

# CHAOTIC REHEATING

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## Abstract

We discuss the structure of parametric resonance which occurs in the process of reheating after inflation with two interacting scalar fields. It is found that, for the case of a not too large coupling constant, a quasi-homogeneous part of the second, initially subdominant scalar field may not be neglected due to its stochastic growth during inflation. This fact has strong consequences for the reheating stage: dynamics of background fields becomes chaotic after inflation – it consists of subsequent chaotic and regular eras, and a tachyonic instability for inhomogeneous perturbations arises in the quantum reheating problem. This instability may pose a problem for the standard reheating scenario. In order to avoid it, the coupling constant should be either sufficiently small, or very large.

## 1 Introduction

The simplest versions of the inflationary scenario of the early Universe, which produced definite predictions about the total density of matter and the power spectrum of adiabatic perturbations in the present-day Universe (later verified by observations), are based on the hypothesis that our Universe had passed through a period of quasi-exponential expansion (the de Sitter, or inflationary stage) about 14 billion years in the past, before the hot radiation-dominated (RD) Friedmann-Robertson-Walker (FRW) era occurred. This inflationary stage was driven by (at least) one effective scalar field (named inflaton) which was evolving sufficiently slow during inflation. Quantum vacuum fluctuations of this field induced by space-time curvature during the de Sitter stage are the source of spatial inhomogeneities of space-time metric and matter density which have led to formation of all structures in the present Universe including compact objects.

However, to provide required smallness of initial perturbations, an inflaton field should be very weakly coupled to other quantum fields including "usual" ones (i.e., those of the standard model of strong, electromagnetic and weak interactions). Also, immediately after the end of an inflationary stage, all energy density in the Universe is concentrated in the quasi-homogeneous "cold" inflaton field  $\phi$ . So, to make matching with the subsequent hot RD stage possible, it is necessary, first, to transmit the energy density to other quantum fields of matter and, second, to heat matter in the Universe. To achieve the first aim, the inflaton should be somehow coupled to other quantum fields. The simplest way to model a coupling to bosonic fields of "usual" matter is to introduce a second scalar field  $\xi$  interacting with the inflaton  $\phi$ . Of course, fermion fields and boson fields of a higher spin have to be added, too. Further, we assume the quartic interaction  $V_{int} = \frac{g^2}{2} \phi^2 \xi^2$ .

In the perturbative regime, when occupation numbers of inhomogeneous modes of all fields are small, energy transfer from the inflaton to other fields occurs through the process  $\phi\phi \rightarrow \xi\xi$  which is rather slow. The same process, as well the process  $\phi\phi \rightarrow \phi\phi$  (in the case of the  $\frac{\lambda\phi^4}{4}$  inflaton self-interaction) and other non-linear processes, leads to reheating of matter, too. However, it was found in 90th that sometimes the energy transfer and reheating may proceed much quicker if they occur in a regime of the broad parametric resonance, see <sup>1)-8)</sup> (a narrow parametric resonance is usually not effective due to expansion of the Universe). The structure of the resonance is rather complicated <sup>8)</sup>: there is a stability - instability chart in the variables  $(\frac{g^2}{\lambda}, k)$  where  $k$  is the inverse wave-length of a  $\xi$  - mode.

In the present work, we will show that this picture actually takes place for  $\kappa = \frac{g^2}{\lambda} \gg 1$  only. As we will see, in the opposite case the conjecture of a single field driven inflation is not correct. This results in a drastic change of dynamics during a reheating stage. Dynamical chaos arises in the quasi-homogeneous background problem, that is a general feature for Hamiltonian systems with a large number of degrees of freedom (see, e.g., <sup>9),10)</sup> regarding chaotic dynamics in isotropic cosmology). There is even more dramatic change in the quantum inhomogeneous reheating problem: a tachyonic instability arises for sufficiently large  $\kappa$  and long wavelengths. It may present a problem for the standard reheating model because of generation of strong inhomogeneities. To exclude a tachyonic instability without qualitative change of the theory, one need the coupling constant  $\kappa$  to be small as compared with unity.

