ for the youngest stars to about 40 $\mathrm{km\,s^{-1}}$ for the oldest disk stars with increasing stellar ages from $\sim 10^6$ yr to $\sim 4\,10^9$ yr. Wielen (1977) has found that the observed increase of the velocity dispersion of disk stars with increasing age indicates strongly a significant irregular gravitational field in the galactic disk. The irregular field causes a rapid diffusion of stellar orbits in velocity (and positional) space. The nature of this relaxation should be quite different from the violent relaxation. Various mechanisms for the slow relaxation have been proposed. See, e.g., Grivnev & Fridman (1990) for a review of the problem. In the present work, we elaborate upon the idea of the collective relaxation: unstable gravity perturbations in the disk affect the averaged velocity distribution function. The instabilities and subsequent collective relaxation occur near the equilibrium and the perturbations remain relatively small which makes this process very different from what occurs during violent relaxation.

Apparently, Toomre (1964, p. 1237), Goldreich & Lynden-Bell (1965), Barbanis & Woltjer (1967), Kulsrud (1972), and Jenkins & Binney (1990) have first suggested instabilities as a cause of enhanced relaxation in disk-shaped rapidly rotating galaxies. It was stated that because of its long-range Newtonian forces a self-gravitating medium (a stellar "gas," say) would possess collective properties: collective, or cooperative motions in which all the particles of the system participate. These properties would be manifested in the behavior of small gravity perturbations arising against the equilibrium background. Collective processes are analogous to two-body collisions, except that one particle collides with many which are collected together by some coherent process such as a wave or an unstable perturbation. The collective processes are random, and usually much stronger than the ordinary two-body collisions and leads to a random walk of the particles that rapidly takes the complete system toward thermal equilibrium.

We present (for the first time as far as we are aware) a quantitative theory of relatively slow relaxation on the Hubble time $\sim 10^{10}$ yr of self-gravitating, rapidly rotating stellar disks of flat galaxies toward a thermal quasi-steady state by collective effects. In

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the process a star "collides" with inhomogeneities of a galactic gravitational field which result from the development of the Jeans instability. [2] We find that it successfully accounts for several basic observations of the Milky Way.

2. Equilibrium

In the rotating frame of a disk galaxy, the collisionless motion of an ensemble of identical stars in the plane of the system can be described by the Boltzmann equation for the distribution function $f(\mathbf{r}, \mathbf{v}, t)$ without the integral of collisions (Lin, Yuan, & Shu 1969):

$$\frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial r} + \left(\Omega + \frac{v_\varphi}{r}\right) \frac{\partial f}{\partial \varphi} + \left(2\Omega v_\varphi + \frac{v_\varphi^2}{r} + \Omega^2 r - \frac{\partial \Phi}{\partial r}\right) \frac{\partial f}{\partial v_r} - \left(\frac{\kappa^2}{2\Omega} v_r + \frac{v_r v_\varphi}{r} + \frac{1}{r}\frac{\partial \Phi}{\partial \varphi}\right) \frac{\partial f}{\partial v_\varphi} = 0, \quad (1)$$

where r, φ, z are the galactocentric cylindrical coordinates, the total azimuthal velocity of the stars was represented as a sum of the random v_φ and the basic rotation velocity $V_{\rm rot} = r\Omega$, v_r is the velocity in the radial direction, and the epicyclic frequency $\kappa(r)$ is defined by $\kappa = 2\Omega[1 + (r/2\Omega)(d\Omega/dr)]^{1/2}$. The quantity $\Omega(r)$ denotes the angular velocity of galactic rotation at the distance r from the center, and κ varies from 2Ω for a rigid rotation to Ω for a Keplerian one. Random velocities are small compared with $r\Omega$. Collisions are neglected here because the collision frequency is much smaller than the cyclic frequency Ω. In the kinetic equation (1), $\Phi(\mathbf{r}, t)$ is the total gravitational potential determined self-consistently from the Poisson equation $\nabla^2 \Phi = 4\pi G \int f d\mathbf{v} = 4\pi G n$, where n is the volume density.

