

# Pulsating sdB Stars: A New Approach to Probing their Interiors

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## Abstract.

Horizontal branch stars should show significant differential rotation with depth. Models that assume systematic angular momentum exchange in the convective envelope and local conservation of angular momentum in the core produce HB models that preserve a rapidly rotating core. A direct probe of core rotation is available. The nonradial pulsations of the EC14026 stars frequently show rich pulsation spectra. Thus their pulsations probe the *internal* rotation of these stars, and should show the effects of rapid rotation in their cores. Using models of sdB stars that include angular momentum evolution, we explore this possibility and show that some of the sdB pulsators may indeed have rapidly rotating cores.

## 1. Introduction

First studied in detail by R. Peterson in the mid 1980s (Peterson 1983, 1985a,b), the rapid rotation of some horizontal branch stars remains an enigmatic feature of this phase of evolution. Observations by Behr et al. (2000a,b) show a gap in the horizontal branch of M15 and M13, with two distinct rotation rates on either side. Cooler than about 11000 K, HB stars frequently show rapid rotation ( $v \sin i \approx 40$  km/s) while no rotation rates above 10 km/s appear in the hotter stars.

One class of field stars is clearly related to horizontal branch stars. The subdwarf B (sdB) stars are very hot ( $T_{\text{eff}} \gtrsim 25,000\text{K}$ ), and show high gravity ( $\log g \approx 5.3 - 6.1$ ). Models proposed to explain their origin include common envelope binary evolution (eg. Sandquist et al., 2000, Mengel et al., 1976), variations of mass loss efficiency on the RGB (D’Cruz et al., 1996), and even mass stripping by planetary companions (Soker & Harpaz, 2000). Some proposed origin scenarios make clear predictions about their rotational properties; for example, those that involve binary mergers or common envelope evolution should spin up the sdB product. Extrapolation of the results for Pop. II stars suggests that sdB stars should be slow rotators, with  $v \sin i \lesssim 10\text{km/s}$ . The work by Heber et al. (1999) has shown that at least one star, PG 1605+072, is a more rapid rotator – as expected based on an asteroseismic analysis of the pulsations by Kawaler (1998).

Sills & Pinsonneault (2000) (hereafter SP2K) explored HB rotation and single-star evolution with different assumptions about internal an-



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gular momentum transport. Starting with models that are rotating as solid bodies at the departure from the main sequence, SP2K followed evolution of the models through the helium flash and onto the HB. They studied several limiting cases of angular momentum transport, two of which we follow up on in our study of Pop. I hot HB stars. Both conserve specific angular momentum (angular momentum per unit mass,  $j$ ) in convectively stable regions. In convective regions, convective mixing is assumed to produce either constant angular velocity in the convection zone (Case B) or constant  $j$  (Case D); in both cases, the angular momentum contained within the convection zone is preserved.

SP2K concluded that the best fit to observations of Pop. II HB rotation came from precursors with rapidly rotating cores and constant specific angular momentum in convection zones (i.e. Case D). These models could rotate at the rates seen in the cool HB stars. The slow rotation in the hotter HB stars requires choking of angular momentum transport by chemical composition gradients produced by diffusion - such effects are seen in HB stars (and models) at temperatures above 11,000K (SP2K). All rotating models of HB stars show rapidly rotating cores. Similarly, sdB stars that originate through binary mergers or common envelope evolution should show pathological rotation.

A subset of sdB stars are multiperiodic pulsating stars. The pulsating sdBV stars, if they represent “typical” sdB stars and, by extension, typical HB stars, should allow asteroseismology to address the state of internal rotation of HB stars. In fact, one sdBV star, PG 1605+072, was found to have rapid rotation based on the asteroseismic analysis by Kawaler (1998) – this was later partially verified by Heber et al. (1999) through measurement of rotational broadening of spectral lines. Based on this initial success, it is likely that asteroseismology can be a useful tool to explore important questions of the second parameter problem in globular clusters and the origins of field sdB stars.

The sdB stars probably originate from higher mass progenitors than Pop II HB stars. Stars that are now sdB stars could have had much higher initial angular momenta (Kawaler, 1987). Therefore, we have computed models of rotating RGB and sdB stars using Population I progenitors which sample the various structures that might be relevant for the pulsators. We look at the asteroseismic consequences of these rotation profiles on the observed sdB stars pulsations.