## 2 Inflationary stage

Our model of the isotropic flat Universe is similar to that introduced in <sup>8)</sup>:

$$\ddot{\phi} + 3H\dot{\phi} + \lambda\phi^3 + g^2\xi^2\phi = 0, \quad (1)$$

$$\ddot{\xi} + 3H\dot{\xi} + g^2\phi^2\xi = 0, \quad (2)$$

$$H^2 = \frac{8\pi}{3M_P^2} \left( \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\xi}^2 + \frac{\lambda}{4}\phi^4 + \frac{g^2}{2}\phi^2\xi^2 \right). \quad (3)$$

The usual point of view <sup>8)</sup> is that the inflationary stage is driven by the field  $\phi$  and the dynamics of field  $\xi$  is not important until the inflation is finished. Actually a criterion of validity of this statement is given by a value of the effective mass of the field  $\xi$ . If initial conditions for (1 - 3) are such that the mass of  $\xi$  is much larger than the Hubble parameter  $H$ , inflation becomes single-field driven soon after its beginning, and the scenario described in <sup>8)</sup> is realized. In fact, this condition limits the coupling constant  $\kappa = g^2/\lambda$ : if the inequality  $m_\xi^2 = g^2\phi^2 \gg H^2 = 2\pi G\lambda\phi^4/3$  remains valid until the end of inflation (when  $G\phi^2 \sim 1$ ), then  $\kappa \gg 1$  ( $M_P^2 = G^{-1}$ ).

What happens when the coupling constant  $\kappa$  is not large? Let us suppose that  $\xi$  is never dynamically important in this regime, too:

$$g^2\xi^2 \ll \lambda\phi^2. \quad (4)$$

Here  $\phi$  and  $\xi$  may be understood both as operators and stochastic quasi-classical quantities. Furthermore, let us consider the period of inflation when fluctuations in  $\phi$  may be neglected, so that the background field  $\phi$  may be considered as a deterministic quantity. Its evolution as

a function of  $\tau = \ln(a(t)/a_f)$ , where  $a_f$  is the value of the scale factor at the end of inflation (thus,  $\tau < 0$ ), is given by the equation

$$\phi^2 = -\frac{\tau}{\pi G}, \quad |\tau| \gg 1. \quad (5)$$

Then the evolution of  $\xi$  may be calculated in the Gaussian approximation (i.e., assuming that its distribution is Gaussian). The following inequality is valid in the regime under consideration:  $m_\xi^2 = g^2\phi^2 \ll H^2 = 2\pi G\lambda\phi^4/3$ . Since  $G\phi^2 \gg 1$  during inflation, it is satisfied, if  $g^2$  is not too much larger than  $\lambda$  (in fact, this regime takes place everywhere on the "phase diagram" of our theory except for the region  $\kappa \gg 1!$ ). The equation for the evolution of  $z \equiv \pi G\langle\xi^2\rangle$  reads:

$$\frac{dz}{d\tau} = -\frac{2m_\xi^2 z}{3H^2} + \frac{GH^2}{4\pi} = \frac{g^2 z}{\lambda\tau} + \frac{\lambda\tau^2}{6\pi^2}. \quad (6)$$

Let us take  $z = 0$  for  $|\tau| = \tau_0$  as an initial condition. Then

$$z = C|\tau|^{g^2/\lambda} - \frac{\lambda|\tau|^3}{6\pi^2\left(3 - \frac{g^2}{\lambda}\right)}, \quad C = \frac{\lambda\tau_0^{3 - \frac{g^2}{\lambda}}}{6\pi^2\left(3 - \frac{g^2}{\lambda}\right)}. \quad (7)$$

As we can see, there is a stochastic growth of  $\langle\xi^2\rangle$  due to the second term on the right-hand side. Thus, we come to an inconsistency with the conjecture (4): it is impossible to neglect the influence of  $\xi$  on the large-scale inflationary dynamics, and one needs to solve a two-field problem if  $\kappa$  is not large as compared with unity. This fact results in a radical change of reheating dynamics, too. Now we are going to discuss this question in detail.