The equilibrium state is assumed, for simplicity, to be an axisymmetric and spatially homogeneous stellar disk. The distribution function may also be a function of $\mathbf{r}$, for instance, in the case of an inhomogeneous disk, in which case the theory is significantly complicated (Alexandrov, Bogdankevich, & Rukhadze 1984, p. 425). Secondly, in our simplified model, the perturbation is propagating in the plane of the disk. This approximation of an infinitesimally thin disk is a valid approximation if one considers perturbations with a radial wavelength that is greater than the typical disk thickness. We assume here that the stars move in the disk plane so that $v_z = 0$. This allows us to use the two-dimensional distribution function $f = f(v_r, v_\varphi, t)\delta(z)$ such that $\int f dv_r dv_\varphi dz = \sigma$, where σ is the surface density. We expect that the waves propagating in the disk plane have the greatest influence on the development of structures in the disk. The latter suggestion is strongly supported by numerical simulations (Hohl 1978).

The disk in the equlibrium is described by the following equation:

$$\left(2\Omega v_\varphi + \frac{v_\varphi^2}{r}\right) \frac{\partial f_{\rm e}}{\partial v_r} - \left(\frac{\kappa^2}{2\Omega} v_r + \frac{v_r v_\varphi}{r}\right) \frac{\partial f_{\rm e}}{\partial v_\varphi} = 0, \quad (2)$$

where $\partial f_{\rm e}/\partial t = 0$ and the angular velocity of rotation $\Omega(r)$ is such that the necessary centrifugal acceleration is exactly provided by the central gravitational force $r\Omega^2 = \partial \Phi_{\rm e}/\partial r$. Equation (2) does not determine the equilibrium distribution $f_{\rm e}$ uniquely. For the present analysis we choose $f_{\rm e}$ in the form of the anisotropic Maxwellian (Schwartzschild) distribution

$$f_{\rm e} = \frac{\sigma_{\rm e}}{2\pi c_r c_\varphi} \exp\left(-\frac{v_r^2}{2c_r^2} - \frac{v_\varphi^2}{2c_\varphi^2}\right) = \frac{2\Omega}{\kappa}\frac{\sigma_{\rm e}}{2\pi c_r^2} \exp\left(-\frac{v_\perp^2}{2c_r^2}\right). \quad (3)$$

The Schwarzschild distribution function is a function of the two epicyclic constants of motion $\mathcal{E} = v_\perp^2/2$ and $r_0^2\Omega(r_0)$, where $r_0 = r + (2\Omega/\kappa^2)v_\varphi$. These constants of motion are related to the unperturbed star orbits:

$$r = -\frac{v_\perp}{\kappa}\left[\sin(\phi_0 - \kappa t) - \sin\phi_0\right]; v_r = v_\perp \cos(\phi_0 - \kappa t);$$
$$\varphi = \frac{2\Omega}{\kappa}\frac{v_\perp}{r_0\kappa}\left[\cos(\phi_0 - \kappa t) - \cos\phi_0\right]; v_\varphi \approx r_0 \frac{d\varphi}{dt} + r_0 \frac{v_\perp}{\kappa}\frac{d\Omega}{dr}\sin(\phi_0 - \kappa t) \approx \frac{\kappa}{2\Omega} v_\perp \sin(\phi_0 - \kappa t), \quad (4)$$

where $v_\perp$, ϕ_0 are constants of integration, $v_\perp/\kappa r_0 \sim \rho/r_0 \ll 1$, ρ is the mean epicycle radius, and we follow Lin et al. (1969), Shu (1970), and Griv & Peter (1996) making use of expressions for the unperturbed epicyclic trajectories of stars in the equilibrium central field $\Phi_{\rm e}(r)$. In equations (3) and (4), r_0 is the radius of the circular orbit, which is chosen so that the constant of areas for this circular orbit

[2] The classical Jeans-type instability of small-amplitude gravity disturbances is one of the most frequent and most important instabilities in the stellar subsystems of galaxies. The Jeans instability is driven by a strong nonresonant interaction of the gravity fluctuations with the bulk of the particle population, and the dynamics of Jeans perturbations can be characterized as a fluidlike interaction. Combined with the familiar Lin–Shu–Kalnajs dispersion relation this is a venerable suggestion as to why flat galaxies almost always exhibit spiral structure (Binney & Tremaine 1987, p. 336).