## 2. Evolutionary Model Calculations

The evolution of our stellar models was computed using ISUEVO, a standard stellar evolution code that is optimized for producing models

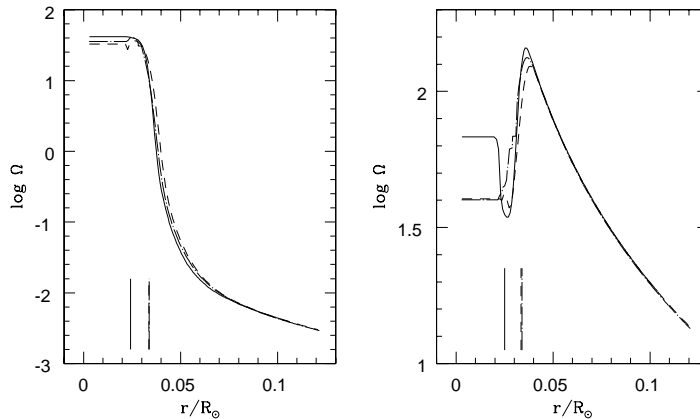
for use in asteroseismology studies. ISUEVO computes models ranging from the ZAMS through the RGB and AGB and beyond that have been used for studies of HB stars (Stobie et al., 1997; van Hoolst et al., 1999; Kawaler, 1998; Kilkenney et al., 1999; Reed et al., 2003).

As we are modeling Population I stars, we assume a metallicity  $Z=0.02$  and an initial mixture that is solar. Model sequences were computed with initial masses of 1.0, 1.5, and 1.8  $M_{\odot}$ , and evolved from the ZAMS until the helium core flash. The core masses at the helium flash for these models was 0.4832 $M_{\odot}$ , 0.4798 $M_{\odot}$ , and 0.4652 $M_{\odot}$  respectively (measured to the midpoint of the hydrogen–burning shell). We follow the conventional technique of computing a ZAHB model (Sweigert & Gross, 1976). For envelopes thinner than 0.0003 $M_{\odot}$ , we had to reduce the thickness of the hydrogen to helium transition zone to preserve the surface abundance of hydrogen as the pre-flash value.

The transport of angular momentum within stars that are evolving up the giant branch is dominated by the deep convective envelope. The distribution of specific angular momentum  $j$  is affected only by slow diffusive processes in the absence of convection (Zahn, 1992). Within convection zones, we follow the procedure of SP2K and examine the two limiting cases of  $j$  transport described above. The angular velocity profiles in our models represent the extreme of no diffusion of  $j$ ; in real stars, the angular velocity profiles will be similar to or shallower than what we report. The total angular momentum within radiative zones is also an upper limit, as diffusive transport of  $j$  usually drains angular momentum from the core into the more slowly spinning envelope. We note that we calculate the angular momentum profile as a separate side calculation, using the local moment of inertia of each mass shell. Since all angular velocities are much smaller than breakup velocities, the structural effects of rotation are second–order perturbations and can safely be ignored in these pilot calculations.

We assume that the initial rotation on the main sequence was as a solid body. The initial rotation rates for our models are taken from Kawaler (1987) – for example, for the 1.8 $M_{\odot}$  model, Kawaler (1987) implies  $v_{\text{rot}} = 185$  km/s,  $J_{\text{init}} = 3.2 \times 10^{50}$ , and  $\Omega = 2.4 \times 10^{-5} s^{-1}$ . Given that our plan is to use these models for asteroseismology, the structure of the internal rotation profile is the focus of our study rather than the predicted surface rotation velocities.

During early RGB evolution, the convective envelope deepens. A steep angular velocity gradient develops quickly as the star reaches the RGB, steepening significantly in a few times  $10^7$  years. Evolution up the RGB means an increase in radius (and moment of inertia) meaning that the outer layers slow down. At the same time, the core is contracting and therefore spinning up. Thus evolution up the RGB steepens



*Figure 1.* Rotation profiles for EHB models. Left: Case B evolution. Right: Case D evolution. Models at three stages of evolution are shown; vertical lines at the bottom corresponding to the transition from pure helium to the helium-depleted core. The profiles show very subtle changes in the outer layers with evolution.

the angular velocity profile. On the RGB the dominant feature of the internal rotation profile is the change in angular velocity that occurs near  $0.27M_{\odot}$  from the center of the model. The rotation profile drops precipitously beyond as the result of draining of angular momentum from the core by the convective envelope.

Horizontal branch stars evolved from Case B RGB models show much smaller surface rotation velocities than those with Case D precursors. All models preserve a rapidly rotating core with roughly constant angular velocity out to about  $0.3M_{\odot}$ . The rotation profiles of models descended from Case D RGB evolution contain relics of the high specific angular momentum of material in the outer layers of the RGB progenitor. The angular velocity profile changes little during EHB evolution - the core does spin down a bit as the angular momentum is mixed, but the inner core continues to spin about a factor of 5 slower than the fastest material. The angular velocity drops from the maximum by a factor of about 10 or so to the surface.