### 3 Reheating: solution of a homogeneous problem

It is convenient <sup>8)</sup> to introduce new "conformal" variables  $\tilde{\phi} = a\phi$ ,  $\tilde{\xi} = a\xi$ ,  $\eta = \int \frac{dt}{a}$  soon after the end of inflation. The equations (1 - 3) take the following form in these variables (a prime denotes the derivative with respect to the conformal time  $\eta$ ):

$$\tilde{\phi}'' - \frac{a''}{a}\tilde{\phi} + \lambda\tilde{\phi}^3 + g^2\tilde{\xi}^2\tilde{\phi} = 0, \quad (8)$$

$$\tilde{\xi}'' - \frac{a''}{a}\tilde{\xi} + g^2\tilde{\phi}^2\tilde{\xi} = 0, \quad (9)$$

$$(a')^2 = \frac{8\pi}{3M_P^2} \left( \frac{1}{2} \left( \tilde{\phi}' - \frac{a'}{a}\tilde{\phi} \right)^2 + \frac{1}{2} \left( \tilde{\xi}' - \frac{a'}{a}\tilde{\xi} \right)^2 + \frac{\lambda}{4}\tilde{\phi}^4 + \frac{g^2}{2}\tilde{\phi}^2\tilde{\xi}^2 \right). \quad (10)$$

After the end of the inflationary stage, terms containing  $\frac{a''}{a}$  and  $\frac{a'}{a}$  may be neglected. Then the system (8-10) is drastically simplified making the problem almost Hamiltonian:

$$\tilde{\phi}'' + \lambda\tilde{\phi}^3 + g^2\tilde{\xi}^2\tilde{\phi} = 0, \quad (11)$$

$$\tilde{\xi}'' + g^2\tilde{\phi}^2\tilde{\xi} = 0, \quad (12)$$

$$(a')^2 = \frac{8\pi}{3M_P^2} \left( \frac{1}{2} (\tilde{\phi}')^2 + \frac{1}{2} (\tilde{\xi}')^2 + \frac{\lambda}{4}\tilde{\phi}^4 + \frac{g^2}{2}\tilde{\phi}^2\tilde{\xi}^2 \right) = \frac{8\pi}{3M_P^2} E. \quad (13)$$

Structure of its solutions can easily be understood using the projection of its phase space onto the plain  $\tilde{\phi}' = 0$ ,  $\tilde{\xi}' = 0$ . The region where the system moves is bounded by the curve

$$\tilde{\xi} = \pm \frac{1}{g|\tilde{\phi}|} \sqrt{2 \left( E - \frac{\lambda}{4} \tilde{\phi}^4 \right)}. \quad (14)$$

There are two essentially different regimes of the system behaviour. The first is realized when  $\tilde{\xi}$  and  $\tilde{\phi}$  are of the same order. Then both fields oscillate. The system moves somewhere in the region near the origin, and its motion is substantially chaotic because of chaotic character of energy redistribution between  $\tilde{\phi}$  and  $\tilde{\xi}$  degrees of freedom. For larger  $\kappa$ , chaos becomes stronger. We call these periods "chaotic eras".

The second regime is realized when a strong fluctuation of  $(\tilde{\xi}')^2$  occurs, and the system leaves the central region percolating into a "tube" under or below this region. This tube is narrow, so the system cannot live inside it during an infinite time. The lower the energy of the system  $E$  and the higher the control parameter  $\kappa$  are, the larger is the number of oscillations inside the tube. We call these periods "regular" (because of the possibility of finding an approximate solution for  $\tilde{\phi}$  and  $\tilde{\xi}$ ) or "double - period eras" (because of the analogy with intermittency in hydrodynamics).