$r_0^2(d\varphi_0/dt)$ is equal to the angular momentum imtegral $M_z = r^2(d\varphi/dt)$, and $v_\perp^2 = v_r^2 + (2\Omega/\kappa)^2 v_\varphi^2$. Also, φ_0 is the position angle on the circular orbit, $(d\varphi_0/dt)^2 = (1/r_0)(\partial\Phi_\mathrm{e}/\partial r)_0 = \Omega^2$. The quantities Ω, κ, and c_r are evaluated at r_0. In equation (3) the fact is used that as follows from equations (4) in a rotating frame the radial velocity dispersion c_r and the azimuthal velocity dispersion c_φ are connected through $c_r \approx (2\Omega/\kappa)c_\varphi$. In the Solar vicinity, $2\Omega/\kappa \approx 1.7$. The distribution function f_e has been normalized according to $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_\mathrm{e} dv_r dv_\varphi = 2\pi(\kappa/2\Omega)\int_0^\infty v_\perp dv_\perp f_\mathrm{e} = \sigma_\mathrm{e}$, where σ_e is the equilibrium surface density. Such a distribution function for the unperturbed system is particularly important because it provides a fit to observations (Lin et al. 1969; Shu 1970). It is this equilibrium that is examined for stability.

3. Collisionless Relaxation

We proceed by applying the standard procedure of the weakly nonlinear (or quasi-linear) approach (Galeev & Sagdeev 1983; Alexandrov et al. 1984, p. 408; Krall & Trivelpiece 1986, p. 512) and decompose the time dependent distribution function $f = f_0(\mathbf{v}, t) + f_1(\mathbf{v}, t)$ and the gravitational potential $\Phi = \Phi_0(r, t) + \Phi_1(\mathbf{r}, t)$ with $|f_1/f_0| \ll 1$ and $|\Phi_1/\Phi_0| \ll 1$ for all $\mathbf{r}$ and t. The functions f_1 and Φ_1 are oscillating rapidly in space and time, while the functions f_0 and Φ_0 describe the slowly developing "background" against which small perturbations develop; $f_0(t = 0) \equiv f_\mathrm{e}$ and $\Phi_0(t = 0) \equiv \Phi_\mathrm{e}$. The distribution f_0 continues to distort as long as the distribution is unstable. Linearizing equation (1) and separating fast and slow varying variables one obtains:

$$\frac{df_1}{dt} = \frac{\partial\Phi_1}{\partial r}\frac{\partial f_0}{\partial v_r} + \frac{1}{r}\frac{\partial\Phi_1}{\partial\varphi}\frac{\partial f_0}{\partial v_\varphi}, \qquad (5)$$

$$\frac{\partial f_0}{\partial t} = \langle\frac{\partial\Phi_1}{\partial r}\frac{\partial f_1}{\partial v_r} + \frac{1}{r}\frac{\partial\Phi_1}{\partial\varphi}\frac{\partial f_1}{\partial v_\varphi}\rangle, \qquad (6)$$

where d/dt means the total derivative along the star orbit (4) and $\langle\cdots\rangle$ denotes the time average over the fast oscillations. To emphasize it again, we are concerned with the growth or decay of small perturbations from an equilibrium state.

In the epicyclic approximation, the partial derivatives in equations (5) and (6) transform as follows (Lin et al 1969; Shu 1970; Griv & Peter 1996):

$$\frac{\partial}{\partial v_r} = v_r\frac{\partial}{\partial\mathcal{E}} - \frac{2\Omega}{\kappa}\frac{v_\varphi}{v_\perp^2}\frac{\partial}{\partial\phi_0}; \frac{\partial}{\partial v_\varphi} = \left(\frac{2\Omega}{\kappa}\right)^2 v_\varphi\frac{\partial}{\partial\mathcal{E}} + \frac{2\Omega}{\kappa}\frac{v_r}{v_\perp^2}\frac{\partial}{\partial\phi_0}. \qquad (7)$$

To determine oscillation spectra, let us consider the stability problem in the lowest WKB approximation: the perturbation scale is sufficiently small for the disk to be regarded as spatially homogeneous. This is accurate for short wave perturbations only, but qualitatively correctly even for perturbations with a longer wavelength, of the order of the disk radius R. In this local WKB approximation in equations (5) and (6), assuming the weakly inhomogeneous disk, the perturbation is selected in the form of a plane wave (in the rotating frame):

$$f_1, \Phi_1 = \frac{1}{2}\delta f, \delta\Phi\left(e^{ik_r r + im\varphi - i\omega_* t} + \mathrm{c.\,c.}\right), \qquad (8)$$

where δf, $\delta\Phi$ are amplitudes that are constant in space and time, m is the nonnegative azimuthal mode number, $\omega_* = \omega - m\Omega$ is the Doppler-shifted wave-frequency, and $|k_r|R \gg 1$ (Griv & Peter 1996). The solution in such a form represents a spiral wave with m arms whose shape in the plane is determined by the relation $k_r(r - r_0) = -m(\varphi - \varphi_0)$. With φ increasing in the rotation direction, we have $k_r > 0$ for trailing spiral patters, which are the most frequently observed among spiral galaxies. A change of the sign of k_r corresponds to changing the sense of winding of the spirals, i.e., leading ones. With $m = 0$, we have the density waves in the form of concentric rings that propagate away from the center when $k_r > 0$ or toward the center when $k_r < 0$.