The internal rotation profile remain nearly fixed through the stage of core helium burning. As shown in Figure 1, the rotation profile (with radius) shows the same general features at the start and end of HB evolution. Further evolution results in little additional angular velocity change with radius. This relatively small change in angular velocity profile on the EHB could allow efficient parameterization for studies of their seismic influence.

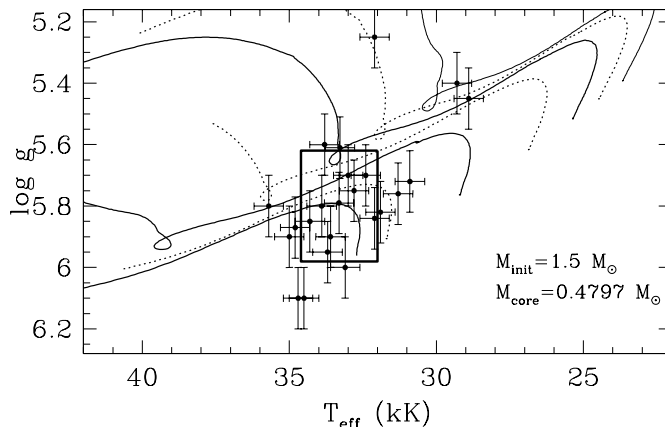


Figure 2. Evolutionary tracks of EHB models. Models derived from the post-helium flash core of a  $1.5 M_{\odot}$ . Alternating tracks can be distinguished by the different line types. Envelope masses are 0.00001, 0.00004, 0.00032, 0.00122, 0.00232, 0.00322, 0.00422  $M_{\odot}$ , running from lower left to upper right. Data from Kilkenny (2001).

### 3. Asteroseismology Probes

Nonradial pulsations in sdB stars provide a window into their interiors, and may test the hypothesis that horizontal branch stars have rapidly rotating interiors. Here we illustrate the process by which we can detect rapid rotation of the stellar interior using currently observed sdBV stars. A future paper will describe our asteroseismic results in more detail and apply the analysis to individual pulsators.

Figure 2 shows an H–R diagram that includes the known sdB stars and sample evolutionary tracks from our sets of models. Given the convergence of the tracks in this plane, it is clear that H–R diagram position alone is insufficient to judge their evolutionary state, with stars that are close to the ZAEHB mingling with stars that have depleted helium in their cores. The lowest gravity EC 14026 stars fall sufficiently far from the ZAEHB that they cannot be core helium–burners for core masses that are consistent with single–star evolution.

Many of the high–gravity pulsating sdB stars show complex pulsations that require a large number of periodicities. Examples include PG 1047+003 Kilkenny et al. (2001) and PG 0014+067 Brassard et al. (2001). Low  $\ell$  nonradial modes have a limited frequency distribution, meaning that some of the pulsators cannot be understood without resorting to either high values of  $\ell$  or to rapid rotation.

Nonradial oscillations are characterised by the eigenfrequency  $\sigma_{nlm}$  corresponding to a spheroidal mode with quantum numbers  $n$ ,  $\ell$ , and  $m$ . The values  $\ell$  and  $m$  refer to the spherical harmonic  $Y_{\ell}^m$ . The sign

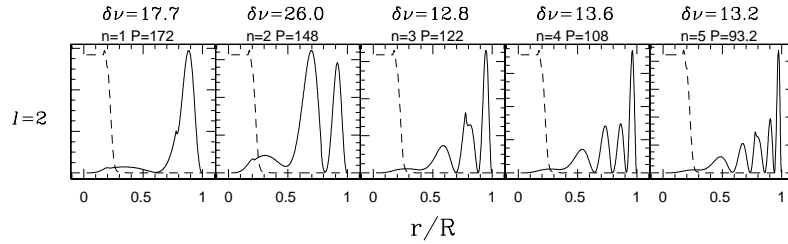


Figure 3. Kernels for a sample of low  $\ell$  nonradial modes in a Case B sdB model. The solid lines represent the kernels, and the dashed line shows the (scaled) value of the angular rotation velocity as a function of fractional radius. The rotational splitting for each mode is given above each panel

of  $m$  gives the direction of propagation of the surface running wave, and therefore there are  $2\ell + 1$  possible values of  $m$  for a given  $\ell$ . For perfect spherical symmetry, the oscillation frequencies are independent of  $m$  and depend only on  $n$  and  $\ell$ . But if rotation is present, the value of the oscillation frequency will depend on  $m$ . For *solid body* rotation the result of rotation is to split modes of different values of  $m$  by a constant frequency that is proportional to the rotation rate.