## 4 Special solution with synchronous oscillations

Let us consider a particular case when initial conditions are chosen in such a way that homogeneous components of  $\tilde{\phi}$  and  $\tilde{\xi}$  are proportional to each other at the end of inflation. Of course, generically it is not so. However, we will see below that this special solution correctly reproduces almost all features characteristic for a generic case.

Examining the equations (11,12) it is easy to find that if  $\tilde{\phi} = \alpha \tilde{\xi}$ , then the constant  $\alpha$  is fixed:

$$\alpha^2 = \frac{g^2}{g^2 - \lambda} = \frac{\kappa}{\kappa - 1}, \quad (15)$$

and we have the following equation for  $\tilde{\phi}$ :

$$\tilde{\phi}'' + g^2 \tilde{\phi}^3 = 0 \quad (16)$$

Of course, there is no chaos for this solution. Now we consider fluctuations of  $\tilde{\phi}$  and  $\tilde{\xi}$  on this background. Introducing the Fourier mode decomposition as usual, we get:

$$\delta \tilde{\xi}_k'' + (k^2 + g^2 \tilde{\phi}^2) \delta \tilde{\xi}_k + 2g^2 \tilde{\xi} \tilde{\phi} \delta \tilde{\phi}_k = 0, \quad (17)$$

$$\delta \tilde{\phi}_k'' + (k^2 + 3\lambda \tilde{\phi}^2 + g^2 \tilde{\xi}^2) \delta \tilde{\phi}_k + 2g^2 \tilde{\xi} \tilde{\phi} \delta \tilde{\xi}_k = 0. \quad (18)$$

When  $\delta \tilde{\phi}_k$  and  $\delta \tilde{\xi}_k$  modes are strongly coupled to each other, it is not clear how to determine the number of particles as well as Floquet indices. Nevertheless, it is possible to introduce new decoupled perturbations:

$$\Sigma_k = A \delta \tilde{\xi}_k + B \delta \tilde{\phi}_k, \quad (19)$$

$$\Delta_k = C \delta \tilde{\xi}_k + D \delta \tilde{\phi}_k \quad (20)$$

where  $A, B, C, D$  are some constants to be determined. Omitting a simple calculation, we present final equations for these rotated perturbations:

$$\Sigma_k'' + (k^2 + 3g^2 \phi^2) \Sigma_k = 0, \quad (21)$$

$$\Delta_k'' + (k^2 + \frac{2 - \kappa}{\kappa} g^2 \phi^2) \Delta_k = 0. \quad (22)$$

Therefore, first, the presence of  $\xi \neq 0$  results in rotation in the space of field perturbations. Now correct variables are the rotated ones. It should be noted that the definition of fields  $\Sigma_k$ ,  $\Delta_k$  strongly depends on  $\kappa$ , so there is no hope to construct something like a stability-instability chart for arbitrary initial fields  $\tilde{\phi}$  and  $\tilde{\xi}$ .

Second, the field  $\Sigma_k$  can be interpreted as an "inflaton" with the wave vector  $k$ , because the structure of (21) is the same as for the inflaton mode if  $\tilde{\xi} = 0$ .

Third, there is a tachyonic instability for  $\Delta_k$  when  $\kappa > 2$ . It means that there is an avalanche-like creation of excitations for this mode, but these excitations are quite unlike usual particles. Instead, we have a chaotic growth of spatial inhomogeneities. It is interesting to note that this special toy solution gives an opportunity to investigate the system behaviour near the boundary of chaotic instability. The boundary for the instability for this toy solution is sharp – there is no instability for all modes  $k$  if  $\kappa \leq 2$ . On the other hand,  $\kappa > 1$  is necessary for this solution to exist at all.

It turns out that almost all these consequences are valid in a generic case (background fields  $\tilde{\phi}$  and  $\tilde{\xi}$  are both nonzero and independent) if we consider the WKB regime of an inhomogeneous problem. There is only one difference – it is hardly possible to determine an exact boundary of instability in this case. This will be discussed elsewhere.