In equation (5) using the transformation of the derivatives $\partial/\partial v_r$ and $\partial/\partial v_\varphi$ given by equations (7), one obtains the solution

$$f_1 = \int_{-\infty}^{t} dt'\mathbf{v}_\perp\frac{\partial\Phi_1}{\partial\mathbf{r}}\frac{\partial f_0}{\partial\mathcal{E}}, \qquad (9)$$

where $f_1(t' = -\infty) \to 0$. In this equation making use of the time dependence of perturbations in the form of equation (8) and the unperturbed trajectories of stars given by equations (4) in the exponential factor, it is straightforward to show that

$$f_1 = -\Phi_1(r_0)\frac{\partial f_0}{\partial\mathcal{E}}\sum_{l=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} l\kappa\frac{e^{i(n-l)(\phi_0-\zeta)}J_l(\chi)J_n(\chi)}{\omega_* - l\kappa}, \qquad (10)$$

where $J_l(\chi)$ is the Bessel function of the first kind of order l, $\chi = k_* v_\perp/\kappa \sim k_*\rho$, $k_* = k\{1 + [(2\Omega/\kappa)^2 - 1]\sin^2\psi\}^{1/2}$ is the effective wavenumber, ψ is the pitch angle of perturbations, $\tan\psi = k_\varphi/k_r = m/rk_r$, and we used the expansion

$$\exp(\pm ib\sin\phi_0) = \sum_{n=-\infty}^{\infty} J_n(b)\exp(\pm in\phi_0)$$

and the usual Bessel function recursion relation

$$J_{l+1}(\chi) + J_{l-1}(\chi) = (2l/\chi)J_l(\chi).$$

In the equation above the denominator vanishes when $\omega_* - l\kappa \to 0$. This occurs near corotation ($l = 0$) and other resonances ($l = \pm 1, \pm 2, \cdots$). The Lindblad resonances occur at radii where the field $(\partial/\partial\mathbf{r})\Phi_1$ resonates with the harmonics $l = -1$ (inner resonance) and $l = 1$ (outer resonance) of the epicyclic (radial) frequency of equilibrium oscillations of stars κ. Clearly, the location of these resonances depends on the rotation curve and the spiral pattern speed; the higher the m value, the closer in radius the resonances are located (Lin et al. 1969; Shu 1970). In this paper, only the main part of the galactic disk is studied which lies sufficiently far from the resonances: below in all equations $\omega_* - l\kappa \neq 0$.

We substitute the solution (10) into equation (6). Taking into account that the terms $l \neq n$ vanish for axially symmetric functions f_0, after averaging over ϕ_0 we obtain the equation for the slow part of the distribution function:

$$\frac{\partial f_0}{\partial t} = i\pi \sum_{\mathbf{k}} \sum_{l=-\infty}^{\infty} |\Phi_{1,\mathbf{k}}|^2 \frac{\partial}{\partial v_\perp} \frac{k_*\kappa}{v_\perp \chi} \frac{l^2 J_l^2(\chi)}{\omega_* - l\kappa} \frac{\partial f_0}{\partial v_\perp}. \quad (11)$$

As usual in the weakly nonlinear theory, in order to close the system one must engage an equation for $\Phi_{1,\mathbf{k}}$. Averaging over the fast oscillations, we have

$$(\partial/\partial t)|\Phi_{1,\mathbf{k}}|^2 = 2\Im\omega_*|\Phi_{1,\mathbf{k}}|^2, \quad (12)$$

where suffixes $\mathbf{k}$ denote the $\mathbf{k}$th Fourier component.