If a star undergoes *differential* rotation (i.e.  $\Omega = \Omega(r)$ ) then the splitting is given by

$$\sigma_{n\ell m} = \sigma_{n\ell 0} - m \int_0^R \Omega(r) K_{n\ell}(r) dr \quad (1)$$

where  $K_{n\ell}(r)$  is the rotation kernel corresponding to a mode with  $n$  and  $\ell$ . The observed splitting for a mode with a given  $n$  and  $\ell$  represents an average of  $\Omega(r)$  weighted by the rotation kernel  $K_{n\ell}(r)$  (Kawaler et al., 1999). If the kernel function of a mode is nonzero where the star has very rapid rotation, then the resulting splitting can reveal the inner rapid rotation despite slow rotation at the surface layers.

Figure 3 shows sample rotation kernels for an sdB model representative of the pulsating EC 14026 stars. The modes illustrated span the period range seen in the pulsating sdB stars. Note that higher  $n$  modes have more peaks, but that in all modes the kernel has nonzero value close to the stellar core. Also indicated in Figure 3 are lines representing  $\log \Omega(r)$  for Case B evolution of rotation in convection zones. Note that even though the amplitude of  $K_{n\ell}$  is small in the inner regions, the large value of  $\log \Omega(r)$  in the same region suggests that the value of the rotational splitting will be dominated by the core rotation.

For several  $\ell = 2$  modes in this model, the rotation kernel is significant in the core. This results from resonant mode trapping by the He/C+O composition transition zone. At this evolutionary stage, the

model shows some mixed-character modes for  $\ell = 2$  which can have significant amplitude in the core (Charpinet et al., 2002).

Figure 3 lists the computed splittings a Case B sdB model. The surface rotation velocity, if representative of the entire star, would produce a splitting of less than  $0.01\mu\text{Hz}$  for all modes (surface rotation velocity of only 6 m/s), yet the rapid internal rotation results in splittings ranging from  $1\mu\text{Hz}$  ( $\ell = 1, n = 1$ ) to  $80\mu\text{Hz}$  ( $\ell = 2, n = 0$ ).

In all cases, the rotational splitting is much larger than expected from the surface rotation rate. Also, the splitting changes (sometimes dramatically) from one mode to the next in a sequence of the same  $\ell$  but different  $n$ . The kernels (see Figure 3) sweeping across the steep drop in  $\Omega$  produce these fluctuations. Mode trapping by the composition transition zones produces some strong core localization of the kernels for certain modes, and these also show larger rotational splitting.

#### 4. Conclusions

The sdB stars may retain rapidly rotating cores as a relic of their evolution on the RGB. Such rapid rotation could serve as a reservoir of angular momentum which, when tapped, can produce anomalously fast rotation at the surface on the HB.

Nonradial pulsations can reveal the rapidly rotating cores via rotational splitting of nonradial modes. The rotational splitting will be much larger than expected from the surface rotation velocity. This may be happening in PG 1605+072; the rotational splitting identified in that star by Kawaler (1998) is about 3 times larger than the  $v \sin i$  measured via spectroscopy (Heber et al., 1999). Also, the splittings seen in Feige 48 (Reed et al., 2003) are significantly larger than expected from the upper limits to  $v \sin i$  by Heber et al. (1999).

The fact that the splittings can be quite large, coupled with the large differences in expected splittings from mode to mode, can make observational identification of rotational splitting quite difficult. Generally, one looks for a sequence of equally-spaced triplets in a Fourier transform, and identifies them as  $\ell = 1$  for example. Even if all members of such triplets are not seen, finding several pairs of modes split by the same amount would suggest rotational splitting. For sdBV stars, with significant mode-to-mode differences in splittings expected, a series of rotationally split frequencies would be indistinguishable from modes of different  $\ell$  and  $n$ . Without considering differential rotation, such a rich mode spectrum would require appealing to other mechanisms.

Finally, we note that the periods seen in the PG1716+426 (“Betsy”) stars are about the same periods that we expect for the rotation rates

of the cores of sdB stars that evolve as single stars from the RGB. Could this mean that the mechanism responsible for the pulsation of these stars might be connected with rotation ( $r$ -modes or overstable convective modes)?

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