## 5 Discussion

We see that the structure of resonance in the standard reheating theory (1 - 3) can be more complicated than the picture presented in <sup>8)</sup>. The stability - instability chart exists for  $\kappa \gg 1$  only (i.e., in the broad resonance regime), and there is no possibility of its constructing for not too large  $\kappa$  because of the appearance of chaos in the homogeneous background problem and the necessity to rotate modes. The underlying reason is the stochastic growth of  $\langle \xi^2 \rangle$  during inflation. As a result, quasi-homogeneous backgrounds for both scalar fields  $\phi$  and  $\xi$  are nonzero by the end and after inflation. Thus, chaos occurs in the homogeneous reheating problem, which is a usual situation for Hamiltonian system with a sufficiently large dimension of the phase space. Note that the Toda-Brumer necessary criterion for the global dynamical chaos (a negative determinant of the matrix  $||\partial^2 U(\phi, \xi)/\partial\phi\partial\xi||$ ) is fulfilled for our model. Generically, chaotic mixing in the  $(\tilde{\phi}, \tilde{\xi})$  plane rapidly provides equipartition of energy density between the inflaton ( $\phi$ ) and the other matter field ( $\xi$ ). Thus, the first aim of reheating is achieved.

Evolution of conformally transformed fields  $\tilde{\phi}$  and  $\tilde{\xi}$  consists of consequent chaotic ( $|\tilde{\phi}| \sim |\tilde{\xi}|$ ) and regular ( $|\tilde{\phi}| \ll |\tilde{\xi}|$ ) eras. This phenomenon is similar to intermittency in the theory of nonlinear dynamics. There, it is well known that the number of chaotic eras grows while increasing an external parameter of a system ( $1/\kappa$  in our case). This analogy leads to a conjecture that a critical  $\kappa_c$  exists such that regular eras disappear for  $\kappa < \kappa_c$ .

Dynamics of quantum reheating in the case of nonzero  $\xi(t)$  differs from the picture <sup>8)</sup> even without dynamical chaos in the problem (1-3). The first new feature is a time-dependent rotation in the space of modes  $\delta\tilde{\phi}_k$  and  $\delta\tilde{\xi}_k$ . This effect is not unexpected: while our work was in preparation, papers discussing the same effect have appeared <sup>11),12)</sup>. Its necessity is clear from the physical point of view: the Universe should be filled up with particle-like excitations after a reheating stage. It is well known that the term "particle" is correctly defined for an oscillator with a variable frequency in the WKB-regime only. The correct WKB-solution may not be constructed without mode rotation in our case.

The second, more important, feature is the appearance of tachyonic instability for sufficiently large  $\kappa$  and long wavelengths. This effect is different from that recently found in the theory of reheating in the course of spontaneous symmetry breaking after the end of the hybrid inflation <sup>13),14)</sup>. The latter effect occurs in case the oscillation frequency of background fields immediately after inflation is much more than the Hubble parameter  $H$  at this moment. In our model, this is not the case and there is no spontaneous symmetry breaking. So, initially it was hardly possible to expect a vacuum instability with respect to the creation of tachyonic excitations.

The tachyonic instability in our model is more like an instability in chaotic dynamical systems with a positive largest Lyapunov exponent <sup>1)</sup>. The question arises immediately: is it a desirable or undesirable property? On one hand, this instability is closely connected with chaotic mixing leading to a rapid transfer of energy density from the inflaton to other fields. On the other hand, it produces strong spatial inhomogeneities in the Universe (and inhomogeneously distributed high-energy particles as was in the case of the broad parametric resonance type preheating <sup>1),8)</sup>). Special investigation is needed to track the further fate of these inhomogeneities. At this moment, we want to state only that, to avoid a strong tachyonic instability, the coupling constant  $\kappa$  should be either sufficiently small, or very large (in the latter case, there is no chaos in the homogeneous reheating problem, and the results of <sup>8)</sup> remain valid).

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<sup>1</sup>Regarding long wavelength perturbations, a close point of view was expressed in the recent paper <sup>15)</sup> which appeared when our paper was prepared for publication.