Equations (11) and (12) form the closed system of weakly nonlinear equations for Jeans oscillations of the rotating homogeneous disk of stars, and describe a diffusion in velocity space. The spectrum of oscillations and their growth rate are (Griv, Rosenstein, Gedalin, & Eichler 1999a; Griv, Gedalin, Eichler, & Yuan 2000a)

$$\frac{k^2 c_r^2}{2\pi G\sigma_0|k|} = -\sum_{l=-\infty}^{\infty} l\kappa \frac{e^{-x} I_l(x)}{\omega_* - l\kappa}, \quad (13)$$

and $\Im\omega_{*,\mathrm{J}} \approx \sqrt{4\pi G\sigma_0 e^{-x} I(x)/x} \lesssim \Omega$, respectively. In the Solar vicinity, $\Omega \approx 3\,10^{-8}$ yr^{-1}. Here, $I_l(x)$ is a Bessel function of an imaginary argument with its argument $x \approx k_*^2\rho^2$ and $\rho = c_r/\kappa$ is now the mean epicyclic radius. A very important feature of the instability under consideration is the fact that it is aperiodic (the real part of the wavefrequency vanishes in a rotating frame we are using). Usually the quasi-linear theory is applied when the growth rate is small compared with the real part of the wavefrequency as for the case of the resonance interaction $\omega = \mathbf{k}\cdot\mathbf{v}$, where $\mathbf{v}$ is the velocity of the particle involved in the interaction. However, the theory can be applied also to aperiodic instabilities (Shapiro & Shevchenko 1963; Alexandrov et al. 1984, p. 420; Krall & Trivelpiece 1986, p. 531). A further simplification results from restricting the frequency range of the waves examined by taking the low frequency limit ($|\omega_*|$ less than the epicyclic frequency of any disk star). In the opposite case of the high perturbation frequencies, $|\omega_*| > \kappa$, the effect of the disk rotation is negligible and therefore not relevant to us. This is because in this "rotationless" case the star motion is approximately rectilinear on the time and length scales of interest which are the wave growth/damping periods and wavelength, respectively (Alexandrov et al 1984, p. 113). Thus, the terms in series (10)–(13) for which $|l| \geq 2$ may be neglected, and consideration will be limited to the transparency region between the turning points in a disk (between the inner and outer Lindblad resonances). In this case, in equations (10)–(13) the function $\Lambda(x) = \exp(-x)I_1(x)$ starts from $\Lambda(0) = 0$, reaches a maximum $\Lambda_{\max} < 1$ at $x \approx 0.5$, and then decreases. Hence, the growth rate has a maximum at $x < 1$ (see Griv, Yuan, & Gedalin 1999b, Fig. 2 in their paper).

In general, the growth rate of the Jeans instability is high $|\Im\omega_{*,\mathrm{J}}| \sim \Omega$; perturbations with wavelength $\lambda_\mathrm{J} \approx 2\pi\rho$ have the fastest growth rate (Morozov 1981; Griv & Peter 1996; Griv et al. 1999a). In the Solar vicinity of the Galaxy, $\lambda_\mathrm{J} = 2 - 4$ kpc.

4. Astronomical Implications

As an application of the theory we investigate the relaxation of low frequency and Jeans-unstable, $|\omega_*| < \kappa$ and $\omega_*^2 < 0$, respectively, oscillations in the homogeneous galactic disk. Indeed, already in the 1940s it was observed that in the Solar neighborhood the random velocity distribution function of stars with an age $t \gtrsim 10^8$ yr is close to a Schwarzschild distribution — a set of Gaussian distributions along each coordinate in velocity space, i.e., close to equilibrium along each coordinate (Chandrasekhar 1960; Ogorodnikov 1965; Binney & Tremaine 1987, p. 471). In addition, older stellar populations have a higher velocity dispersion than younger ones.[3] On the other hand, a simple calculation of the relaxation time of the local disk of the Milky Way due to pairwise star–star encounters brings the standard value $\sim 10^{14}$ yr (Chandrasekhar 1960), which considerably exceeds the lifetime of the universe. According to our approach, collisionless relaxation does play a determining role in the evolution of stellar populations of the Galactic disk.

Evidently, the unstable Jeans oscillations must influence the distribution function of the main, nonresonant part of stars in such a way as to hinder the wave excitation, i.e., to increase the velocity dispersion. This is because the Jeans instability, being essentially a gravitational one, tends to be stabilized by random motions (Toomre 1964; Shu 1970; Bertin 1980; Morozov 1981; Griv & Peter 1996). Therefore, along with the growth of the oscillation amplitude, random velocities must increase at the expense of circular motion, and finally in the disk there can be established a quasi-stationary distribution so that the Jeans-unstable perturbations are completely vanishing and only undamped Jeans-stable waves remain.[4]

In the following, we restrict ourselves to the most "dangerous," in the sense of the loss of gravitational stability, long-wavelength perturbations, χ^2 and $x^2 \ll 1$ (see the explanation after eqn. [13]). Then in equations (10)–(13) one can use the expansions $J_1^2(\chi) \approx \chi^2/4$ and $e^{-x}I_1(x) \approx (1/2)x - (1/2)x^2 + (5/16)x^3$. Equation (11) takes the simple form

$$\frac{\partial f_0}{\partial t} = D\frac{\partial^2 f_0}{\partial v_\perp^2}, \tag{14}$$

where $D = (\pi/16\kappa^2)\sum_{\mathbf{k}} k_*^2 \Im\omega_{*,\mathrm{J}}|\Phi_{1,\mathbf{k}}|^2$, $\Im\omega_{*,\mathrm{J}} > 0$, and both $\Im\omega_{*,\mathrm{J}}$ and $\Phi_{1,\mathbf{k}}$ are functions of t. As is seen, the velocity diffusion coefficient for nonresonant stars D is independent of $v_\perp$ (to lowest order). This is a qualitative result of the nonresonant character of the star's interaction with collective aggregates.

By introducing the standard definitions $d\tau/dt = D(t)$ and $d/dt = (d\tau/dt)(d/d\tau)$, equations (12) and (14) are rewritten as follows:

$$\frac{\partial f_0}{\partial \tau} - \frac{\partial^2 f_0}{\partial v_\perp^2} = 0, \quad \frac{\partial D}{\partial \tau} = 2\Im\omega_{*,\mathrm{J}}, \tag{15}$$

which has the solution

$$f_0(v_\perp, \tau) = \frac{\mathrm{const}}{\sqrt{\tau}} \exp\left(-\frac{v_\perp^2}{4\tau}\right). \tag{16}$$

(We have taken into account the observations that the distribution of newly born stars is close to the δ-function distribution, $f_0(\mathbf{v}_\perp, t=0) = \delta(\mathbf{v}_\perp)$; Grivnev & Fridman 1990.) As is seen from equation (16), during the development of the Jeans instability, the Schwarzschild distribution of random velocities (a Gaussian spread along v_r, v_φ coordinates in velocity space) is established. The energy of the oscillation field $\propto \sum_{\mathbf{k}} |\Phi_{1,\mathbf{k}}|^2$ plays the role of a "temperature" T in the nonresonant-particle distribution. As the perturbation energy increases, the initially monoenergetic distribution spreads (f_0 becomes less peaked), and the effective temperature grows with time (a Gaussian spread increases): $T = 2\tau \propto \int D(t)dt \propto \int \sum_{\mathbf{k}} k_*^2 \Im\omega_{*,\mathrm{J}}|\Phi_{1,\mathbf{k}}|^2 dt$.

From the above, this mechanism increases the velocity dispersion of stars in Milky Way's disk after they are born. Subsequently, sufficient velocity dispersion prevents the Jeans instability from occuring. The "diffusion" of nonresonant stars takes place because they gain mechanical (oscillatory) energy as the instability develops. The velocity diffusion, however, presumably tapers off as Jeans stability is approached: the radial velocity dispersion c_r becomes greater than the critical one $c_{r,\mathrm{crit}} \approx (2\Omega/\kappa)c_\mathrm{T}$, where c_T is the well-known Toomre's critical velocity dispersion to suppress the instability of axial symmetric gravity perturbations (Morozov 1981; Griv & Peter 1996; Griv et al. 1999a). Thus, the true time scale for relaxation in the Milky Way may be much shorter than its standard value $\sim 10^{14}$ yr for the classical Chandrasekhar–Ogorodnikov collisional relaxation; it may be of the order $(\Im\omega_{*,\mathrm{J}})^{-1} \gtrsim \Omega^{-1} \gtrsim 10^9$ yr, i.e., comparable with 10 periods of the Milky Way rotation in the Solar vicinity. The above relaxation time is in fair agreement with both observations (Wielen 1977; Knude, Winther, & Schnedler-Nielsen 1987; Gilmore

[3]The age dependence of velocity dispersions for stellar populations has always been of particular interest, because the form of this relationship allows us to judge whether was any relaxation in the galactic disk and even to determine the mechanism that was responsible for increase in random star velocities.

[4]In turn, the Jeans-stable perturbations are subject to a resonant Landau-type instability (Griv et al. 2000a).

et al. 1990; Grivnev & Fridman 1990; Meusinger, Stecklum, & Reimann 1991) and N-body simulations of Milky Way's disk (Hohl 1971; Sellwood & Carlberg 1984; Griv, Gedalin, Liverts et al. 2000b).

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